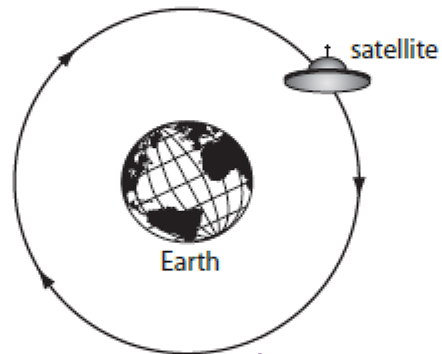
Lesson 4.4: Gravitational Potential Energy

Unit 4: Circular Motion and Gravitation

Gravitational Potential Energy:

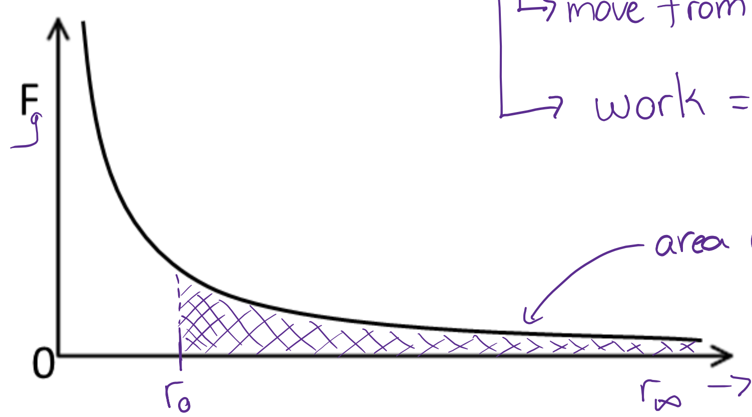
Newton's law of universal gravitation tells us the **force** between a spaceship and planet Earth at a distance of R between the centres of mass. If that spaceship wants to explore other parts of space, it needs to escape Earth's gravitational field.



To remove a satellite from orbit, **work** needs to be done on it.

→ move from $r_0 \rightarrow r_\infty$ where $F_g \rightarrow 0$

→ work = force × distance



area under F vs r curve is work

$$W = F_g \cdot r$$
$$= \frac{GMm}{r^{\cancel{2}}} \cdot \cancel{r}$$
$$W = G\frac{Mm}{r}$$

* We can say the G. potential energy is zero at $r = \infty$.

* It's like making the ground = 0 when calculating $E_p = mgh$

$$W = \Delta E_p$$
$$W = E_{p\infty} - E_p$$
$$W = 0 - E_p$$

$$\therefore E_p = -G\frac{Mm}{r}$$

Example 1: A monkey of mass 2.0 kg is sent into orbit at 2 Earth radii. Find the gravitational potential energy of the monkey at this radius. ($m_e = 5.98 \times 10^{24}$ kg and $r_e = 6.83 \times 10^{7}$ m)

$$E_p = -G\frac{Mm}{r}$$
$$= -G\frac{m_e m_m}{2r_e}$$
$$= -\frac{(6.67\times10^{-11})(5.98\times10^{24})(2)}{2(6.83\times10^{7}\,m)} = \boxed{-6.25\times10^{7}\,J}$$

$m \downarrow$ Earth $= M$

Example 2: Find the work required to move a 320 kg satellite from 2 Earth radii to 6 Earth radii.

$2r_e \qquad 6r_e$

$$W = \Delta E_p$$
$$= E_p - E_{po}$$
$$= \left(-G\frac{Mm}{6r_e}\right) + \left(+G\frac{Mm}{2r_e}\right)$$
$$= G\frac{Mm}{r_e}\left(\cancel{\tfrac{1}{2} - \tfrac{1}{6}}\right)^{\frac{1}{3}}$$
$$= G\frac{Mm}{3r_e}$$

$$= \frac{(6.67\times10^{-11})(5.98\times10^{24})(320)}{3(6.38\times10^{6})}$$

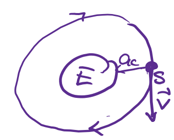
$$\boxed{= 6.67\times10^{9}\ J}$$

Example 3: What is the total orbital energy of an earth satellite of mass 200 kg orbiting at 3 earth radii?

* need both its gravitational potential + kinetic energy.

$$E_T = E_p + E_k$$
$$= -G\frac{Mm_s}{r} + \frac{1}{2}m_s v^2$$

Here we need to find an eq. for $\vec{v}^{\,2}$



$$F_c = F_g$$
$$\frac{\cancel{m_s}v^2}{\cancel{r}} = G\frac{M\cancel{m_s}}{r^{\cancel{2}}}$$
$$v^2 = \frac{GM}{r}$$
$$\therefore \frac{1}{2}m_s v^2 = \boxed{G\frac{Mm_s}{2r}}$$

$$E_T = -G\frac{Mm_s}{r} + G\frac{Mm_s}{2r}$$
$$= G\frac{Mm_s}{r}\left(\cancel{\tfrac{1}{2} - 1}\right)^{-\frac{1}{2}}$$
$$E_T = -G\frac{Mm_s}{2r}$$

*recall $r = 3r_e$

$$= \frac{-(6.67\times10^{-11})(5.98\times10^{24})(200)}{2[3(6.38\times10^{6})]}$$

$$\boxed{E_T = -2.08\times10^{9}\ J}$$

*Note: You could 1st calc. v^2 then sub it in instead.

Escape Velocity:

In order for an object to escape the gravitational field of a large body (i.e., a spaceship escaping Earth's gravitational field), it must have enough kinetic energy to over come the gravitational potential energy. In other words, the kinetic energy must, at least, balance the potential energy.

Derivation

$$E_p + E_k = 0$$

$$-G\frac{Mm}{r} + \frac{1}{2}mv^2 = 0$$

$$\frac{1}{2}mv^2 = G\frac{Mm}{r}$$

$$v^2 = \frac{2GM}{r}$$

$$v = \sqrt{\frac{2GM}{r}} = \vec{V}_{escape}$$

https://youtube.com/shorts/raqUc1QoWYM?si=nVZ4-E20xSnhYVle

Example 4: The escape velocity for Earth is around 11.2 km/s, which is the velocity a spacecraft would need to escape Earth's gravitational field. How does the requirements to escape Earth compare to Mars if Mars has a mass of 6.42 x 10^{23} kg and a radius 3.39 x 10^{6} m?

$$\vec{V}_e = \sqrt{\frac{2GM}{R}}$$

$$\vec{V}_e = \sqrt{\frac{2(6.67\times10^{-11})(6.42\times10^{23})}{3.39\times10^{6}}}$$

$$\boxed{\vec{V}_e = 5.0\times10^{3}\ m/s \quad \text{or} \quad 5.0\ km/s}$$

Assignment: Sander-Gore Pg. 144 #1-2 | Pg. 146 #10-12 | WS 4.4