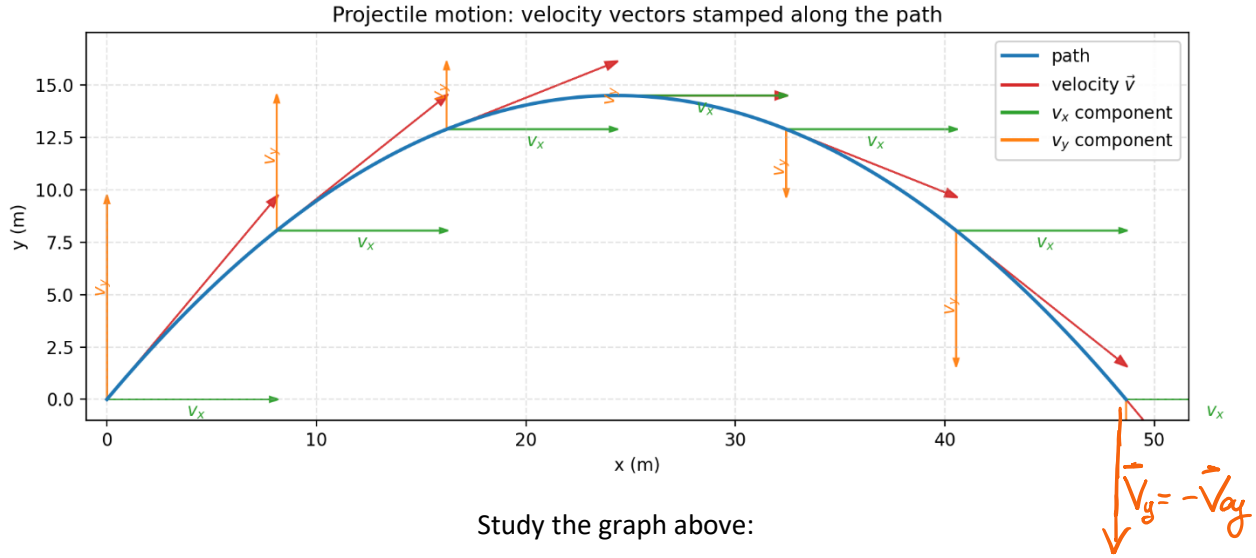
Lesson 1.7: Projectile Motion II

Unit 1: Kinematics, Vectors, and Mathematics



$\vec{V}_y = -\vec{V}_{ay}$

Study the graph above:

What can be said about the velocity components in a 2D projectile?

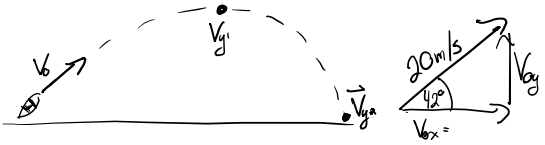
$\vec{V}_x$ is constant

$\vec{V}_{fy} = -\vec{V}_{oy}$ decreases to max + proportionally increases ↓ to min.

What is the relationship between the time it takes for the projectile to reach its max height and drop from its max height?

They are equal

Example 1: A football is kicked with an initial velocity of 20. m/s with a 42° elevation above horizonal.



x

$\vec{V}_{ox} = 20\cos 42$

$= 14.9\,m/s$

y

$\vec{V}_{oy} = 20\sin 42$

$= 13.4\,m/s$

$\vec{a}_y = -9.8\,m/s$

a) How long is it in the air?

Option 1: ½ time (max height)

$\vec{V}_{y1} = 0\,m/s$

$\vec{V}_y = \vec{V}_{oy} + \vec{a}_y t$

$0 = 13.4 + (-9.8)t$

$t = 1.37\,s$ (half way)

$\therefore t_{total} = 1.37(2)$

$= 2.73\,s$

Option 2: Full time

$\vec{V}_{y2} = -13.4\,m/s$

$V_{y2} = V_{oy} + \vec{a}_y t$

$-13.4 = 13.4 + (-9.8)t$

$t = 2.73\,s$

Option 3: Full time

$\vec{d}_y = 0\,m$

$d_y = V_{oy}t + \frac{1}{2}\vec{a}_y t^2$

$0 = 13.4t + \frac{1}{2}(-9.8)t^2$

$4.9t^2 = 13.4t$

$t = 2.73\,s$

$d_y = \frac{V_{y'}^2 - V_{oy}^2}{2\vec{a}_y}$

b) How high does it go? (two choices)

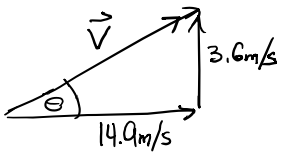
Option #1	Option #2
$\vec{d}_y = ?$ $t = 1.37s$ $V_{oy} = 13.4 m/s$ $d_y = \vec{V}_{oy}t + \frac{1}{2}\vec{a}_y t^2$ $d_y = 13.4(1.37) + \frac{1}{2}(-9.8)(1.37)^2$ $= \boxed{9.16m} \approx 9.2m$	$V_{y'}^2 = \vec{V}_{oy}^2 + 2\vec{a}_y\vec{d}_y$ $0 = (13.4)^2 + 2(-9.8)\vec{d}_y$ $\vec{d}_y = 9.16m \approx 9.2m$

c) How far does it go?

$\vec{d}_x = ?$
$\vec{V}_{ox} = 14.9 m/s$
$t = 2.73s$
$\vec{d}_x = \vec{V}_{ox}t$
$= 14.9(2.73)$
$= 40.677m \approx 41m$

d) Find the velocity after 1.0 s.

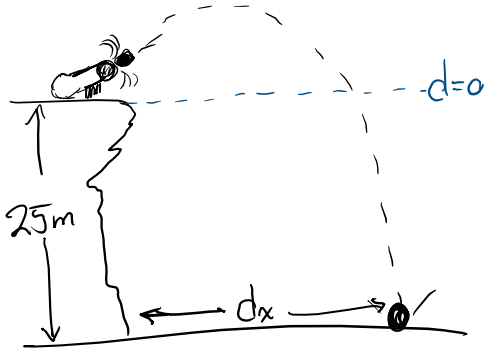
$t = 1.0s$
$V_x = 14.9 m/s$
$V_y = V_{oy} + \vec{a}_y t$
$= 13.4 + (-9.8)(1)$
$= 3.6 m/s$

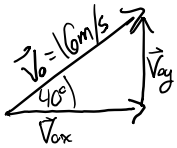


$\vec{V} = \sqrt{(3.6)^2 + (14.9)^2}$
$\theta = \arctan\left(\frac{3.6}{14.9}\right)$

$\vec{V} = 15.3 m/s$ at $13.6°$
$= 15 m/s$ at a $14°$ inclination.

Example 2: A cannonball is shot off a 25 m cliff with a velocity of 16 m/s at 40° above the cliff. How far (horizontally) will it travel?





x
$\vec{V}_{ox} = 16\cos 40$
$= 12.26 m/s$
$d_x = ?$
$t = ?$

y
$\vec{V}_{oy} = 16\sin 40$
$= 10.28 m/s$
$\vec{a}_y = -9.8 m/s^2$
$\vec{d}_y = -25m$
$t = ?$

$d_y = V_{oy}t + \frac{1}{2}\vec{a}_y t^2$
$-25 = 10.28t + \frac{1}{2}(-9.8)t^2$
$4.9t^2 - 10.28t - 25 = 0$
a b c

$t = \frac{10.28 \pm \sqrt{(-10.28)^2 - 4(4.9)(-25)}}{2(4.9)}$

$t = 3.539s$ or $t = \cancel{-1.44s}$

$d_x = V_x t$
$= 12.26 m/s\,(3.539s)$
$d_x = 43.4m$
or $\boxed{43m}$