PRECi2
11.1 B

1. Find the number of different arrangements of the word FRIGHTEN if:
   a) there are no restrictions
   b) the first letter must be G
   c) the first letter is not a G
   d) the first letter must be G and the last letter must be H
   e) the arrangements must end in TH
   f) the arrangements begin and end in a vowel

2. How many three-letter "words" can be formed from the word GROUP if:
   a) there are no restrictions
   b) the words contains no vowels
   c) the words contain at least one vowel
   d) the words begin and end in a vowel

3. A book collector has five Italian, three Spanish, and three Greek books. He also has just purchased a Portuguese book. In how many ways can he arrange these 12 books on a shelf, if the books of the same language must be kept together?

4. Eight boys are arranged in a row. Hans and David are two boys in this group. In how many ways can these eight boys be arranged in this row if:
   a) Hans and David are not permitted to sit together
   b) Hans and David are not permitted to sit together nor is either boy permitted to stand at either end

5. Four couples attend a movie together and sit in a row with eight seats. In how many ways can they be seated if:
   a) each girl sits beside her date
   b) the boys and girls alternate positions

6. How many different "words" can be made from the letters of the word LOGARITHM if:
   a) the last letter must be a vowel
   b) the letters L, O, and G must be together
   c) the letters L, O, and G must be together and the L must precede the G

7. How many permutations are there of the word:
   a) SCHOOLS
   b) BOOKKEEPERS
   c) CANADIAN

8. In how many distinct ways can three red flags, two blue flags, two green flags, and four yellow flags be arranged in a row?

9. If $a^3b^2c^4$ is written with exponents of one, how many arrangements of the nine letters are possible?

10. How many different five-digit numbers can be formed, using the digits 1, 2, 3, 4, 5, if:

a) the odd digits occupy the odd places

b) the odd digits occupy the odd places in ascending order

11. Using the digits 2, 2, 2, 3, 3, 4, 5, how many:

a) seven-digit numbers can be formed

b) seven-digit numbers can be formed if the number is greater than 3 400 000

c) seven-digit numbers can be formed if the number is greater than 3 400 000 and divisible by 5

12. The letters of the word BARRIER are arranged all at a time. Find the number of arrangements:

a) beginning with the letter R

b) beginning with two Rs

c) beginning with three Rs

d) beginning with exactly one R

e) beginning with exactly two Rs

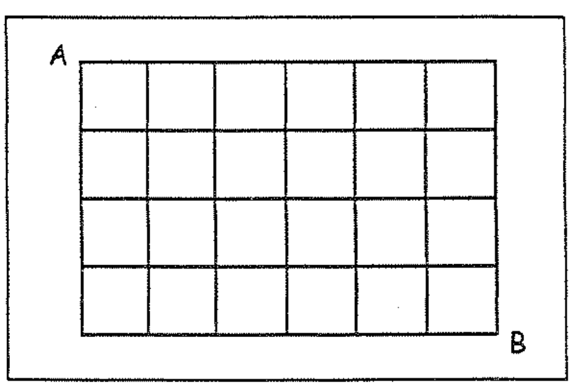
13. How many different seven-digit numbers can you make using the digits 3, 4, 5, 6, 7, 8, 9:

a) if the odd digits are in descending order

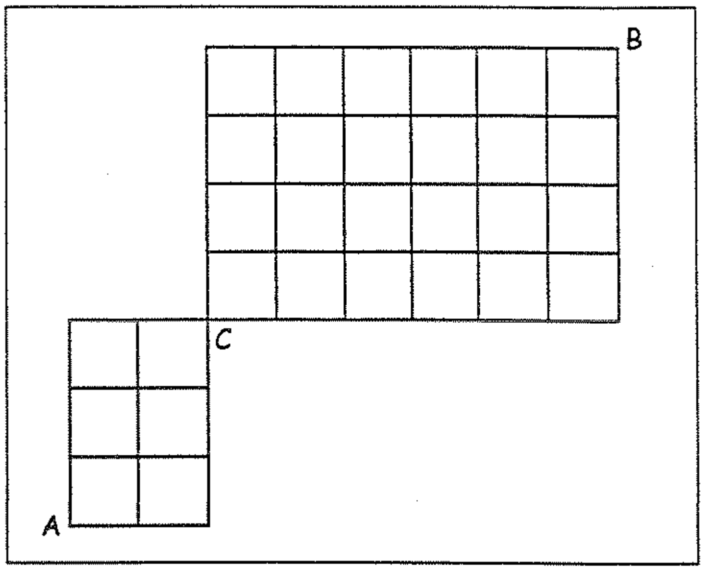
b) if the even digits are in ascending order

14. How many arrangements can be made using all the letters of BABBLING BABY?

15. In how many different ways can person A walk to person B if the trip takes exactly 10 blocks?



16. In how many different ways can person A walk to person B, passing through street intersection C, if the trip takes exactly 15 blocks?



17. A coin is flipped eight times. In how many ways could the result be five heads and three tails?

18. In how many ways can the letters of the word MAXIMUM be arranged if:

a) the three Ms must be together

b) the three letters MUM must be together

19. How many arrangements can be made using all the letters of:

a) CINCINNATI OHIO

b) TORONTO ONTARIO

KEY

1. a) ${}_8P_8 = \frac{8!}{0!} = 8! = 40\ 320$

b) $\underline{1} \cdot {}_7P_7 = 1\left(\frac{7!}{0!}\right) = 7! = 5040$

(Notice how the formula may be used for doing part of the question in part (b).)

c) All possible — the number that begins with G = 40 320 − 5040 = 35 280

d) $\underline{1} \cdot {}_6P_6 \cdot \underline{1} = 1\left(\frac{6!}{0!}\right)1 = 6! = 720$

e) ${}_6P_6 \cdot \underline{1} \cdot \underline{1} = \left(\frac{6!}{0!}\right)1 \cdot 1 = 6! = 720$

f) $\underline{2} \cdot {}_6P_6 \cdot \underline{1} = 2\left(\frac{6!}{0!}\right)1 = 2(6!) = 1440$

#2)

3. ~~4.~~ There are four groups. Answer: $4!5!3!3!1! = 103\,680$

4. a) All possible – Hans and David together =
$8! - 7!2! = 30\,240$.

b) Do the ends first and then the middle.
$\underline{\;6\;} \cdot (6! - 5!2!) \cdot \underline{\;5\;} = \cancel{1200}$ 14,400

5. They are four pairs.

a) $4!(2!\,2!\,2!\,2!) = 384$.

b) $2(4!)(4!) = 1152$

6. a) Last letter has three choices. Total is: ${}_8P_8 \cdot \underline{\;3\;} = 120\,960$.

b) Seven groups with a triple. Total is: $7!3! = 30\,240$.

c) The triple LOG cannot switch entirely, since L must precede the G.

Case 1: L is first

$\underline{\;1\;} \cdot \underline{\;2\;} \cdot \underline{\;1\;} = 2$

Case 2: L is second

$\underline{\;1\;} \cdot \underline{\;1\;} \cdot \underline{\;1\;} = 1$

Total is: $7!2 + 7!(1) = 7!(3) = 15\,120$.

7. a) $\dfrac{7!}{2!2!} = 1260$

b) $\dfrac{11!}{2!2!3!} = 1\,663\,200$

c) $\dfrac{8!}{3!2!} = 3360$

8. $\dfrac{11!}{3!2!2!4!} = 69\,300$

2. a) ${}_5P_3 = \dfrac{5!}{2!} = 60$

b) $\underline{\;3\;} \cdot \underline{\;2\;} \cdot \underline{\;1\;} = 6$

c) All possible — number of words with no vowels
$= 60 - 6 = 54$

d) $\underline{\;2\;} \cdot {}_3P_3 \cdot \underline{1} = \cancel{2\left(\frac{3!}{0!}\right)1 = 2(3!) = 12}$ = 6

9. $\dfrac{9!}{3!2!4!} = 1260$

10. a) $\underline{\;3\;} \cdot \underline{\;2\;} \cdot \underline{\;2\;} \cdot \underline{\;1\;} \cdot \underline{\;1\;} = 12$

b) Since the odd numbers cannot change order, it follows that you must divide by the number of ways of ordering the odd numbers — by $3! = 6$. Therefore, $12 \div 6 = 2$.

11. a) $\dfrac{7!}{3!2!} = 420$

b) *Case 1:* if first digit is 3

$\underline{1} \cdot \underline{2} \cdot \dfrac{5!}{3!} = 40$

Case 2: if first digit is 4 or 5

$\underline{2} \cdot \dfrac{6!}{3!2!} = 120$

Total number of numbers is: $40 + 120 = 160$.

c) Last digit must be 5.

Case 1: if first digit is 3

$$\underline{1}\cdot\underline{1}\cdot\frac{4!}{3!}\cdot\underline{1}=4$$

Case 2: if first digit is 4

$$\underline{1}\cdot\frac{5!}{3!2!}\cdot\underline{1}=10$$

Total number of numbers is: 4 + 10 = 14.

12. a) $\underline{1}\cdot\frac{6!}{2!}=360$

b) $\underline{1}\cdot\underline{1}\cdot\frac{5!}{0!}=120$

c) $\underline{1}\cdot\underline{1}\cdot\underline{1}\cdot\frac{4!}{0!}=24$

d) Exactly one R means that the first letter is an R and the second is not an R.

$$\underline{1}\cdot\underline{4}\cdot\frac{5!}{2!}=240$$

e) Exactly two Rs means that the first two letters are Rs and the third is not an R.

$$\underline{1}\cdot\underline{1}\cdot\underline{\cancel{3}\,4}\cdot\frac{4!}{0!}=\cancel{72}\ 96$$

13. a) The odd digits cannot change order. Therefore, $\frac{7!}{4!}=210$.

b) The even digits cannot change order. Therefore, $\frac{7!}{3!}=840$.

14. $\frac{12!}{5!2!}=1\ 995\ 840$

15. The ways of ordering six Rights and four Ups. Therefore, $\frac{10!}{6!4!}=210$.

16. To get to C you must order two Rs and three Us which is $\frac{5!}{3!2!}=10$ ways.

Once at C, A must walk to B by ordering six Rs and four Us which is $\frac{10!}{6!4!}=210$ ways.

By the Fundamental Principle, A can walk to B in 10(210) = 2100 ways.

17. $\dfrac{8!}{5!3!} = 56$

18. a) The Ms cannot change order since they are indistinguishable. They are treated as a grouped object with no internal order changes.
Therefore, $5! = 120$ ways.

b) *Case 1:* if the order is MUM, then you have to order five objects in $5! = 120$ ways.

Case 2: if the order is MMU and MMU is the last object, you have $4! = 24$ ways.

Case 3: if the order is MMU and MMU is not the last object, the object right after this triple cannot be an M or you will repeat MMUM from Case 1. Therefore, there are three choices for the letter right after MMU and these three objects can be ordered in 3! ways. Total number for case 3 is $3(3!) = 18$ ways.

Case 4 and *Case 5* with order UMM are similar to Cases 2 and 3. Therefore, $24 + 18 = 42$ ways.

Total of all possibilities with MUM together is:
$120 + 24 + 18 + 24 + 18 = 204$ ways.

19. a) $\dfrac{14!}{2!4!3!2!} = 151\ 351\ 200$

b) $\dfrac{14!}{3!5!2!2!} = 30\ 270\ 240$