

1.5: Proofs That Are Not Valid

Curricular Competencies

I can solve problems with persistence and a positive attitude

I can explain and justify math ideas and decisions

I can use mistakes as opportunities to advance learning

Invalid proof A proof that contains an error in reasoning or contains an invalid assumption.

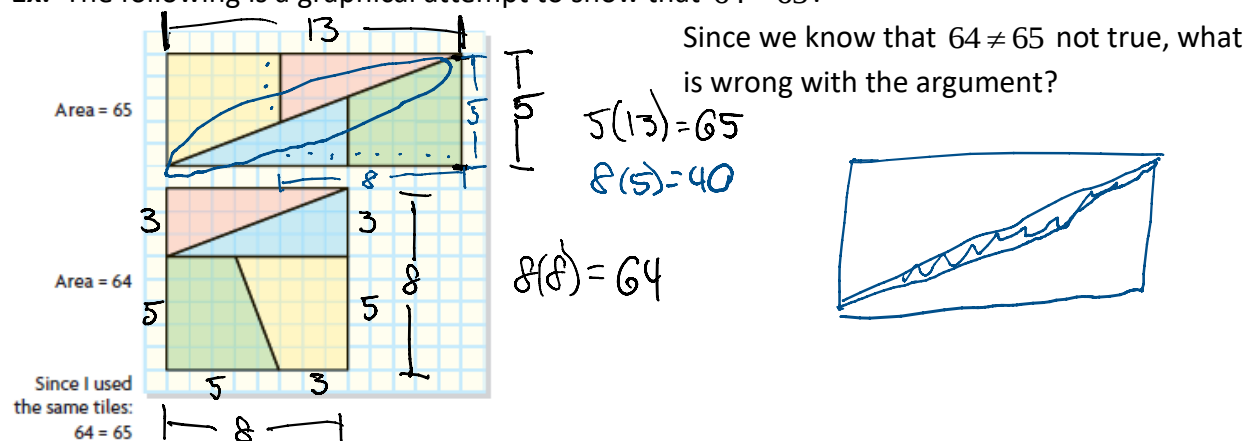
Circular reasoning An argument that is incorrect b/c it makes use of the conclusion to be proven.

Consider the following statement: There are three errors ss in this sentence. Is the statement valid? Explain.

2 errors → makes statement invalid. Also, claims 3 errors when 2 → that's the 3^d error. *Invalid.

This is an example of circular reasoning.

Ex: The following is a graphical attempt to show that $64 = 65$.



Ex. Students at ~~DPS~~ ^{STKR} attend French classes. Jack is a student at STKR. Therefore Jack attends French class. What is the error in the reasoning?

Invalid assumption, not all students take French.

Ex: Bev claims she can prove that $3=4$. She starts with 3 variables such that $a+b=c$ and then performs the following steps. Explain what she is doing at each step and identify where she went wrong.

$a+b=c$ → expanded the equation.
 $4a - 3a + 4b - 3b = 4c - 3c$ → rearranged eq. using opposite operations.
 $4a + 4b - 4c = 3a + 3b - 3c$
 $\frac{4(a+b-c)}{(a+b-c)} = \frac{3(a+b-c)}{(a+b-c)}$ → Factor out the 4 and 3.
 $4 = 3$ ← divided by $a+b-c$ to cancel out
 (Note: $a+b-c=0$ is circled in green, indicating division by zero.)
 Bev divide by ZERO ∴
 Not OK Bev.

Ex: Hossai is trying to prove the following number trick:

Choose any number. Add 3. Double it. Add 4. Divide by 2. Take away the number you started with.

Each time Hossai tries the trick, she ends up with 5. Her proof, however, does not give the same result.

Hossai's Proof	
n	Choose any number.
$n + 3$	Add 3.
$2n + 6$	Double it.
$2n + 10$	Add 4.
$\rightarrow 2n + 5$	Divide by 2.
$n + 5$	Take away the number you started with.

$$\frac{2n+10}{2} = \frac{2n}{2} + \frac{10}{2}$$

$$= n + 5$$

Where is the error in Hossai's proof?

Hossai didn't divide by 2 properly.

In Summary:

- A single error in reasoning will break down the logical argument of a deductive proof. This will result in an invalid conclusion, or a conclusion that is not supported by the proof.
- Division by zero always creates an error in a proof.
- Circular reasoning must be avoided! Be careful not to assume a result that follows from what you are trying to prove.
- The reason you write a proof is so that others can read and understand it. After you write a proof, have someone else read it; if they get confused, fix it!

Practice pg 42 # 1, 2, 3, 5, 6, 7, 9