PREC12
11.1 A

ON THE BUTTON

TO DECODE THE BUTTON:

Solve any equation at the right and find the solution around the rim of the button. Each time the solution appears on the button, write the letter of that equation above the solution. Keep working and you will decode the button.

Ⓔ $4! = n$	Ⓤ $\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = n$
Ⓐ $5! = n$	Ⓜ $\frac{6!}{4!} = n$
Ⓝ $7! = n$	Ⓣ $\frac{7!}{6!} = n$
Ⓘ $3!4! = n$	Ⓕ $\frac{8!}{5!} = n$
Ⓢ $2!5! = n$	Ⓞ $\frac{6!}{4!2!} = n$
Ⓡ $6 \cdot 5! = n!$	Ⓩ $\frac{9!}{6!3!} = n$
Ⓗ $11 \cdot 10! = n!$	Ⓦ $\frac{7!}{2!5!} = n$
Ⓛ $9 \cdot 8 \cdot 7! = n!$	Ⓓ $\frac{5!}{3!2!} = n$

CODE LINE

DIRECTIONS:

Figure out the answer to any question below. Then find your answer in the coded line at the bottom of the page.

Each time the answer appears in the code, write the letter of that question above it.

KEEP WORKING AND YOU WILL DECODE THE LINE.

Ⓐ How many arrangements of the letters M, A, T, and H are possible if each letter can be used only once in each arrangement?

Ⓢ Six people are to be seated in a row of six chairs. How many different seating arrangements are possible?

Ⓓ There are 3 roads connecting Towns A and B, and 4 roads connecting Towns B and C. How many different routes are there from Town A to Town C?

Ⓞ The GT Dragger offers 5 different engines, 4 different paint jobs, and 2 different radios. How many different "packages" are possible?

Ⓘ How many different batting orders are possible for the 9 men on a baseball team?

Ⓥ Orgo has 5 pairs of pants, 6 sport shirts, and 3 belts. How many different outfits can he make using these items?

Ⓛ How many different 2-letter arrangements can be selected from the set {S,H,A,R,K}?

Ⓟ How many 3-letter arrangements are possible using the 26 letters of the alphabet if no letter can be used more than once?

Ⓡ If a school offers 9 different subjects, how many different schedules of 5 classes are possible?

Ⓒ In how many different ways can a president, vice president, and secretary be elected from a class of 22 students?

Ⓔ How many different 4-digit numerals are there? (Hint: zero cannot be used as the first digit.)

TITLE: BIG DRIPS

362,880 9240 362,880 9240 20 9000 720 24 15,120 9000

9000 24 90 9000 720 12 15,120 40 15,600 15,600 9000 15,120 720

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4. Using the numbers 1, 2, 3, 5, 6, 8, 0, with no repetition allowed, how many four-digit numbers can be constructed if:
   a) there is no other restriction other than no repeated digits
   b) the number is even
   c) each digit of the number is even
   d) the number is divisible by 5

5. How many four-digit numbers greater than 5364 are possible using the digits 1, 2, 3, 5, 7, 8 and:
   a) repetition of digits is allowed
   b) repetition of digits is not allowed

6. Winnipeg Stadium has five gates. In how many ways can you enter the stadium and leave the stadium:
   a) by a different gate
   b) by any gate

7. In how many way can a school president, a vice-president, and a secretary be selected from the 23 students comprising the student council?

8. In how many ways can five different books be arranged on a bookshelf?

9. In how many ways can four women and three men be seated on a bench if the men and women must alternate seats?

10. In how many ways can four women and four men be seated on a bench if the men and women must alternate seats?

11. In how many ways can two letters be placed in five mail boxes if:
    a) the two letters must be placed in different mail boxes
    b) the letters may be placed in any mail box

KEY

4. a) The first digit cannot be a zero or it would be considered a three-digit number.

   $\underline{6} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} = 720$

   0 re-enters the selection set after the first digit has been selected.

   b) *Case 1:* The last digit is 0.

   $\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{1} = 120$

   Case 2: The last digit is 2, 6, or 8.

   $\underline{5} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} = 300$

   The number of even four-digit numbers will be:
   $120 + 300 = 420$.

   c) *Case 1:* The last digit is 0.

   $\underline{3} \cdot \underline{2} \cdot \underline{1} \cdot \underline{1} = 6$

   Case 2: The last digit is 2, 6, or 8.

   $\underline{2} \cdot \underline{2} \cdot \underline{1} \cdot \underline{3} = 12$

   The number of all even four-digit numbers will be:
   $6 + 12 = 18$.

   d) *Case 1:* The last digit is 0.

   $\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{1} = 120$

   Case 2: The last digit is 5.

   $\underline{5} \cdot \underline{5} \cdot \underline{4} \cdot \underline{1} = 100$

   The number of four-digit numbers divisible by 5 will be:
   $120 + 100 = 220$.

5. a) We will list the possible replacements under each blank.

   Case 1: The first digit is 7 or 8.

   $\underline{2} \cdot \underline{6} \cdot \underline{6} \cdot \underline{6} = 432$

   {7, 8} {any digit}

   Case 2: The first digit is 5 and second is 5, 7, or 8.

   $\underline{1} \cdot \underline{3} \cdot \underline{6} \cdot \underline{6} = 108$

   {5} {5,7,8} {any digit}

   Case 3: The first digit is 5 and second is 3.

   $\underline{1} \cdot \underline{1} \cdot \underline{2} \cdot \underline{6} = 12$

   {5} {3} {7, 8} {any digit}

   The total number is $432 + 108 + 12 = 552$.

   b) *Case 1:* The first digit is 7 or 8.

   $\underline{2} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} = 120$

   {7, 8} {losing one digit each time}

   Case 2: The first digit is 5 and second is 7 or 8.

   $\underline{1} \cdot \underline{2} \cdot \underline{4} \cdot \underline{3} = 24$

   {5} {7, 8} {any unused digit}

   Case 3: The first digit is 5 and second is 3.

   $\underline{1} \cdot \underline{1} \cdot \underline{2} \cdot \underline{3} = 6$

   {5} {3} {7,8} {7 or 8, 1, 2}

   The total number is $120 + 24 + 6 = 150$.

6. a) $\underline{5} \cdot \underline{4} = 20$

   b) $\underline{5} \cdot \underline{5} = 25$

7. $\underline{23} \cdot \underline{22} \cdot \underline{21} = 10\,626$

8. $\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 120$

9. The first person must be a woman, the second a man, etc.

   $\underset{w}{\underline{4}} \cdot \underset{m}{\underline{3}} \cdot \underset{w}{\underline{3}} \cdot \underset{m}{\underline{2}} \cdot \underset{w}{\underline{2}} \cdot \underset{m}{\underline{1}} \cdot \underset{w}{\underline{1}} = 144$

10. The first person can be a woman or a man. After the first person they alternate.

    $\underline{8} \cdot \underline{4} \cdot \underline{3} \cdot \underline{3} \cdot \underline{2} \cdot \underline{2} \cdot \underline{1} \cdot \underline{1} = 1152$

11. Let the blanks always represent the smaller set, the letters instead of the mail boxes. There are two letters and five mail boxes.

    a) $\underline{5} \cdot \underline{4} = 20$

    b) $\underline{5} \cdot \underline{5} = 25$

PRECI2
11.1 A

1. Simplify without using a calculator.

a) $\frac{7!}{6!}$

b) $\frac{31!}{30!}$

c) $\frac{12!}{10!}$

d) $\frac{9!}{6!}$

e) $\frac{10!}{6!4!}$

f) $\frac{6!}{4!2!}$

g) $6!7$

h) $3!2!$

i) $3!4!$

j) $\frac{5!8!}{4!7!}$

2. Simplify as much as possible.

a) $\frac{(k+3)!}{(k+2)!}$

b) $\frac{(n+7)!}{(n+6)!}$

c) $\frac{n!}{(n-2)!}$

d) $\frac{7!(r+2)!r}{(r-1)!6!}$

3. Solve for n.

a) $\frac{(n+2)!}{(n+1)!}=20$

b) $\frac{n!}{(n-1)!}=10$

c) $\frac{n!}{(n-2)!}=42$

d) $\frac{(n+2)!}{8!(n-2)!}=\frac{57}{16}$ * Bonus

e) $\frac{(n+1)!}{(n-1)!}-30=0$

1. a) $\frac{7!}{6!}=\frac{7\cdot 6!}{6!}=7$

b) $\frac{31!}{30!}=\frac{31\cdot 30!}{30!}=31$

c) $12(11)=132$

d) $9(8)7=504$

e) $\frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1}=210$

f) $\frac{6\cdot 5}{2\cdot 1}=15$

g) $7!=5040$

h) $6(2)=12$

i) $6(24)=144$

j) $\frac{5!8!}{4!7!}=\frac{5\cdot 4!\,8\cdot 7!}{4!7!}=5\cdot 8=40$

2. a) $\frac{(k+3)!}{(k+2)!}=\frac{(k+3)(k+2)!}{(k+2)!}=k+3$

b) $\frac{(n+7)!}{(n+6)!}=\frac{(n+7)(n+6)!}{(n+6)}=n+7$

c) $\frac{n(n-1)(n-2)!}{(n-2)!}=n(n-1)$

d) $\frac{7!(r+2)!r}{(r-1)!6!}=\frac{7\cdot 6!(r+2)(r+1)r\cdot(r-1)!\cdot r}{(r-1)!6!}=7(r+2)(r+1)r^2$

3. a) $n+2=20$

$n=18$

b) $n=10$

c) $n(n-1)=42$

$n^2-n-42=0$

$(n-7)(n+6)=0$

$n=7$ or $n=-6$ (reject)

e) $n=5$

d) $\frac{(n+2)(n+1)\cdot n\cdot(n-1)(n-2)!}{8!(n-2)!}=\frac{57}{16}$

$(n+2)(n+1)n(n-1)=\frac{57\cdot 8\cdot 7!}{16}$

$(n+2)(n+1)n(n-1)=\frac{57\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{16}$

$(n+2)(n+1)n(n-1)=57(7)(6)(5)(4)3$

$(n+2)(n+1)n(n-1)=19(3)(6)(5)(4)(7)3$

$(n+2)(n+1)n(n-1)=18(19)(20)21$

$n=19$ (match the numbers)