1.4 Proving Conjectures: Deductive Reasoning

Curricular Competencies

I can use play, inquiry and problem solving to gain understanding
I can explain and justify math ideas and decisions
I can apply flexible and strategic approaches to problems

Recall: Inductive Reasoning: drawing conclusion by observing patterns and "ID'ing" properties in specific ex.

identifying

Deductive Reasoning drawing specific conclusion through logical reasoning by starting w general assumptions.

Transitive Property: $A \rightarrow B$, $B \rightarrow C$
$A \rightarrow C$

Ex. Make a deduction in each of the following cases.

a. Paul lives in Medicine Hat. Medicine Hat is in Alberta.
Paul lives in Alberta.

b. Every animal has a heart. All dogs are animals.
All dogs have hearts.

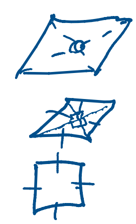
$1+2=3$

$11+12=23$

c. The sum of any two consecutive whole numbers is an odd number. The whole numbers 11 and 12 are consecutive.
The sum of 11 and 12 is odd.

d. The diagonals of a parallelogram bisect each other. *PQRS* is a parallelogram.
The diagonals of PQRS will bisect each other.

e. The diagonals of a rhombus intersect at right angles. *KLMN* is a rhombus.
The diagonals of KLMN intersect at right angles.



Proof: mathematical argument showing that a statement is valid in all cases, no counterexamples exist.

We can use deductive reasoning to PROVE a statement is true.

Remember: We can use inductive reasoning to make conjectures and find evidence to support our conjecture. We may be able to find a counterexample to prove our conjecture is false but we can NEVER prove a conjecture is true.

$\overset{2}{\leftarrow} \quad \overset{1}{} \quad \overset{2}{\rightarrow}$

$1 \; 2 \; \boxed{3} \; 4 \; 5$

Ex 2: Example 2: Jon discovered a pattern when adding integers:

$$1 + 2 + \boxed{3} + 4 + 5 = 15 = 5(3) \checkmark$$
$$(-15) + (-14) + \boxed{(-13)} + (-12) + (-11) = -65 = (-13)5 \checkmark$$
$$(-3) + (-2) + \boxed{(-1)} + 0 + 1 = -5 = 5(-1) \checkmark$$

He claims that whenever you add five consecutive integers, the sum is always 5 times the median of the numbers. Prove Jon's conjecture using an algebraic method.

↳ Middle #

Let x be the median value.

General Case: $1(x-2) + 1(x-1) + x + (x+1) + (x+2)$

$$x \cancel{-2} + x \cancel{-1} + x + x \cancel{+1} + x \cancel{+2}$$

$$\underbrace{5x}$$

↳ 5 times median value.

Ex 3: Prove that the sum of any two odd numbers is an even number.

$7 + 5 = 12 \checkmark$

$1 + 3 = 4 \checkmark$

$$\boxed{\begin{array}{c}\text{To represent even \#'s} \\ 2x \\ \text{To represent odd \#'s.} \\ 2x+1\end{array}}$$

Proof:

$(2x+1) + (2y+1)$

$2x + 1 + 2y + 1$

$2x + 2y + 2$

$2(x+y+1)$

$(2x+1) + (2x+1)$

$4x + 2$

$2(2x+1)$

↳ since 2 is a factor, this is always even.

Ex 5: The following is an example of a number trick. Use inductive reasoning and three trials to determine the answer each time.

Choose a number. Double it. Add 5. Add your original number. Add 7. Divide by 3. Subtract your original number.

	1	2	3
Original Number	3	8	84
Double	6	16	168
Add 5	11	21	173
Add Original #	14	29	257
Add 7	21	36	264
Divide by 3	7	12	88
Subtract Original #	4	4	4

Prove deductively what the answer should be each time.

Let x be the original #.

Original $\quad x$

double $\quad 2x$

add 5 $\quad 2x+5$

add original # $\quad 2x+5+x = 3x+5$

add 7 $\quad 3x+5+7 = 3x+12$

divide by 3 $\quad \frac{3x+12}{3} = x+4$

subtract original $\quad x+4-x = \boxed{4}$

A two-column proof is used in mathematics to prove a statement is true. One of the columns contains ___facts___ that are known to be true and the other column contains ___evidence___ describing why the corresponding statement is true. (refer to p 29 ex 4)

Practice: pg 31 #1, 2, 4, 5, 7, 8, 10, 15