

### Strategies

What other strategy could you use to solve the problem?

$$\begin{aligned} \text{b)} \quad & 40 + 0.05s > 55 + 0.03s \\ & 40 + 0.05s - 40 > 55 + 0.03s - 40 \\ & 0.05s > 15 + 0.03s \\ & 0.05s - 0.03s > 15 + 0.03s - 0.03s \\ & 0.02s > 15 \\ & \frac{0.02s}{0.02} > \frac{15}{0.02} \\ & s > 750 \end{aligned}$$

Sarah's pay will be higher at store B when her sales are greater than \$750. If she thinks that her sales will be more than \$750 on most days, then store B is the better offer.

### Show You Know

Danny started his own computer repair business. He offers his customers two payment options. Option A has a base fee of \$40 plus \$8 per hour. Option B has no base fee but costs \$15 per hour. How many hours does a repair job have to take in order for option B to be less expensive?

- Model the problem using an inequality.
- After how many hours will option B be less expensive?



### Key Ideas

- To solve a multi-step inequality, isolate the variable.

$$\begin{aligned} & -3(x - 5) \leq 3x + 9 \\ & -3x + 15 \leq 3x + 9 \\ & -3x + 15 - 3x \leq 3x + 9 - 3x \\ & -6x + 15 \leq 9 \\ & -6x + 15 - 15 \leq 9 - 15 \\ & -6x \leq -6 \\ & \frac{-6x}{-6} \geq \frac{-6}{-6} \\ & x \geq 1 \end{aligned}$$

Remember to reverse the inequality symbol, when multiplying or dividing by a negative number.

- Problems that involve comparing different options can often be modelled and solved using inequalities.

## Check Your Understanding

### Communicate the Ideas

- Describe the similarities and differences between the process for solving a multi-step linear equation and a multi-step linear inequality. Discuss your answer with a classmate.
- Consider the inequality  $3x + 10 > 5x + 22$ . Lindsay started to solve the inequality by subtracting  $5x$  from both sides. Victoria told her to start by subtracting  $3x$  from both sides.
  - Use Lindsay's approach to solve the inequality.
  - Use Victoria's approach to solve the inequality.
  - Are the solutions the same? Explain.
  - Explain why you think Victoria gave her advice. Is her reasoning helpful in solving the inequality? Explain.
  - Which method of solving the inequality do you prefer? Explain why.

### Practise

For help with #3 to #7, refer to Example 1 on pages 361–363.

- Solve each inequality and verify the solution.

- $5x - 19 < 36$
- $27 + 2x > -13$
- $3 \leq \frac{x}{5} - 7$

- Determine the solution of each inequality.

- $-5y + 92 \geq 40$
- $2.2 > 10.6 + 4y$
- $\frac{y}{-6} - 2 < 16$
- $\frac{3}{2}x + 6 \leq 10\frac{4}{5}$

- Verify that  $x \geq 8$  is the correct solution to the inequality  $3x + 11 \geq 35$ .
  - Verify that  $x < -3$  is the correct solution to the inequality  $24 - 5x > 39$ .

- Solve each inequality and verify the solution.

- $7x < 2x + 30$
- $10x - 22 \geq 8x$
- $-12x + 10 > 19 - 4x$
- $\frac{1}{2}(x + 5) > 22$

- Determine each solution.

- $-2y > 8y - 20$
- $9y - 17 \leq 8 + 6.5y$
- $3.4 - 1.3y < 0.5y - 2.2$
- $\frac{3}{4}y - 1 \geq -\frac{1}{4}(1 - 2y)$

For help with #8 and #9, refer to Example 2 on pages 363–364.

- For each situation
  - choose a variable and explain what it represents
  - write an inequality
- A basketball team wants to buy new jerseys. Uniforms R Us charges \$50 per jersey. Jerseys Unlimited charges \$40 per jersey plus \$80 for a logo design. How many jerseys does the team need to buy for Jerseys Unlimited to be the better option?
- Ann uses her cell phone to send text messages. The monthly charge for text messaging is currently \$15 plus \$0.05 per message sent. The company is offering a new plan that costs a flat rate of \$0.12 per text message. How many text messages does Ann need to send in order for the new plan to be the better option?

9. John is considering two paper delivery jobs. The *Advance* will pay \$10 plus \$0.05 for each paper delivered daily, and the *Times* will pay \$15 plus \$0.04 for each paper delivered daily. How many papers delivered each day would make the *Advance* the better offer?

- Write an inequality to model the problem.
- Solve the inequality and interpret the solution.



### WWW Web Link

To learn how to solve inequalities using a graphing calculator, go to [www.mathlinks9.ca](http://www.mathlinks9.ca) and follow the links.

### Apply

10. Kim is comparing the rates at two car rental companies for a one-day rental. She wants to determine how many kilometres she would need to drive for ABC Rentals to be the better rental option.

**ABC Rentals**  
\$25 per day plus  
\$0.14 per kilometre

**It's a Deal Rentals**  
\$55 per day

- Estimate the number of kilometres that would make ABC Rentals the better option.
- Represent the situation using an inequality.
- Solve the inequality and interpret the solution.
- Compare the solution with your estimate.

11. Kevin is comparing job offers at two stores. Dollar Deal offers \$8/h plus 10% commission. Great Discounts offers \$18/h with no commission. What do Kevin's weekly sales need to be in order for Dollar Deal to pay more? Assume that he works an 8-h day five days per week.

12. The student council is considering two different companies to print the school's yearbooks. Great Graphics charges \$250 plus \$12.25 per book. Print Express charges \$900 plus \$9.50 per book. How many orders for yearbooks would make Print Express the better option?

13. Greenway Golf Course offers two plans for paying for buckets of balls at the driving range. How many buckets of balls used per month make the members' plan the better deal?



#### Greenway Golf Course Rates

**Standard Plan:** \$6 per bucket

**Member's Plan:** \$98 monthly fee plus \$1.50 per bucket

14. Molly has a business making candles. Her business costs are \$200 plus \$0.70 per candle made. She charges her customers \$3.50 for each candle. If she sells all of the candles she makes, how many candles sold would allow her to make a profit?



15. Two full water storage tanks are being drained for maintenance. The first tank holds 800 L of water and drains at a rate of 18 L/min. The second tank holds 500 L of water and drains at a rate of 7 L/min. Use an inequality to determine when the first tank will contain less water than the second tank.

16. Rob and Ashley are riding their bicycles uphill. Currently, Rob is 5.7 km from the top and climbing at 0.24 km/min. Ashley is 4.5 km from the top and riding at 0.17 km/min.

- Estimate when Rob will be closer to the top than Ashley.
- Use an inequality to determine when Rob will be closer to the top than Ashley.

### Extend

17. Solve  $\frac{2}{3}(2x - 5) < \frac{1}{2}(x + 2)$ .

Show the solution on a number line.

18. If  $2x + 5 > 10$  and  $5x - 4 < 20$ , determine the possible values of  $x$ . Show your solution on a number line.

19. Lauren charges \$12 to cut lawns for neighbours. It takes her 25 min to cut each lawn and 40 min per month to maintain her lawn mower. She wants to earn \$400 each month without working more than 16 h cutting lawns. How many lawns can Lauren cut in a month and stay within her guidelines? Use two inequalities to determine the range for the number of lawns that she can cut.

20. Ella's teacher asked which is greater,  $x$  or  $-x$ ? Ella said that  $x$  is always greater than  $-x$ .

- Write an inequality to represent Ella's response and solve it. When, if ever, is Ella correct?

- Ella's teacher explained that her response is correct for some values of  $x$  only. For what values of  $x$  is Ella incorrect? Give one specific solution where Ella is correct and one where she is incorrect.

21. Solve  $-13 \leq 5 - 2x$  and  $5 - 2x \leq 9$ .

22. Given that  $b < 0$ , solve the inequality  $3 > bx + 3$ .

### Math Link

An amusement park manager needs to ensure that the park is profitable. For the park to make a profit, the total revenue needs to be more than the total expenses.

There are fixed expenses and revenues that remain the same. There are also variable expenses and revenues that depend on the number of visitors.

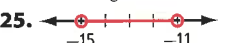
The manager estimates operating expenses and revenues for the park per visitor. These are shown in the table. Assuming the park offers ten rides, fill in the missing information.

- What is the total of the variable expenses per visitor? What are the total fixed costs? Write an expression to represent the total expenses.
- What is the total of the variable revenues per visitor? What are the total fixed revenues? Write an expression to represent the total revenues.
- Develop and solve an inequality to determine the number of visitors needed per day to make a profit. Justify your solution mathematically.

| Daily Expenses                               |        |
|----------------------------------------------|--------|
| Total variable operating costs per visitor   | \$15   |
| Total fixed costs (\$5000 + \$1200 per ride) |        |
| Daily Revenues                               |        |
| Admission (includes ride pass) per visitor   | \$38   |
| Food per visitor                             | \$25   |
| Souvenirs per visitor                        | \$10   |
| Parking per visitor                          | \$10   |
| Total variable revenues per visitor          |        |
| Fixed revenue from sponsorship               | \$2500 |

## 9.2 Solving Single-Step Inequalities, pages 357–359

5. a)  $x \geq 29$  b)  $-7 < x$  c)  $x > -13.8$  d)  $35 \leq x$   
 6. a)  $y \geq 9$  b)  $-14.5 < y$  c)  $y \leq -4$  d)  $y > 6.25$   
 7. a)  $x > 150$  b)  $x \leq 36$  c)  $2.4 \geq x$  d)  $x > -30$   
 8. a)–c) No, the inequality is not changed because there is no multiplication or division by a negative number.  
 d) Yes, the direction of the inequality is changed because there is division by a negative number.  
 e) Yes, the direction of the inequality is changed because there is multiplication by a negative number.  
 f) No, the inequality is not changed because there is no multiplication or division by a negative number.  
 9. a) yes b) yes c) yes d) no  
 10. a) yes b) no c) no d) yes  
 11. a) yes b) no c) yes d) yes  
 12. a) yes b) No; the correct answer is  $x > -16$ , not  $x > 16$ .  
 13. a) No; the correct solution is  $-9 \geq x$ , not  $-11 \geq x$ .  
 b) yes  
 14. a)  $85f \leq 1400$  b)  $f \leq 16.47$  c) No, the boundary value is not a positive integer, which is required when discussing the number of fence sections.  
 15. a)  $6w > 50$  b)  $w > 8.3$ . Megan must win 9 or more races to move up to the next racing category. c) No, the number of races won will be a non-negative integer.  
 16. a) Example: Three solutions are  $-6$ ,  $-20.2$ , and  $-10$ . Three non-solutions are  $0$ ,  $-4$ , and  $8.6$ .  
 b) Example: Three solutions are  $0$ ,  $-2$ , and  $5.5$ . Three non-solutions are  $-11$ ,  $-8$ , and  $-16$ .  
 17. Example: The inequality sign is reversed because each side was divided by a negative number,  $-5$ .  
 18. a) Example: The single sharpening cost is about \$6. When this is divided by 48, the answer is 8. So, if the skates need to be sharpened more than 8 times, the monthly charge would be a better option.  
 b)  $5.75s > 49$ ;  $s > 8.52$ . It would be better to take advantage of the monthly special if the skates were sharpened more than 8 times. The estimate and solution to the inequality are the same.  
 19. a)  $0.03p \geq 250$ ;  $p \geq 8333.\bar{3}$ . The owner would need to make a profit of at least \$8333.33 in order to donate at least \$250 to the local charity. b) Example: Check the boundary point,  $8333.\bar{3}$ , and then check that both sides of the inequality are equal:  $0.03(8333.\bar{3}) = 250$ . Check a number that is larger than the boundary value (9000) and see that it is a solution:  $0.03(9000) = 270$ . Since 270 is larger than 250, then 9000 is a solution to the inequality.  
 20.  $0.084k \leq 57$ ;  $k \leq 678.57$ ; They can travel no more than 678.57 km, assuming the car consumes the average amount of fuel.  
 21. a) Natalie must run 8 laps in order to complete the 3200-m distance:  $3200 \div 400 = 8$ . The total time of 9 min 23 s is equivalent to 563 s:  $9 \times 60 + 23 = 563$ . The expression  $8x$  represents her total time, where  $x$  is her average time per lap. Consequently, her total time must be less than the current record of 563 s:  $8x < 563$ .  
 b)  $x < 70.375$

22.  $\frac{s}{5} \geq 120$ ;  $s \geq 600$ . She will have to spend at least \$600 to get at least 120 points.  
 23. a)  $115d \geq 1000$ ;  $4d \leq 50$  b)  $d \geq 8.70$ ;  $d \leq 12.5$   
 c) Chris can build between 9 and 12 doghouses and stay within his guidelines.  
 24.  $x > -\frac{5}{6}$   
 25.   
 26. The mass of the energy bar must be between 66.67 g and 76.92 g.  
 27. a)  $x \leq 5$  b)  $x \geq 5$   
 28. a)  $-14 \leq x$  and  $x \leq -1$  b)  $-1 < x$  and  $x < 6$   
 c)  $\frac{5}{2} \geq x$  and  $x > -\frac{3}{2}$

## 9.3 Solving Multi-Step Inequalities, pages 365–367

3. a)  $x < 11$  b)  $x > -20$  c)  $50 \leq x$   
 4. a)  $y \leq 10.4$  b)  $-2.1 > y$  c)  $y > -108$  d)  $x \leq 3\frac{1}{5}$   
 5. a) Check the boundary value:  $3(8) + 11 = 35$ . Check another number in the solution set,  $x = 10$ :  $3(10) + 11 = 41$ . Since 41 is greater than 35, the solution is correct. b) Solve the inequality:  $24 - 5x - 24 > 39 - 24$ . Finally, simplify  $\frac{-5x}{-5} < \frac{15}{-5}$  to get the solution,  $x < -3$ .  
 6. a)  $x < 6$  b)  $x \geq 11$  c)  $x < -\frac{9}{8}$  d)  $x > 39$   
 7. a)  $y < 2$  b)  $y \leq 10$  c)  $y > \frac{28}{9}$  d)  $y \geq 3$   
 8. a) Example: Let  $j$  represent the number of jerseys;  $40j + 80 < 50j$  b) Example: Let  $n$  represent the number of text messages sent in one month;  $0.12n < 0.05n + 15$   
 9. a)  $0.05p + 10 > 0.04p + 15$  b) John will have to deliver more than 500 papers to make the *Advance* the better offer.  
 10. a) Example: ABC Rentals would be a better deal if you travel less than 200 km per day ( $30 \div 0.15$ ).  
 b)  $0.14k + 25 < 55$  c) ABC Rentals will be the better option if Kim travels less than 214.3 km per day.  
 11. Kevin's weekly sales must be at least \$4000 for Dollar Deal to pay more.  
 12. Print Express would be the better option if more than 236 yearbooks are ordered.  
 13. The member's plan is a better deal when 22 or more buckets of balls are used per month.  
 14. Molly must sell at least 72 candles in order to make a profit.  
 15. The first tank will contain less water after  $27\frac{3}{11}$  minutes have passed.  
 16. a) Example: Estimate: 20 min b) Rob will be closer to the top after 17.14 min have passed.  
 17.  $x < 5.2$ ;   
 18. 

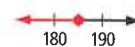
19. Lauren can cut 34, 35, or 36 lawns per month and stay within her guidelines.  
 20. a) Ella is correct when  $x > 0$ .  
 b) Ella is incorrect when  $x \leq 0$ . Example: Ella is correct when  $x = 2$ , but she is incorrect when  $x = -11.2$ .  
 21.  $9 \geq x$  and  $x \geq -2$   
 22.  $x > 0$

## Chapter 9 Review, pages 368–369

1. inequality  
 2. graphically; algebraically  
 3. open circle  
 4. solution  
 5. boundary point  
 6. closed circle  
 7. a) Example: Savings  $\leq 40\%$   
 b) Example: Free shipping for purchases  $\geq \$500$   
 c) Example: Number of items on sale  $> 80$   
 8. a) Example: The bicycle must be at least 6.8 kg in mass.

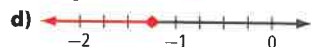


- b) Example: The bicycle must be less than or equal to 185 cm.



9. a)  $x > 13$ ; A number greater than 13.

- b)  $x \leq 8.6$ . A number less than or equal to 8.6.



11. Example:

| Linear Inequality     | One Solution Value | One Non-solution Value |
|-----------------------|--------------------|------------------------|
| $r > -4$              | 3                  | -10                    |
| $0 \leq s \leq 7$     | 3.4                | 11                     |
| $9.5 > t$             | 8                  | 22                     |
| $v \leq -\frac{5}{4}$ | -2                 | 0                      |

12. a)  $d > -3$  b)  $5.4 < a$  c)  $-33 \geq b$  d)  $c < -16$   
 13. a) No, the solution is incorrect. The boundary value of 8 is correct but the direction is incorrect.  
 b) The solution is correct.  
 14. a)  $14.5t \geq 600$  b) Tim must work at least 41.4 h per week to achieve his goal.  
 15. Danielle can buy a maximum of 13 scoops of ice cream and stay within her budget.  
 16. No, the solution is incorrect. The boundary value is correct but the direction is incorrect.  
 17. a) Yes, the solution is correct. b) Example: First, check that the boundary value creates an equation. Then, check a solution value in the original inequality and see if it results in a true inequality statement.

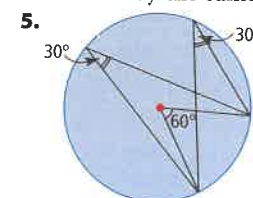
18. a)  $x < 45$  b)  $7.5 > x$  c)  $x \geq -1$  d)  $x < -20$   
 e)  $1.4 \leq x$  f)  $x < -\frac{9}{8}$

19. The maximum number of people that can attend the banquet is 64.  
 20. Greg would need to purchase at least 201 tracks per month to make Plan A the better option.

## Chapter 10

### 10.1 Exploring Angles in a Circle, pages 382–385

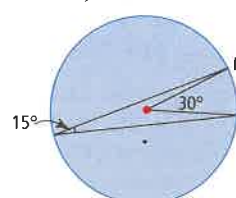
3. ADB and AEB are inscribed angles that are subtended by the same arc as the central ACB. The measure of ACB is  $82^\circ$ . Therefore, ADB and AEB have measures that are half the measure of ACB. Half of 82 is 41. So, the measure of ADB is  $41^\circ$  and the measure of AEB is  $41^\circ$ .  
 4. a)  $23^\circ$ . Example: The inscribed angles subtended by the same arc of a circle are equal. b)  $46^\circ$ . Example: A central angle is twice the measure of an inscribed angle subtended by the same arc.



5.   
 6. a)  $90^\circ$ . Example:  $\angle ABD$  is an inscribed angle subtended by the diameter of the circle. b) 8 cm

7. a)  $90^\circ$  b) 11.3 cm  
 Example: Since  $\triangle CFG$  is a right triangle, by the Pythagorean relationship,  
 $8^2 + 8^2 = FG^2$   
 $64 + 64 = FG^2$   
 $128 = FG^2$   
 $\sqrt{128} = FG$   
 $11.3 \approx FG$

8. Example: Jacob could place his flashlight anywhere on the major arc MN.



9. Example: In the diagram, X is the ideal location.

