PREC12
11.2

4. In how many ways can four policemen be selected for special duty from a group of 12 policemen?

5. Tasha's wardrobe includes five slacks, eight blouses, and five pairs of shoes. She wants to select three slacks, four blouses, and three pairs of shoes for her camping trip. What is the number of selections Tasha can make?

6. In a class of 30 students, each student shakes hands with each of the other students once. How many handshakes are there?

7. a) How many different five-card hands can be dealt from an ordinary deck of 52 cards?

   b) How many of these hands contain five cards of the same suit?

8. Lotto 6-49 is a lottery in which a person selects six numbers from 1 to 49. How many ways can a selection be made?

9. A woman gives a dinner party for six of her nine friends.

   a) In how many ways can she choose her six friends?

   b) In how many ways can she choose her six friends if Dorothy and Oksana will not attend together?

10. a) In how many ways can three strangers choose hotels in a town with seven hotels?

    b) In how many ways can the three strangers choose the hotels if two of them are from different parties and refuse to stay at the same hotel?

    c) If one of the hotels has room for only two more people, in how many ways could the three strangers find accommodations?

11. On a Senior 4 Mathematics examination, students must answer exactly five of the first six questions and three out of the last five questions. In how many ways can this be done?

12. How many ways can a committee of five be chosen from 10 girls and eight boys so that the girls have a majority on the committee?

16. A tennis club has 10 boys and eight girls as members. From amongst these members how many different matches are possible with:

    a) a boy against a girl

    b) two boys against two girls

    c) a boy and a girl against another boy and a girl

20. A committee of five is to be selected from nine persons. In how many ways can this selection be made if:

    a) Jack must be on the committee

    b) Jack must not be on the committee

    Find the sum of (a) and (b) and compare this sum to ${}_9C_5$. Give a reason for your findings.

21. Seven points are placed in a circle. A triangle is formed every time three of these points are joined by segments. How many different triangles of this type are possible?

22. How many "words" of five different letters can be made from the English alphabet if each word: (*Note:* Treat Y as a consonant.)

    a) consists of two vowels and three consonants

    b) contains at most three consonants

    c) contains at least one consonant and one vowel

    d) begins with a consonant and ends in a vowel

23. ~~A group has a repertoire of 30 songs. In how many ways can they select seven songs for a particular segment of their concert? If they must open with a particular song and close with another particular song, in how many ways can they arrange this seven-song segment?~~

Lesson 5

Answer Key

4. ${}_{12}C_4 = 495$

5. ${}_5C_3 \cdot {}_8C_4 \cdot {}_5C_3 = 10(70)10 = 7000$

6. Two students must be selected for every handshake. The order of selection is unimportant. Therefore, there are ${}_{30}C_2 = 435$ handshakes.

7. a) You must select five cards in any order. ${}_{52}C_5 = 2\,598\,960$ different poker hands.

   b) There are four suits and there are 13 cards in each suit. Therefore, there are ${}_4C_1 \cdot {}_{13}C_5 = 5148$ flushes.

8. ${}_{49}C_6 = 13\ 983\ 816$ choices

9. a) ${}_9C_6 = 84$

   b) *Case 1:* with Dorothy or Oksana invited:
   $\underline{\ 2\ } \cdot {}_7C_5 = 2(21) = 42$
   Case 2: without Dorothy or Oksana: ${}_7C_{\cancel{6}} = \cancel{21}\ 7$
   Total number of ways is: $42 + \cancel{21}\ 7 = \cancel{63}$. $\boxed{49}$

   or
   ${}_9C_6 - {}_7C_4 = \boxed{49}$

10. a) Each person can choose any hotel.
    ${}_7C_1 \cdot {}_7C_1 \cdot {}_7C_1 = 7(7)7 = 343$ ways

    b) Place the special people first.
    ${}_7C_1 \cdot {}_6C_1 \cdot {}_7C_1 = 7(6)7 = 294$ ways

    c) All possible – the time they all selected this two-vacancy hotel $= 343 - 1 = 342$.

11. ${}_6C_5 \cdot {}_5C_3 = 6(10) = 60$ ways

12. *Case 1:* five girls
    ${}_{10}C_5 = 252$

    Case 2: four girls and one boy
    ${}_{10}C_4 \cdot {}_8C_1 = 210(8) = 1680$

    Case 3: three girls and two boys
    ${}_{10}C_3 \cdot {}_8C_2 = 120(28) = 3360$
    The number of choices with girls in the majority is:
    $252 + 1680 + 3360 = 5292$.

16. a) Choose one of each: ${}_{10}C_1 \cdot {}_8C_1 = 10(8) = 80$.

    b) Choose two of each: ${}_{10}C_2 \cdot {}_8C_2 = 45(28) = 1260$.

    c) Same as part (b) except they can switch partners:
    $1260(2) = 2520$

20. a) Jack and four others: $\underline{\ 1\ } \cdot {}_8C_4 = 70$ ways.

    b) Jack is rejected: ${}_8C_5 = 56$ ways.

    ${}_9C_5 = 126$ and $70 + 56 = 126$. They are equal. Parts (a) and (b) cover all the possible ways of selecting five persons from nine. Jack is on the committee or he is not; there is no other possibility.

21. You must select three points to form a triangle. The order of selection is unimportant. Therefore, ${}_7C_3 = 35$ such triangles are possible.

22. The order of the letters is important, since each different order produces a new word. Choose the required letters first and then order the five letters. There are 26 letters in the alphabet containing five vowels.

a) ${}_5C_2 \cdot {}_{21}C_3(5!) = 10(1330)120 = 1\ 596\ 000$ words.

b) *Case 1:* five vowels: $5! = 120$

Case 2: four vowels and one consonant:
${}_5C_4 \cdot {}_{21}C_1(5!) = 5(21)120 = 12\ 600$

& case 4: 3 vowels & 2 consonants: ${}_5C_3 \cdot {}_{21}C_2 \cdot 5!$ $= 252000$

Case 3, Part (a), above: 1 596 000

Total number of words is: $120 + 12\ 600 + 1\ 596\ 000$ $+ 252000$
~~1 608 720~~. $\boxed{1860720}$

c) All possible – all vowels – all consonants
${}_{26}P_5 - 5! - {}_{21}P_5 = 7\ 893\ 600 - 120 - 2\ 441\ 880 =$
5 451 600 words

d) $\underline{\ 21\ } \cdot {}_{24}P_3 \cdot \underline{\ 5\ } = 21(12\ 144)5 = 1\ 275\ 120$ words

23. ${}_{30}C_7 = 2\ 035\ 800$ ways. They must choose five songs for the middle part of their program: ${}_{28}C_5 = 98\ 280$ ways.