PREC12
11.3B

1. Expand and simplify.

   a) $(a+b)^{10}$
   b) $(x-y)^{9}$
   c) $(2x+1)^{7}$
   d) $(2x^{2}-x)^{5}$
   e) $\left(3a^{3}+\frac{3}{a}\right)^{4}$
   f) $\left(2+\frac{1}{x}\right)^{3}$

2. Write and simplify the first three terms of the following:

   a) $\left(x-\frac{1}{x}\right)^{12}$
   b) $\left(3ax+\frac{x^{2}}{3}\right)^{6}$
   c) $\left(\frac{x}{2}-\frac{2}{x}\right)^{7}$

3. Find and simplify the seventh term of $(a+b)^{8}$.

4. Find the fifth term of $\left(\frac{y}{4}-\frac{2}{y}\right)^{7}$.

5. Find and simplify:

   a) the sixth term of $(2y+x)^{11}$
   b) the middle term of $\left(2x-\frac{1}{2x}\right)^{12}$
   c) the term containing x^{20} in $(2x-x^{4})^{14}$
   d) the term containing x^{2} in $\left(\frac{x^{5}}{2}-\frac{2}{x^{3}}\right)^{10}$
   e) the constant term in $\left(2x^{4}-\frac{1}{2x^{2}}\right)^{12}$
   f) the term containing x^{14} in $(2x+x^{2})^{11}$
   g) the fourth term in $\left(\frac{x^{3}}{3}-2\right)^{9}$
   h) the (r+1) term of $\left(3a-\frac{1}{6a^{2}}\right)^{9}$

6. Write the first four terms of $\left(2x^{3}-\frac{1}{4x}\right)^{11}$. Find the eighth term.

7. Find the middle term of $\left(3x-\frac{1}{3x}\right)^{8}$.

Lesson 6

Answer Key

Since $\binom{n}{0}$ and $\binom{n}{n}$ are both equal to 1 they will not be stated in the expansions.

1. a) $a^{10}+\binom{10}{1}a^9b+\binom{10}{2}a^8b^2+\binom{10}{3}a^7b^3+\binom{10}{4}a^6b^4+$

$\binom{10}{5}a^5b^5+\binom{10}{6}a^4b^6+\binom{10}{7}a^3b^7+\binom{10}{8}a^2b^8+$

$\binom{10}{9}ab^9+b^{10}$

$=a^{10}+10a^9b+45a^8b^2+120a^7b^3+210a^6b^4+252a^5b^5+$

$210a^4b^6+120a^3b^7+45a^2b^8+10ab^9+b^{10}$

b) $x^9+\binom{9}{1}x^8(-y)^1+\binom{9}{2}x^7(-y)^2+\binom{9}{3}x^6(-y)^3+$

$\binom{9}{4}x^5(-y)^4+\binom{9}{5}x^4(-y)^5+\binom{9}{6}x^3(-y)^6+\binom{9}{7}x^2(-y)^7$

$+\binom{9}{8}x(-y)^8+(-y)^9$

$=x^9-9x^8y+36x^7y^2-84x^6y^3+126x^5y^4-126x^4y^5+$

$84x^3y^6-36x^2y^7+9xy^8-y^9$

c) $(2x)^7+\binom{7}{1}(2x)^6 1+\binom{7}{2}(2x)^5 1^2+\binom{7}{3}(2x)^4 1^3+\binom{7}{4}(2x)^3 1^4$

$+\binom{7}{5}(2x)^2 1^5+\binom{7}{6}(2x)1^6+1^7$

$=128x^7+448x^6+672x^5+560x^4+280x^3+84x^2+14x+1$

d) $(2x^2)^5 + \binom{5}{1}(2x^2)^4(-x) + \binom{5}{2}(2x^2)^3(-x)^2 + \binom{5}{3}(2x^2)^2(-x)^3$

$+\binom{5}{4}(2x^2)(-x)^4 + (-x)^5$

$= 32x^{10} - 80x^9 + 80x^8 - 40x^7 + 10x^6 - x^5$

e) $(3a^3)^4 + \binom{4}{1}(3a^3)^3\left(\frac{3}{a}\right) + \binom{4}{2}(3a^3)^2\left(\frac{3}{a}\right)^2 + \binom{4}{3}(3a^3)\left(\frac{3}{a}\right)^3 + \left(\frac{3}{a}\right)^4$

$= 81a^{12} + 324a^8 + 486a^4 + 324 + \frac{81}{a^4}$

f) $2^3 + \binom{3}{1}2^2\left(\frac{1}{x}\right) + \binom{3}{2}2\left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^3 = 8 + \frac{12}{x} + \frac{6}{x^2} + \frac{1}{x^3}$

2. a) $x^{12} + \binom{12}{1}x^{11}\left(-\frac{1}{x}\right) + \binom{12}{2}x^{10}\left(-\frac{1}{x}\right)^2 + \ldots$

$= x^{12} - 12x^{10} + 66x^8 \ldots$

b) $(3ax)^6 + \binom{6}{1}(3ax)^5\left(\frac{x^2}{3}\right) + \binom{6}{2}(3ax)^4\left(\frac{x^2}{3}\right)^2 + \ldots$

$= 729a^6x^6 + 486a^5x^7 + 135a^4x^8 \ldots$

c) $\left(\frac{x}{2}\right)^7 + \binom{7}{1}\left(\frac{x}{2}\right)^6\left(-\frac{2}{x}\right) + \binom{7}{2}\left(\frac{x}{2}\right)^5\left(-\frac{2}{x}\right)^2 + \ldots$

$= \frac{x^7}{128} - \frac{7x^5}{32} + \frac{21x^3}{8} \ldots$

3. $\binom{8}{6}a^{8-6}b^6 = 28a^2b^6$

4. $\binom{7}{4}\left(\frac{y}{4}\right)^{7-4}\left(-\frac{2}{y}\right)^4 = 35\left(\frac{y^3}{64}\right)\left(\frac{16}{y^4}\right) = \frac{35}{4y}$

5. a) $\binom{11}{5}(2y)^{11-5}x^5 = 462(64y^6)x^5 = 29\,568y^6x^5$

b) Since there are 13 terms, the middle term will be the seventh term : $\binom{12}{6}(2x)^6\left(-\frac{1}{2x}\right)^6 = 924$

c) The pattern of the exponent of x is: $x^{14},\ x^{13}x^4,\ x^{12}x^8\ldots$ or $x^{14}, x^{17}, x^{20}, \ldots$. The requested term is the third term which is: $\binom{14}{2}(2x)^{12}\left(-x^4\right)^2 = 372736x^{20}$.

d) The pattern of the exponent of x is: $\left(x^5\right)^{10}, \left(x^5\right)^9\left(\frac{1}{x^3}\right)$, $\left(x^5\right)^8\left(\frac{1}{x^3}\right)^2 \ldots$ or $x^{50}, x^{42}, x^{34}, \ldots$. The exponent is decreasing by 8. Therefore, to reach x^2 from x^{50}, you need $\frac{48}{8} = 6$ decreases. The requested term is the seventh term: $\binom{10}{6}\left(\frac{x^5}{2}\right)^4\left(-\frac{2}{x^3}\right)^6 = 840x^2$.

e) The pattern of the exponent of x is : $(x^4)^{12}, (x^4)^{11}\left(\frac{1}{x^2}\right)$, $(x^4)^{10}\left(\frac{1}{x^2}\right)^2 \ldots$ *or* $x^{48}, x^{42}, x^{36}, \ldots$. The exponent is decreasing by 6. To reach the constant term, you need $\frac{48}{6} = 8$ decreases. The requested term is the ninth term: $\binom{12}{6}(2x^4)^4\left(-\frac{1}{2x^2}\right)^8 = \frac{495}{16}$.

f) The pattern of the exponent of x is: x^{11}, $x^{10}x^2$, $x^9\left(x^2\right)^2 \ldots$ or x^{11}, x^{12}, x^{13}, ... The requested term is the fourth term which is: $\binom{11}{3}(2x)^8\left(x^2\right)^3 = 42240x^{14}$.

g) $\binom{9}{3}\left(\frac{x^3}{3}\right)^6(-2)^3 = 84\left(\frac{x^{18}}{3^6}\right)(-8) = \frac{84(-8)x^{18}}{729} = \frac{-224x^{18}}{243}$

h) $\binom{9}{r}(3a)^{9-r}\left(-\frac{1}{6a^2}\right)^r$

6. $(2x^3)^{11} + \binom{11}{1}(2x^3)^{10}\left(-\frac{1}{4x}\right) + \binom{11}{2}(2x^3)^9\left(-\frac{1}{4x}\right)^2 +$
$\binom{11}{3}(2x^3)^8\left(-\frac{1}{4x}\right)^3$
$= 2048x^{33} - 2816x^{29} + 1760x^{25} - 660x^{21}$

The eighth term is $\binom{11}{7}(2x^3)^{11-7}\left(-\frac{1}{4x}\right)^7 = 330(2^4x^{12})\left(-\frac{1}{2^{14}x^7}\right)$
$= -\frac{330}{1024}x^5 = -\frac{165}{512}x^5$.

7. The middle term will be the fifth term:
$\binom{8}{4}(3x)^4\left(-\frac{1}{3x}\right)^4 = 70.$