PREC12 Combinatorics Provincial Package

1. When you play lotto 5-30, you must choose 5 different integers from 1 to 30. How many combinations are possible?

   A. $\frac{30!}{5!\,25!}$

   B. $\frac{30!}{25!}$

   C. $25!$

   D. $\frac{30!}{5!}$

2. Determine the 4th term of $(x-2)^6$.

   A. $120x^2$
   B. $240x^2$
   C. $-160x^3$
   D. $-320x^3$

3. Determine the number of different arrangements of all the letters in APPLEPIE.

   A. 3 360
   B. 6 720
   C. 40 312
   D. 40 320

4. Assume a car license plate consists of 7 characters. The first 3 characters can be any of the letters from A to F, but no letter can be repeated. The next 3 characters can be any of the digits from 1 to 9, but no digit can be repeated. The last character can be any of the letters X, Y or Z. An example of this format is: BFA648Y. How many license plates are possible?

   A. 5 040
   B. 181 440
   C. 472 392
   D. 4 084 080

5. Suppose you play a game of cards in which only three cards are dealt from a standard 52-card deck. How many ways are there to obtain one pair? (2 cards of the same rank and 1 card of a different rank.)

   An example of a hand that contains one pair is 2 jacks and 1 five. 

   A. 1 014
   B. 1 872
   C. 3 744
   D. 3 900

6. A soccer coach must choose 3 out of 10 players to kick tie-breaking penalty shots. Assuming the coach must designate the order of the 3 players, determine the number of different arrangements she has available.

A. $\frac{10!}{7!}$

B. $\frac{10!}{3!}$

C. $\frac{10!}{3!\,7!}$

D. $\frac{10!}{3!\,3!\,4!}$

7. Determine the 4th term in the expansion of $(x-2y)^5$.

A. $-80x^2y^3$
B. $-40x^3y^2$
C. $40x^3y^2$
D. $80x^2y^3$

8. Express ${}_{33}C_5$ using factorial notation.

A. $\frac{33!}{5!}$

B. $\frac{33!}{28!}$

C. $\frac{33!}{5!\,28!}$

D. $28!$

9. Determine the 3rd term in the expansion of $(x-y)^{10}$.

A. $-45x^8y^2$
B. $-120x^7y^3$
C. $45x^8y^2$
D. $120x^7y^3$

10. How many different pasta meals can be made from 4 choices of pasta and 2 choices of sauces, if only one pasta and one sauce is selected for each meal?

A. 4
B. 6
C. 8
D. 16

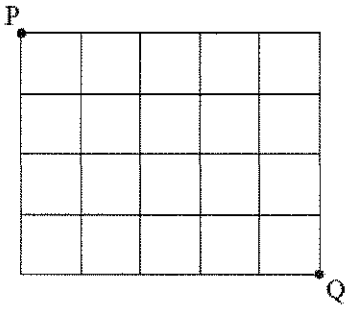
11. A man has 7 different pets and wishes to photograph them 3 at a time arranged in a line. How many different arrangements are possible?

A. 21
B. 35
C. 210
D. 840

12. Determine the 3rd term of $(2x+y)^6$.

A. $15x^4y^2$
B. $240x^4y^2$
C. $120x^3y^3$
D. $160x^3y^3$

13. Moving only to the right or down, how many different paths exist to get from point P to point Q?



A. 120
B. 126
C. 180
D. 480

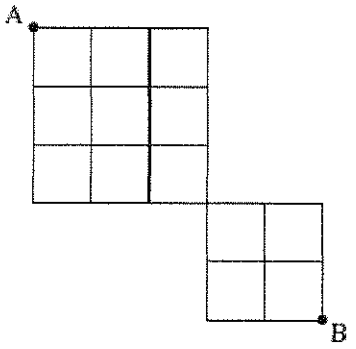
14. Which expression is equivalent to ${}_nC_2$?

A. n^2-2n
B. n^2-n
C. $\frac{1}{2}(n^2-2n)$
D. $\frac{1}{2}(n^2-n)$

15. Simplify: $\frac{n(n+1)!}{(n-1)!}$

A. $2n!$
B. $n!(n^2+n)$
C. $2n$
D. n^3+n^2

16. Moving only to the right or down, how many different paths are there from A to B?



A. 26
B. 52
C. 120
D. 252

17. In the expansion of $(2a-3b)^6$, determine the coefficient of the term containing a^4b^2.

A. −4 320
B. 864
C. 2 160
D. 2 880

18. When playing the 6/49 lottery, a customer must choose 6 different numbers from 1 to 49 inclusive. How many combinations are possible?

A. $49!$

B. $\frac{49!}{6!\,43!}$

C. $\frac{49!}{43!}$

D. $\frac{49!}{6!}$

19. Twelve buttons differ only by colour. There are 4 red buttons, 4 green buttons and 4 yellow buttons. If the buttons are placed in a row, how many different arrangements are possible?

A. 11 880
B. 34 650
C. 19 958 400
D. 479 001 600

20. How many odd 3-digit whole numbers are there? For example, 203 is acceptable but 023 is not.

A. 360
B. 450
C. 500
D. 900

21. How many terms are in the expansion of $(2x+y)^9$?

A. 8
B. 9
C. 10
D. 11

22. Simplify: $\frac{(n-2)!}{(n-1)!}$

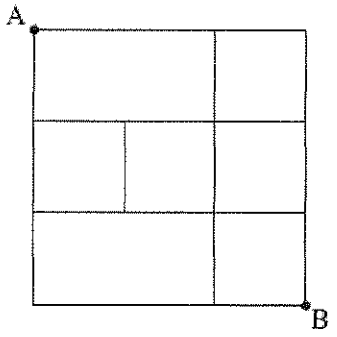
A. $\frac{n-3}{n-1}$

B. $n-2$

C. $\frac{1}{n-1}$

D. $\frac{1}{n(n-1)}$

23. Moving only to the right or down, how many different routes are there from A to B?



A. 10
B. 12
C. 14
D. 18

24. Car license plates consist of 6 characters. Each of the first 3 characters can be any letter from A to Z inclusive except I or O. Each of the last 3 characters can be any digit from 2 to 9 inclusive. If repetitions of letters and digits are not allowed, how many different license plates are possible? An example of this format is G R T 4 9 2.

A. 4 080 384
B. 5 241 600
C. 7 077 888
D. 11 232 000

25. Determine the first three terms in the expansion of $(x+2y)^{10}$.

A. $x^{10}+10x^9y+90x^8y^2$
B. $x^{10}+20x^9y+180x^8y^2$
C. $x^{10}+10x^9y+45x^8y^2$
D. $x^{10}+20x^9y+45x^8y^2$

26. From a class of 12 boys and 10 girls a committee of 3 people is selected. How many different committees have at least 1 boy?

A. 120
B. 540
C. 1 420
D. 1 540

27. Solve algebraically: $\frac{(n-1)!}{(n-3)!} = 30$

28. A class has 30 students.

a) How many ways can a committee of 3 people be selected from the class?
b) How many ways can an executive committee consisting of 3 people (president, vice-president, secretary) be selected from the class? **(1 mark)**
c) If there are 10 boys and 20 girls in the class, how many ways can a committee of 3 people be selected from the class if the committee must contain 1 boy and 2 girls? **(1 mark)**

29. A toy box contains 4 different cars and 6 different trucks.

a) In how many ways can a collection of 5 toys be chosen if the collection must consist of 2 cars and 3 trucks? **(2 marks)**
b) In how many ways can a collection of 5 toys be chosen if the collection must consist of at least 3 cars? **(2 marks)**

30. Determine the first 3 terms of the expansion: $(x-2y)^7$

31. There are 7 boys and 5 girls in a group of students.

a) Calculate the number of ways that a committee of 4 students can be chosen from this group if the committee must have exactly 1 boy. **(2 marks)**
b) If the committee of 4 students must have a female president, a male vice-president, and 2 other members chosen from the remaining students, how many ways can such a committee be chosen? **(2 marks)**

32. a) How many groups of 3 chairs can be chosen from 7 chairs if the chairs are all different colours?
b) How many different ways can 7 chairs be arranged in a row if 2 of the chairs are blue, 3 are yellow, 1 is red and 1 is green? (Assume that all of the chairs are identical except for colour.)

KEY

1. A	4. B	7. A	10. C	13. B	16. C	19. B	22. C	25. B
2. C	5. C	8. C	11. C	14. D	17. C	20. B	23. B	26. C
3. A	6. A	9. C	12. B	15. D	18. B	21. C	24. A	

27. 7
28. a) 4060
b) 24360
c) 1900
29. a) 120
b) 66
30. $x^7 - 14x^6y + 84x^5y^2$
31. a) 70
b) 1575
32. a) 35
b) 420