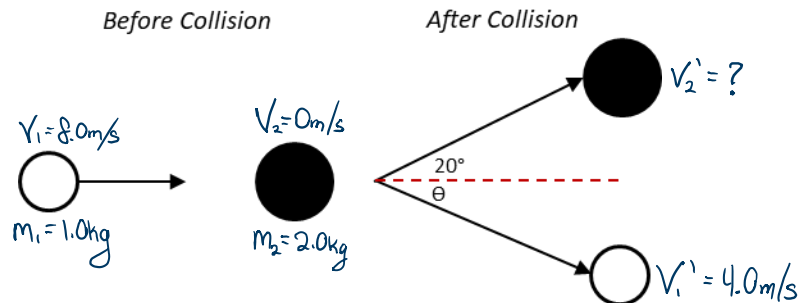


Lesson 3.5: Momentum in 2D

Unit 3: Momentum and Energy

Conservation of momentum still holds true in 2-dimensions

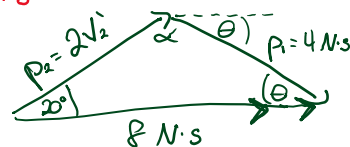
Example 1: 2D elastic collision solving by component and trig methods: A 1.0 kg ball travelling 8.0 m/s collides with a 2.0 kg ball at rest. After the collision, the 1.0 kg ball is travelling at 4.0 m/s at angle Θ with respect to the original path. The 2.0 kg ball is travelling at v_2' at 20° with respect to the original path. Find v_2' and Θ using the trig method.



$$\begin{aligned}
 p_{\text{before}} &= p_{\text{after}} \\
 p_1 + \cancel{p_2} &= p_1' + p_2' \\
 m_1 v_1 &= m_1 v_1' + m_2 v_2' \\
 (1)(8) &= (1)(4) + 2v_2' \\
 8 &= 4 + 2v_2'
 \end{aligned}$$

* Draw vector diagrams:

$$p_1 = 8 \text{ N}\cdot\text{s} \rightarrow$$



$$\begin{aligned}
 \Theta &= 180 - 20 - 136.8^\circ \\
 \Theta &= 23.2^\circ
 \end{aligned}$$

$$\frac{2v_2'}{\sin 23.2} = \frac{4}{\sin 20}$$

$$v_2' = \frac{2 \sin 23.2}{\sin 20}$$

$$v_2' = 2.30 \text{ m/s}$$

Find α (sine law)

$$\frac{8}{\sin \alpha} = \frac{4}{\sin 20}$$

$$\sin \alpha = 2 \sin 20$$

$$\alpha = \arcsin(2 \sin 20)$$

$$\alpha = 43.2^\circ \text{ or}$$

$$180 - 43.2^\circ$$

$$= 136.8^\circ \checkmark$$

* 8 Ns is the largest side so pick largest $\angle$

Find v_2' w cosine law:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$p_1'^2 = p_2'^2 + p_1'^2 - 2p_2'p_1' \cos 20$$

$$(4)^2 = (2v_2')^2 + 8^2 - 2(2v_2')(8) \cos 20$$

$$16 = 4v_2'^2 + 64 - 32v_2' \cos 20$$

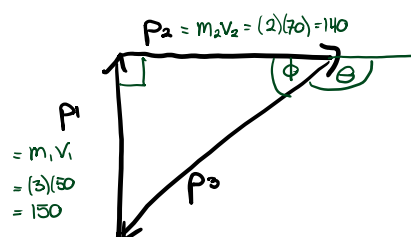
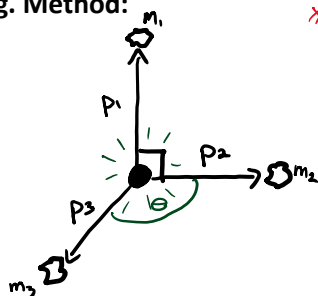
$$0 = 4v_2'^2 - 30.1v_2' + 48$$

$$v_2' = \cancel{5.22 \text{ m/s}} \text{ or } 2.30 \text{ m/s} \checkmark$$

$$p_2 = 2(5.22) = 10.44 \text{ N}\cdot\text{s} \times \text{(only 8 available)}$$

Example 2 – An explosion in 2D: A landmine explodes into three pieces in the horizontal plane. The 1st piece of mass 3.0 kg travels at 50 m/s. The 2nd piece of mass 2.0 kg travels at 70 m/s perpendicular to the 1st piece. Find the velocity of the 3rd piece if its mass is 2.5 kg. Solve using both components and trig.

Trig. Method:



* In explosions $E_{\text{explosion}} = E_{\text{kt}}$ so energy is conserved.

$$p_{\text{before}} = p_{\text{after}}$$

$$0 = p_1 + p_2 + p_3$$

$$p_3 = \sqrt{150^2 + 140^2}$$

$$= 205.2 \text{ N}\cdot\text{s}$$

$$p_3 = m_3 v_3$$

$$v_3 = \frac{p_3}{m_3} = \frac{205.2}{2.5 \text{ kg}}$$

$$v_3 = 82.1 \text{ m/s}$$

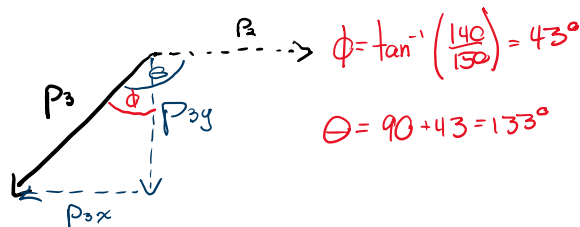
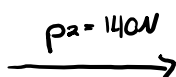
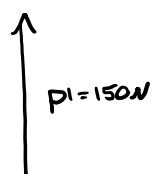
$$\phi = \tan^{-1}\left(\frac{150}{140}\right)$$

$$\phi = 47^\circ$$

$$\theta = 180 - 47 = 133^\circ$$

$$v_3 = 82.1 \text{ m/s}, 133^\circ \text{ from 2nd piece}$$

Component Method:



x-direction

$$p_i = p_f$$

$$0 = p_2 + p_{3x}$$

$$p_{3x} = p_2$$

$$p_{3x} = 140 \text{ N}\cdot\text{s}$$

y-direction

$$0 = p_1 + p_{3y}$$

$$p_{3y} = p_1$$

$$p_{3y} = 150 \text{ N}\cdot\text{s}$$

$$p_3 = \sqrt{p_{3x}^2 + p_{3y}^2}$$

$$= \sqrt{140^2 + 150^2}$$

$$= 205.2 \text{ N}\cdot\text{s}$$

$$v_3 = \frac{205.2}{2.5} = 82.1$$

$$v_3 = 82.1 \text{ m/s}, 133^\circ \text{ from piece 2.}$$