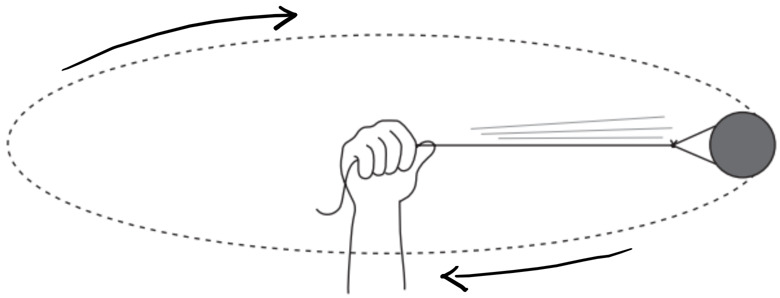
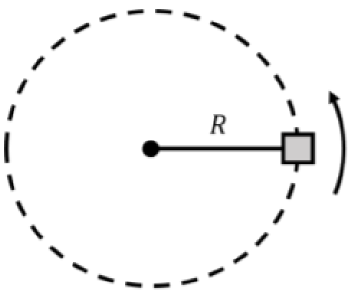
Lesson 4.1: Centripetal Force and Acceleration

Unit 4: Circular Motion and Gravitation

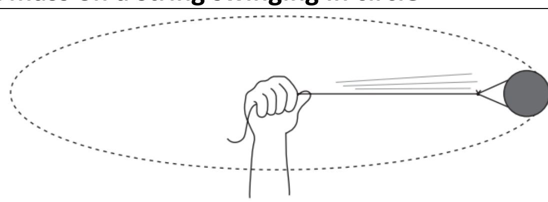
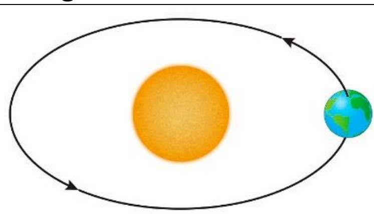
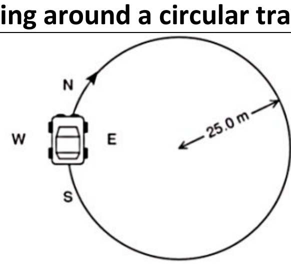
The image shows a ball attached to a string being swung in a circular path. Imagine the string breaks at the exact point illustrated. ***Draw a velocity vector on the diagram to illustrate the direction the ball will move.***



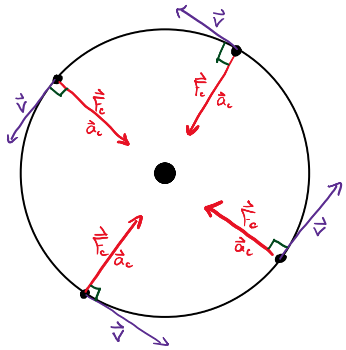


Top view

Imagine you are driving a car around a circular track, maintaning the same speed all the way around the track. Any object that moves in a circle at a steady speed is said to be in uniform circular motion. ***Is the car accelerating?***

Diagram	What Produces the Centripetal Force?
A mass on a string swinging in circle	
	Tension in the string supplies a net force towards the centre = centripetal force
Earth traveling around the sun	
	Gravity supplies a net force towards the centre = centripetal force
A car travelling around a circular track	
	Friction of the tires supplies a net force towards the centre = centripetal force

Circular Motion:



- Velocity always points tangential to the circular path
- $\mathbf{F_c}$ (centripetal force) and $\mathbf{a_c}$ (centripetal acceleration) are towards the centre
- $\mathbf{F_c}$ and $\mathbf{a_c}$ are perpendicular to velocity

Centripetal Acceleration:

When a body moves in a circle with uniform speed, it accelerates towards the centre of the circle with the following magnitude:

$$\boxed{a_c = \frac{v^2}{r}}$$

a_c = centripetal acceleration (m/s^2)
v = linear velocity (m/s)
r = radius (m)

Another form to represent a_c:

* recall $v = \frac{d}{t}$

For a circle $v = \frac{\text{circumference}}{t} = \frac{2\pi r}{T}$ where T = period (time to complete a revolution).

$$\therefore a_c = \frac{v^2}{r}$$

$$= \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$a_c = \frac{4\pi^2 r^{\cancel{2}}}{T^2 \cancel{r}}$$

$$\boxed{a_c = \frac{4\pi^2 r}{T^2}}$$

Centripetal Force:

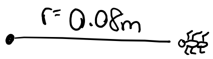
The net force caused by centripetal acceleration.

We can use newton's 2nd law to calculate F_c as follows:

$$\vec{F}_c = m\vec{a}_c$$

$$\boxed{F_c = \frac{mv^2}{r} = \frac{4\pi^2 mr}{T^2}}$$

Example 1: An ant of mass 0.0026 g stands 8 cm from the centre of a vinyl record rotating at $33\frac{1}{3}\ RPM$.
Find the centripetal force acting on the ant.



$r = 8cm = 0.08m$

$T = 1.8s/rev.$

$m = 2.6\times10^{-6} kg$

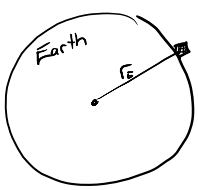
$33\frac{1}{3}$ RPM = frequency (revs. per minute)

$T = \frac{1}{f}$ $\therefore T = \frac{1}{33\frac{1}{3}} = \frac{0.03 min.}{1\ rev.} \times \frac{60s}{1 min} = 1.8s/rev.$

$$F_c = ma_c = m \cdot \frac{4\pi^2 r}{T^2} = (2.6\times10^{-6}) \cdot \frac{4\pi^2(0.08)}{(1.8)^2}$$

$$\boxed{F_c = 2.5\times10^{-6} N}$$

Example 2: Find the centripetal acceleration on an object on the earth due to the Earth's rotation if the radius of the earth is 6.38 x 10^6 m. Then find the associated linear velocity.



$r_E = 6.38\times10^6 m$

$T = 24h = 86400s$

$$a_c = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2(6.38\times10^6)}{(86400)^2}$$

$$\boxed{\vec{a}_c = 0.0337 m/s^2}$$

Linear Velocity:

$$a_c = \frac{v^2}{r}$$

$$v = \sqrt{a_c r}$$

$$v = \sqrt{(0.0337)(6.38\times10^6)}$$

$$\boxed{v = 464 m/s}$$

Assignment: Sander-Gore Pg. 120 #1-3 | Pg 121 #1-3 | Pgs. 125-127 #1-12