



4.2 Gravity and Kepler's Solar System

Falling Objects

Objects that are acted upon only by the force of gravity are said to be in **free-fall**.



All three of these situations show **free-falling** objects.

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The Moon in Free Fall

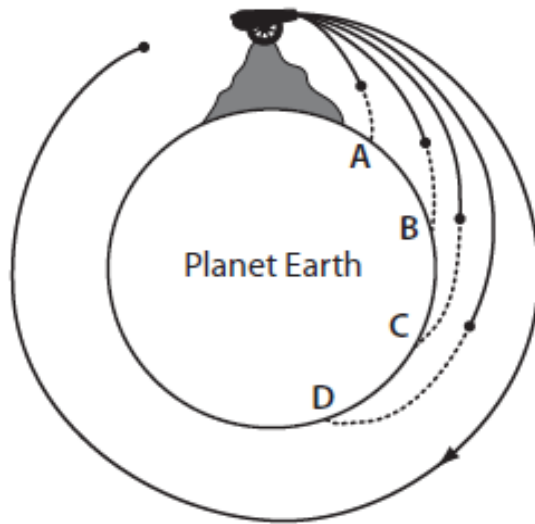


Figure 4.2.2 Newton's diagram explaining why the Moon doesn't fall to Earth.

Newton understood that the **Moon** was in free-fall around the Earth. He created diagrams to explain why the Moon never falls to surface of the Earth.

He described firing a cannon horizontally off the top of a mountain at ever **increasing** speeds. Eventually the cannon ball is travelling so fast that it travels horizontally as much as the Earth's is curving away. The cannon ball has become a **satellite**. Our Moon was our first satellite.





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Gravitational Force, Earth, and the Moon

There must be **centripetal** force to keep the Moon is **orbiting** the Earth. Newton explained that a **gravitation** force was required to provide this centripetal force.

Using known values for the **period** of the Moon and it's **distance** from the Earth Newton was able to determine the centripetal acceleration of the Moon as it orbits the Earth:

$$a_c = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2 (3.84 \times 10^8 \text{ m})}{(27.3 \cancel{\text{d}} \times 24 \cancel{\text{h}} \cancel{\text{d}} \times 3600 \text{ s} \cancel{\text{h}})^2} = 2.7 \times 10^{-3} \text{ m/s}^2$$

The centripetal acceleration is actually the acceleration of **gravity**, g , due to the **Earth** at the **Moon's** location. Newton was absolutely **convinced** that it was the **gravitation** force that provided the necessary **centripetal** force keep the Moon in its orbit about the Earth and that the gravitational force must rapidly decrease as the distance from the Earth increases.



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Johannes Kepler (1571 – 1630)

Johannes Kepler worked in Tycho Brahe's (1546 – 1601) laboratory. Brahe is famous for making precise **observations** of over 800 stars and accurate records of planets over a period of 2 decades.

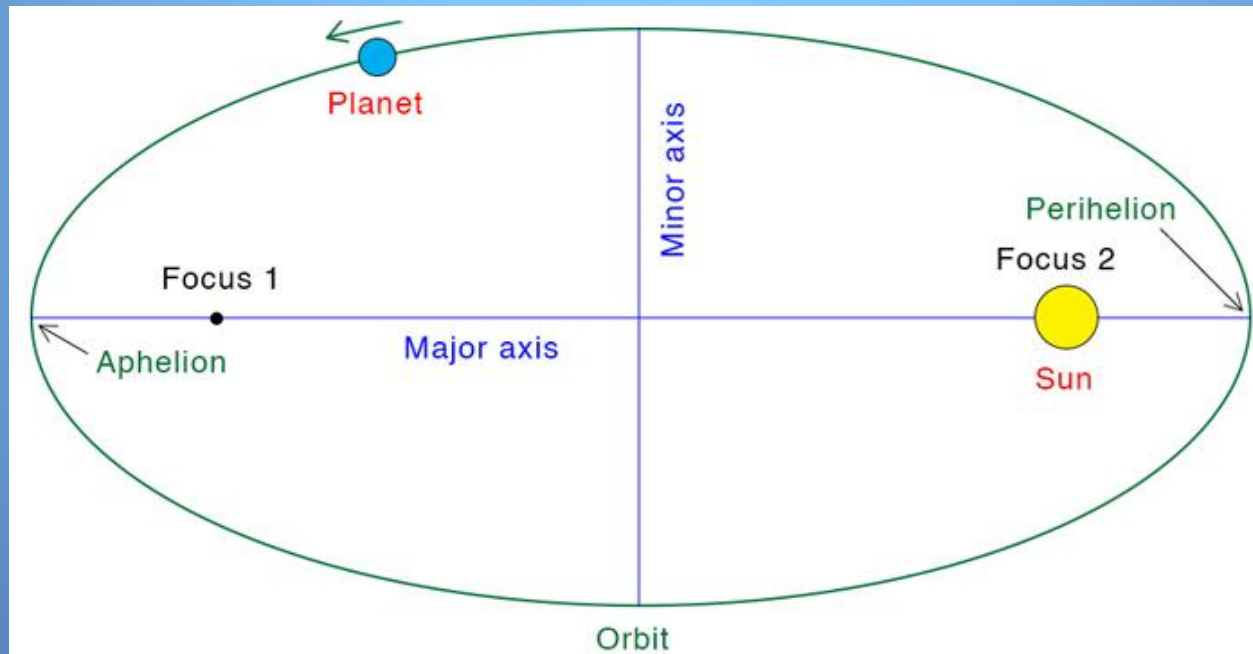
Kepler used these precise observations to search for **mathematical** patterns in the motions of the planets. Kepler formulated **three** laws describing the orbits of the planets around the Sun.

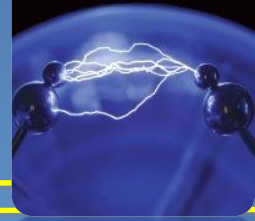


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Kepler's First of Three Laws of Planetary Motion

1. Each planet orbits the Sun in a **elliptical** path, with the **Sun** at one of the two **foci** of the ellipse.





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Kepler's Second Law of Planetary Motion

2. A line joining the center of the Sun and the center of any planet will trace out equal **areas** in equal interval of **time**. If time intervals $T_2 - T_1$ and $T_4 - T_3$ are **equal** the **areas** traced out by the orbital radius of a planet during this interval will be equal. (This really is a version of the Law of Conservation of Energy!)

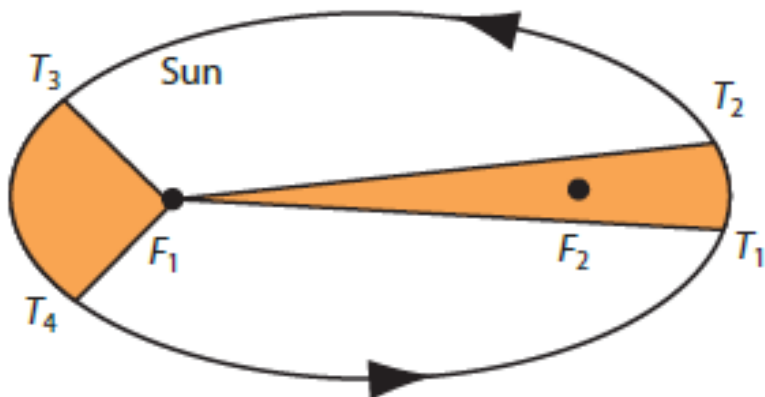


Figure 4.2.3 Kepler's second law

In order for this to be true a Planet during $T_4 - T_3$ has to be travelling **faster** than during interval $T_2 - T_1$.



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Kepler's Third Law of Planetary Motion

3. For any planet in the solar system, the **cube** of its mean orbital radius divided by the **square** of its **period** of revolution is a constant.

$$K = \frac{R^3}{T^2}$$

Note:

This is **not** Universal constant. Change the solar system or orbiting system in question and you would have a **new** value for Kepler's constant.



Example 1

Planet Xerox (a close copy of planet Earth) was discovered by Superman on one of his excursions to the far extremes of the solar system. Its distance from the Sun is $1.50 \times 10^{13} \text{ m}$. How long will this planet take to orbit the sun?

Earth's orbital radius, $R_E = 1.5 \times 10^{11} \text{ m}$

Earth's period of revolution, $T_E = 1.0a$

Xerox's orbital radius, $R_X = 1.50 \times 10^{13} \text{ m}$

$$\frac{R_X^3}{T_X^2} = \frac{R_E^3}{T_E^2} = K_{\text{sun}}$$

*** solve for T_X

$$T_X^2 = \frac{R_X^3}{R_E^3} \cdot T_E^2$$

$$T_X^2 = \frac{[1.5 \times 10^{13} \text{ m}]^3}{[1.5 \times 10^{11} \text{ m}]^3} \cdot [1.0a]^2$$

$$T_X^2 = 1.0 \times 10^6 a^2$$

$$T_X = 1.0 \times 10^3 a$$



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In this section, you should understand how to solve the following key questions.

Page #131	Practice Problems 4.2.1 Kepler's Laws#1– 2
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