Lesson 4.3: Universal Gravitation

Unit 4: Circular Motion and Gravitation

Newton's Law of Universal Gravitation:

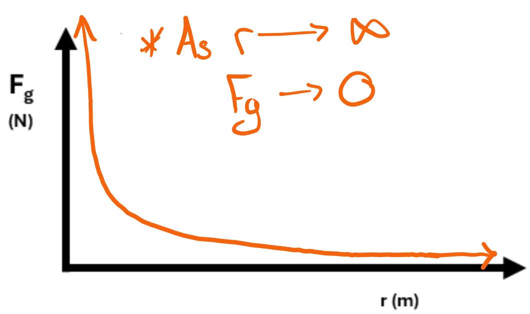
Every body in the universe **attracts** every other body with a force that is:

a) **directly** proportional to the product of the **masses** of the two bodies and
b) **inversely** proportional to the squre of the **distance** between the centres of mass of two bodies

$$F_g = \frac{Gm_1m_2}{r^2} = G\frac{Mm}{r^2}$$

F_g = gravitational force b/w two objects (N)
m_1, m_2 = masses of the objects (kg)
r = distance b/w centres of mass of 2 objects (m)

$$\boxed{G = 6.67\times10^{-11}\ N\cdot m^2/kg^2 = \text{Universal gravitation constant}}$$



How is F_g affected by changes in mass and distance?

Mass 1	Mass 2	Radius	Force of g
$2M_1$	M_2	r	$\mathbf{2F_g}$
$2M_1$	$2M_2$	r	$\mathbf{4F_g}$
M_1	M_2	2r	$\mathbf{\frac{1}{4}F_g}$
M_1	M_2	⅓ r	$\mathbf{9F_g}$

Henry Cavendish's Experiment to Measure G:

In 1797, Henry Cavendish performed a very sensitive [experiment] which both confirmed Newton's Gravitation Law and gave a value for the Gravitational constant: **6.67×10^{-11} Nm²/kg²**

A 2.0 m long rod was suspended from the ceiling by a wire. Two small lead spheres were fixed to the end of the rod. Two large masses were placed near the lead spheres. Cavendish was able to measure the period of the rotation of the rod due to the gravitation force between the lead spheres and the large masses.

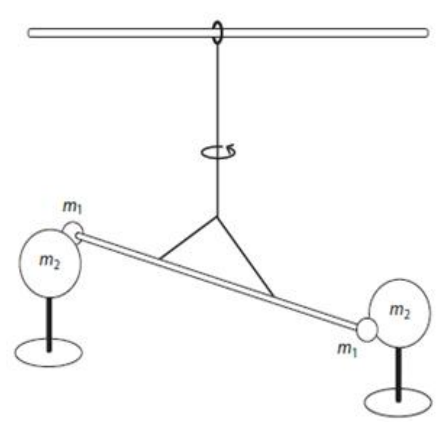


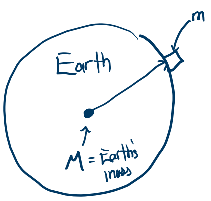
Figure 4.3.1 *Cavendish's apparatus for confirming Newton's law of universal gravitation*

The Gravitational Field Strength of the Earth:

g is the gravitational field <u>intensity</u> (N/kg)

- the amount of **<u>force</u>** per unit **<u>mass</u>** acting on an object toward the source of the field
- every object has its **<u>own</u>** g

Show that the gravitational attraction between any mass, m, and earth, M collapses down to Fg = m(9.8 N/kg) provided m is on the surface of the earth.



$G = 6.67\times10^{-11}\ N\cdot m^2/kg^2$

$r_E = 6.371\times10^{6}\ m$

$M = 5.972\times10^{24}\ kg$

m = test/object mass.

$$F_g = G\frac{Mm}{r^2}$$

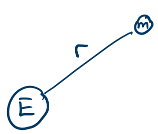
$$\cancel{m}g = G\frac{M\cancel{m}}{r^2}$$

$$\boxed{g = G\frac{M}{r^2}}$$

$$g = \frac{(6.67\times10^{-11}\ Nm^2/kg^2)(5.972\times10^{24}\ kg)}{(6.371\times10^{6}\ m)^2}$$

$$g = 9.81\ m/s^2$$

Example 1: Find the gravitational field intensity (g) of the Earth at the Moon if the distance between centres of mass is 3.84 x 10^8 m.



$$g_E = G\frac{M_E}{r^2}$$

$$g_E = \frac{(6.67\times10^{-11})(5.98\times10^{24})}{(3.84\times10^{8})^2}$$

$$g = \boxed{2.70\times10^{-3}\ N/kg}$$

Satellites in Circular Orbits and Orbital Velocity:

For a satellite orbiting an object, the centripetal force is the gravitational force.

On satellite

$$F_c = F_g$$

$$ma_c = F_g$$

$$\cancel{m}\frac{v^2}{\cancel{r}} = G\frac{M\cancel{m}}{r^{\cancel{2}}}$$

$$\boxed{v = \sqrt{\frac{GM}{r}}}$$

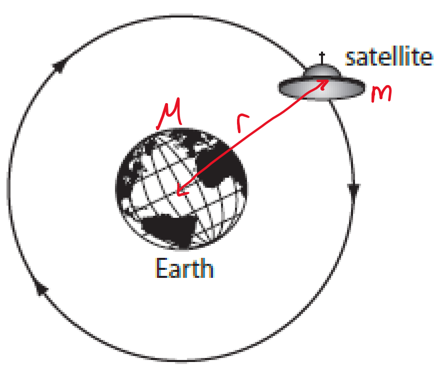


Figure 4.3.4 *A space vehicle must achieve and maintain a minimum speed to remain in orbit.*

<u>OR</u>

$$F_c = F_g$$

$$\cancel{m}\frac{4\pi^2 r}{T^2} = G\frac{M\cancel{m}}{r^2}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Example 2: A satellite orbits planet X at 1.5 x 10^6 m with a period of 2.0 x 10^5 s. Find the mass of the planet.

$$F_c = F_g$$

$$\cancel{m_s} \cdot \frac{4\pi^2 r}{T^2} = G\frac{M_x \cdot \cancel{m_s}}{r^2}$$

$$M_x = \frac{4\pi^2 r^3}{T^2 G}$$

$$M_x = \frac{4\pi^2 (1.5\times10^6)^3}{(2.0\times10^5)^2 (6.67\times10^{-11})}$$

$$\boxed{= 4.99\times10^{19}\ \text{Kg}}$$

Assignment: Sander-Gore Pg. 138-139 #1-3 | Pg. 142 #1-2 | Pg. 145 #1-7