

Lesson 2.4 – Translational Equilibrium

Unit 2: Dynamics and Equilibrium

Translational Equilibrium: When the sum of all the forces acting on an object is zero.

- * sum of horizontal forces is zero.
- * sum of vertical forces is zero.

$$\sum \vec{F}_x = 0$$

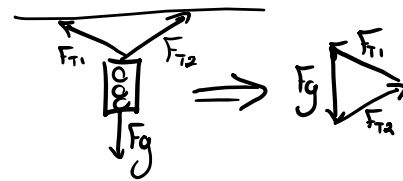
$$\sum \vec{F}_y = 0$$

$\Sigma \leftarrow$ Sigma = sum.

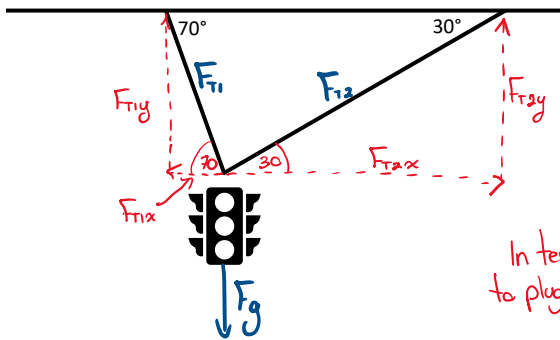
The force vectors will form a closed vector diagram

→ no resultant vector

Ex. Traffic Light



Example 1: Find the tension in the wires ($m = 15 \text{ kg}$)



$$\sum F_x = 0$$

$$F_{T1x} - F_{T2x} = 0$$

$$F_{T1x} = F_{T2x}$$

$$F_{T1} \cos 70^\circ = F_{T2} \cos 30^\circ$$

In terms of F_{T1} to plug into another eq.

$$F_{T1} = \frac{F_{T2} \cos 30^\circ}{\cos 70^\circ}$$

*option to simplify

$$F_{T1} = 2.53 F_{T2}$$

$$\vec{F}_{T1} = \frac{51.1 \cos 30^\circ}{\cos 70^\circ}$$

$$\boxed{\vec{F}_{T1} = 129 \text{ N}}$$

$$\sum F_y = 0$$

$$\text{Expand} \rightarrow F_{T1y} + F_{T2y} - F_g = 0$$

$$F_{T1} \sin 70^\circ + F_{T2} \sin 30^\circ - mg = 0$$

$$\frac{F_{T2} \cos 30^\circ \sin 70^\circ}{\cos 70^\circ} + F_{T2} \sin 30^\circ = mg$$

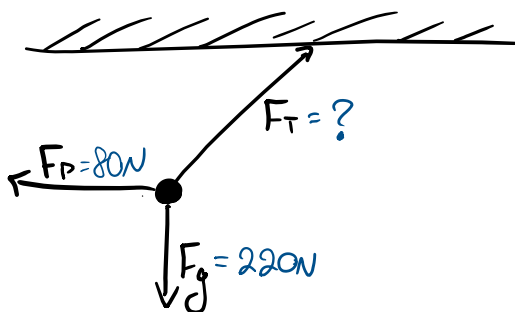
$$F_{T2} \left(\frac{\cos 30^\circ \sin 70^\circ}{\cos 70^\circ} + \sin 30^\circ \right) = mg$$

$$F_{T2} = \frac{mg}{\cos 30^\circ \tan 70^\circ + \sin 30^\circ}$$

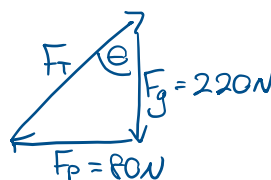
$$F_{T2} = \frac{15(9.8)}{\cos 30^\circ \tan 70^\circ + \sin 30^\circ}$$

$$\boxed{F_{T2} = 51.1 \text{ N}}$$

Example 2: A 220. N mass is hanging from a vertical rope. A person pulls the mass horizontally with a force of 80.0 N. Find the tension in the rope and the angle created.



Use a vector diagram (will sum to 0).



$$F_T = \sqrt{220^2 + 80^2}$$

$$= 234 \text{ N}$$

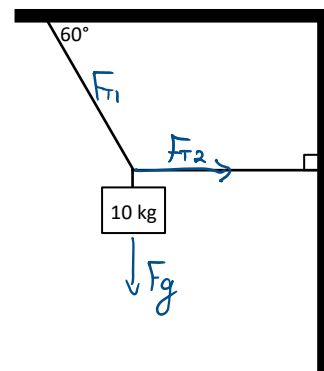
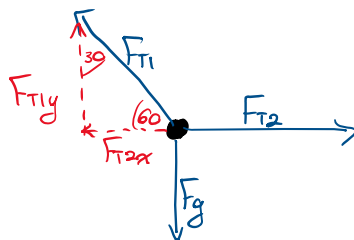
$$\theta = \arctan\left(\frac{80}{220}\right)$$

= 20.0° from vertical
or 80.0° from horizontal.

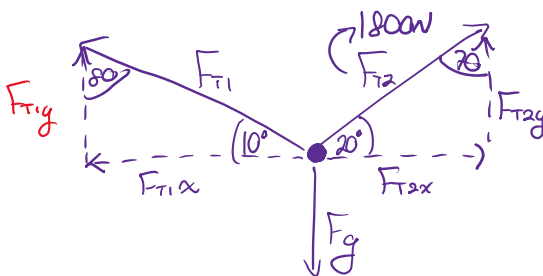
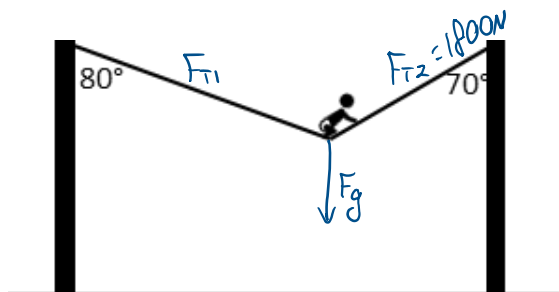
Example 3: Find the tension in each wire:

$$\begin{aligned}\sum \vec{F}_x &= 0 \\ F_{T1x} - F_{T2} &= 0 \\ F_{T1x} &= F_{T2} \\ F_{T1} \cos 60^\circ &= F_{T2} \\ F_{T2} &= 113 \cos 60^\circ \\ \boxed{F_{T2} = 56.6 \text{ N}}\end{aligned}$$

$$\begin{aligned}\sum \vec{F}_y &= 0 \\ F_{T1y} - F_g &= 0 \\ F_{T1} \sin 60^\circ &= mg \\ F_{T1} &= \frac{mg}{\sin 60^\circ} \\ F_{T1} &= \frac{10(9.8)}{\sin 60^\circ} \\ \boxed{F_{T1} = 113 \text{ N}}\end{aligned}$$



Example 4: A circus performer (who is a baby) crawls the tightrope as shown. If the tension in the short length is $1.8 \times 10^3 \text{ N}$, find the mass of the performer.



$$\begin{aligned}\sum F_{Tx} &= 0 \\ F_{T1x} - F_{T2x} &= 0 \\ F_{T1x} &= F_{T2x} \\ F_{T1} \cos 10^\circ &= 1800 \cos 20^\circ \\ F_{T1} &= \frac{1800 \cos 20^\circ}{\cos 10^\circ} \\ F_{T1} &= 1717.54 \text{ N}\end{aligned}$$

$$\begin{aligned}\sum F_g &= 0 \\ F_{T1y} + F_{T2y} - F_g &= 0 \\ F_{T1} \sin 10^\circ + F_{T2} \sin 20^\circ - mg &= 0 \\ m &= \frac{F_{T1} \sin 10^\circ + F_{T2} \sin 20^\circ}{g} \\ m &= \frac{1717.54 \sin 10^\circ + 1800 \sin 20^\circ}{9.8} \\ \boxed{m = 93 \text{ kg.}}\end{aligned}$$