

Lesson 3.3: Momentum and Impulse

Unit 3: Momentum and Energy

Momentum:

- * the product of mass $\times$ velocity
- * Vector quantity (in direction of $\vec{v}$)

$$\vec{p} = m\vec{v}$$

p = momentum * ($\text{kg} \cdot \text{m/s} = \text{N} \cdot \text{s}$)
 m = mass (kg)
 v = velocity (m/s)

Example 1 – Momentum of a single object: Find the momentum of an 1800 kg car travelling at 60 km/h.

$$\frac{60 \text{ km}}{\text{h}} \times \frac{1 \text{ h}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{1 \text{ km}} = 16.7 \text{ m/s}$$

$$\begin{aligned}\vec{p} &= m\vec{v} \\ \vec{p} &= (1800 \text{ kg})(16.7 \text{ m/s}) \\ \vec{p} &= 3.0 \times 10^4 \text{ N} \cdot \text{s}\end{aligned}$$

Concept: Mass and Velocity are directly proportional to momentum

What happens to momentum when... ($p = mv$)

Mass is tripled	3p
Velocity is quadrupled	4p
Mass is doubled and velocity is halved	Moment stays the same

A force acting over a period of time

Impulse

Δ in momentum

Amount of momentum transferred to an object

Avg Force (N)

$$J = \vec{F}_{\text{net}} \cdot \Delta t$$

change in time (s)

Impulse

$$\text{Units: } \text{N} \cdot \text{s} = \text{kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{s} = \text{kg} \cdot \frac{\text{m}}{\text{s}}$$

* Remember the units for momentum

$$\vec{p} = m \cdot \vec{v}$$

Deriving Impulse from Newton's 2nd Law

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$\vec{F}_{\text{net}} = m \left(\frac{\Delta \vec{v}}{\Delta t} \right)$$

$$\vec{F}_{\text{net}} = m \left(\frac{\vec{v} - \vec{v}_0}{\Delta t} \right)$$

$$\vec{F}_{\text{net}} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t}$$

$$\vec{F}_{\text{net}} \Delta t = m\vec{v} - m\vec{v}_0$$

$$J = \vec{p} - \vec{p}_0 = \Delta \vec{p}$$

Concept: Imagine dropping onto a trampoline vs. a hard floor, or catching a baseball with your bare hand vs a padded glove.

The impulse – total momentum change – is the same in each case.

How can this be? What happens to force and time?

Example 2: A 0.5 kg baseball is falling at 20 m/s. A player catches it:

1. **Case A:** catches it with bare hands, stopping it in 0.05 s.
2. **Case B:** catches it in a padded glove, stopping it in 0.20 s.

Question: How does the force on the ball differ? What is the impulse?

Impulse & Force in Catching a Ball

Impulse: $J = \Delta p = 10 \text{ kg} \cdot \text{m/s}$ ($\Delta p = m\Delta v$)

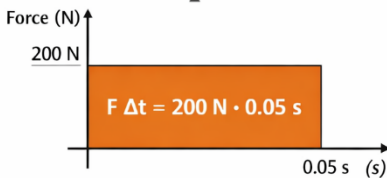
0.5 kg baseball drops from 20 m/s to 0

A: Bare Hands



$\Delta t = 0.05 \text{ s}$

Force: $F = 200 \text{ N}$

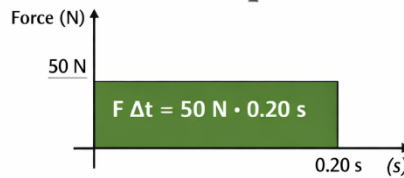


B: Padded Glove



$\Delta t = 0.20 \text{ s}$

Force: $F = 50 \text{ N}$



Same Impulse: Different Force & Time

$$J = F_{\text{net}} \Delta t$$

$$F_{\text{net}} = \frac{J}{\Delta t} = \frac{10 \text{ N} \cdot \text{s}}{0.05 \text{ s}} = 200 \text{ N}$$

$$F_{\text{net}} = \frac{J}{\Delta t} = \frac{10 \text{ N} \cdot \text{s}}{0.2 \text{ s}} = 50 \text{ N}$$

Example 4 – Impulse with angles: A 2000 kg car changes velocity from 16.7 m/s North to 8.3 m/s East. Find the impulse.

$$m = 2000 \text{ kg}$$

$$V_0 = 16.7 \text{ m/s [N]}$$

$$V_f = 8.3 \text{ m/s [E]}$$

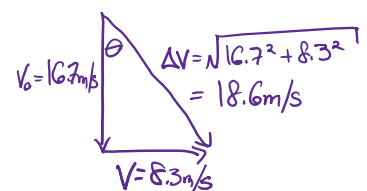
$$\vec{J} = \Delta \vec{p} = m \Delta \vec{v}$$

$$\Delta \vec{p} = 2000(18.6)$$

$$\Delta \vec{p} = 3.7 \times 10^4 \text{ N} \cdot \text{s}, 26^\circ \text{ E of S.}$$

we have 2-D for velocity so let's use vector addition/subtraction

$$\begin{aligned} \Delta \vec{V} &= V - V_0 \\ &= V + (-V_0) \end{aligned}$$



$$\tan^{-1}\left(\frac{8.3}{16.7}\right) = 26^\circ$$

Example 5 – A conservation of momentum problem where one object becomes two: An 1800 kg car is traveling at 6.0 m/s when a 250 kg moose jumps on top of the hood.

$$m_c = 1800 \text{ kg}$$

$$m_m = 250 \text{ kg}$$

$$v_{c0} = 6.0 \text{ m/s}$$

$$v_{m0} = 0 \text{ m/s}$$

Momentum is Conserved $\Rightarrow$

$$p_{m0} + p_{c0} = p_{m+c}$$

$$m_m v_{m0} + m_c v_{c0} = (m_m + m_c) v$$

$$1800(6) = (250 + 1800) v$$

$$v_f = 5.3 \text{ m/s}$$

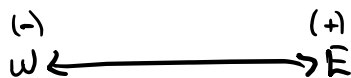
$$F_{A \text{ on } B} = -F_{B \text{ on } A}$$

$$\frac{\Delta(m_A v_A)}{\Delta t} = -\frac{\Delta(m_B v_B)}{\Delta t}$$

$$\Delta p_A = -\Delta p_B$$

$$\Delta p_A + \Delta p_B = 0$$

Example 6 – A conservation of momentum problem where 2 objects collide and stay 2: A 1200 kg car is travelling 30 km/h west and collides with a 300 kg car traveling 40 km/h east. Afterwards the first car is travelling 5.0 km/h east. What is the resulting velocity of the second car?



$$m_1 = 1200 \text{ kg}$$

$$v_{10} = -30 \text{ km/h [W]}$$

$$m_2 = 300 \text{ kg}$$

$$v_{20} = 40 \text{ km/h [E]}$$

$$v_1 = 5.0 \text{ km/h [E]}$$

*Conserved Momentum

$$m_1 v_{10} + m_2 v_{20} = m_1 v_1 + m_2 v_2$$

$$v_2 = \frac{m_1 v_{10} + m_2 v_{20} - m_1 v_1}{m_2}$$

$$v_2 = \frac{1200(-30) + 300(40) - 1200(5)}{300}$$

$$v_2 = -100 \text{ km/h} = 100 \text{ km/h West}$$