

Unit 3: Powers & Exponents

3.2B Negative Exponents

Continue each of the following patterns:

$$\begin{aligned}
 &\rightarrow 2^4 = 16 \quad \div 2 \\
 &\rightarrow 2^3 = 8 \quad \div 2 \\
 &\quad 2^2 = 4 \quad \div 2 \\
 &\quad 2^1 = 2 \quad \div 2 \\
 &\quad 2^0 = 1 \quad \div 2 \\
 &\quad 2^{-1} = \frac{1}{2} = \frac{1}{2^1} \\
 &\quad 2^{-2} = \frac{1}{4} = \frac{1}{2^2} \\
 &\rightarrow 2^{-3} = \frac{1}{8} = \frac{1}{2^3} \\
 &\rightarrow 2^{-4} = \frac{1}{16} = \frac{1}{2^4}
 \end{aligned}$$

$$\begin{aligned}
 &3^4 = 81 \quad \div 3 \\
 &3^3 = 27 \quad \div 3 \\
 &\rightarrow 3^2 = 9 \quad \div 3 \\
 &\quad 3^1 = 3 \quad \div 3 \\
 &\quad 3^0 = 1 \\
 &\quad 3^{-1} = \frac{1}{3} = \frac{1}{3^1} \\
 &\rightarrow 3^{-2} = \frac{1}{9} = \frac{1}{3^2} \\
 &\quad 3^{-3} = \frac{1}{27} = \frac{1}{3^3} \\
 &\quad 3^{-4} = \frac{1}{81} = \frac{1}{3^4}
 \end{aligned}$$

$$7^{-6} = \frac{1}{7^6}$$

How does the result of a negative exponent connect to the result of a positive exponent?

** negative exponents produce reciprocal results*

Another approach.....

What exponent law is likely to lead to a negative exponent?

$$\begin{aligned}
 \frac{3^2}{3^5} &= \frac{18 \cdot \cancel{3}}{18 \cdot \cancel{3} \cdot 3 \cdot 3 \cdot 3} \\
 &= 3^{2-5} \\
 &= 3^{-3} = \frac{1}{3^3}
 \end{aligned}$$

Rule: $b^{-m} = \frac{1}{b^m}$

Ex. Simplify (write with a single positive exponent) & then evaluate:

a) 5^{-3}

$$\begin{aligned}
 &= \frac{1}{5^3} \\
 &= \frac{1}{125}
 \end{aligned}$$

b) 7^{-1}

$$\begin{aligned}
 &= \frac{1}{7^1} \\
 &= \frac{1}{7}
 \end{aligned}$$

c) 10^{-5}

$$\begin{aligned}
 &= \frac{1}{10^5} \\
 &= \frac{1}{100,000}
 \end{aligned}$$

d) -2^{-3}

$$\begin{aligned}
 &= -\frac{1}{2^3} \\
 &= -\frac{1}{8}
 \end{aligned}$$

e) $(-2)^{-4}$

$$\begin{aligned}
 &= \frac{1}{(-2)^4} \\
 &= \frac{1}{16}
 \end{aligned}$$

f) $-(-2)^{-6}$

$$\begin{aligned}
 &= -\frac{1}{(-2)^6} \\
 &= -\frac{1}{64}
 \end{aligned}$$

g) $\frac{1}{3^{-3}}$

$$\begin{aligned}
 &= 3^3 \\
 &= 27
 \end{aligned}$$

h) $\frac{1}{5^{-2}}$

$$\begin{aligned}
 &= 5^2 \\
 &= 25
 \end{aligned}$$

Ex. Simplify using exponent laws:

$$\begin{aligned} \text{a) } 4^3 \times 4^{-1} \\ &= 4^{3+(-1)} \\ &= \boxed{4^2} \end{aligned}$$

$$\begin{aligned} \text{b) } 2 \times 2^{-4} \\ &= 2^{1+(-4)} \\ &= 2^{-3} \\ &= \boxed{\frac{1}{2^3}} \end{aligned}$$

* answers have
+ exponents

$$\begin{aligned} \text{c) } 3^5 \div 3^8 \\ &= 3^{5-8} \\ &= 3^{-3} \\ &= \boxed{\frac{1}{3^3}} \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{9}{9^{-4}} \\ &= 9^{1-(-4)} \\ &= \boxed{9^5} \end{aligned}$$

$$\begin{aligned} \text{e) } \frac{6^{-7}}{6^{-2}} \\ &= 6^{-7-(-2)} \\ &= 6^{-5} \\ &= \boxed{\frac{1}{6^5}} \end{aligned}$$

$$\begin{aligned} \text{f) } (5^{-2})^3 \\ &= 5^{-6} \\ &= \boxed{\frac{1}{5^6}} \end{aligned}$$

$$\begin{aligned} \text{g) } (8^3)^{-1} \\ &= 8^{-3} \\ &= \boxed{\frac{1}{8^3}} \end{aligned}$$

$$\begin{aligned} \text{h) } (7^{-3})^{-4} \\ &= \boxed{7^{12}} \end{aligned}$$

$$\begin{aligned} \text{i) } (5 \times 3)^{-2} \\ &= 5^{-2} \times 3^{-2} \\ &= \boxed{\frac{1}{5^2} \times \frac{1}{3^2}} \end{aligned}$$

$$\begin{aligned} \text{j) } \left(\frac{3}{4}\right)^{-3} \\ &\swarrow \quad \searrow \\ &\frac{3^{-3}}{4^{-3}} \quad \left(\frac{4}{3}\right)^3 \\ &\frac{\frac{1}{3^3}}{\frac{1}{4^3}} \quad \boxed{\frac{4^3}{3^3}} \\ &\frac{1}{3^3} \times \frac{4^3}{1} \\ &\boxed{\frac{4^3}{3^3}} \end{aligned}$$