1. Prove the identity.

$$\frac{\sec\theta - \cos\theta}{\tan\theta} = \sin\theta$$

2. Prove the identity.

$$\frac{\cos\theta + \sin\theta\tan\theta}{\sin\theta\sec\theta} = \csc\theta$$

3. Prove the identity.

$$\frac{1}{1+\sin\theta} = \sec^2\theta - \frac{\tan\theta}{\cos\theta}$$

4. Prove the identity.

$$\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$$

5. Prove the following identity:

$$\sin\theta + \cos\theta\cot\theta = \csc\theta$$

6. Prove the identity.

$$\frac{1-\cos\theta}{\sin^2\theta} = \frac{1}{1+\cos\theta}$$

7. Prove the identity:

$$\frac{\csc\theta}{\tan\theta + \cot\theta} = \cos\theta$$

8. Prove the following identity:

$$\frac{\cot\theta}{\csc\theta - 1} = \frac{\csc\theta + 1}{\cot\theta}$$

9. Prove the identity:

$$\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$$

10. Prove the identity:

$$\frac{1}{\sec\theta + \tan\theta} = \frac{1-\sin\theta}{\cos\theta}$$

11. Prove the identity:

$$\frac{\cot\theta - 1}{1 - \tan\theta} = \frac{\csc\theta}{\sec\theta}$$

12. Prove the identity:

$$(1-\sin\theta)(\sec\theta + \tan\theta) = \frac{1}{\sec\theta}$$

13. Prove:

$$\frac{\sin\theta\cos\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\tan\theta}$$

14. Prove:

$$\frac{\sin 2\theta}{2-2\cos^2\theta} = \cot\theta$$

15. Prove the identity:

$$\frac{\sin\theta+\tan\theta}{1+\cos\theta} \quad = \quad \frac{\sin 2\theta}{2\cos^2\theta}$$

16. Prove the identity:

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} \quad = \quad \csc\theta$$

17. Prove the identity:

$$\frac{\cos 2\theta}{\sin\theta} \quad = \quad \frac{\cot^2\theta-1}{\csc\theta}$$

KEY

1.

Left side	
$\frac{\sec\theta-\cos\theta}{\tan\theta}$	$\sin\theta$
$=\frac{\frac{1}{\cos\theta}-\cos\theta}{\frac{\sin\theta}{\cos\theta}}$	
$=\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{\cos\theta}{\sin\theta}\right)$	
$=\frac{1}{\sin\theta}-\frac{\cos^2\theta}{\sin\theta}$	
$=\frac{1-\cos^2\theta}{\sin\theta}$	
$=\frac{\sin^2\theta}{\sin\theta}$	
$=\sin\theta$	

2.

Left side	
$\frac{\cos\theta+\sin\theta\tan\theta}{\sin\theta\sec\theta}$	$\csc\theta$
$=\frac{\cos\theta+\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)}{\sin\theta\left(\frac{1}{\cos\theta}\right)}$	
$=\frac{\left(\cos\theta+\frac{\sin^2\theta}{\cos\theta}\right)\cos\theta}{\left(\frac{\sin\theta}{\cos\theta}\right)\cos\theta}$	
$=\frac{\cos^2\theta+\sin^2\theta}{\sin\theta}$	

3.

Left Side	
$\frac{1}{1+\sin\theta}$	$\sec^2\theta-\frac{\tan\theta}{\cos\theta}$
	$=\frac{1}{\cos^2\theta}-\frac{\frac{\sin\theta}{\cos\theta}}{\cos\theta}$
	$=\frac{1}{\cos^2\theta}-\frac{\sin\theta}{\cos^2\theta}$
	$=\frac{1-\sin\theta}{\cos^2\theta}$
	$=\frac{1-\sin\theta}{1-\sin^2\theta}$
	$=\frac{\cancel{(1-\sin\theta)}}{(1+\sin\theta)\cancel{(1-\sin\theta)}}$
	$=\frac{1}{1+\sin\theta}$

4.

Left side	Right side
$\tan^2\theta-\sin^2\theta$	$\tan^2\theta\sin^2\theta$
$\left[\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta\right.$	
$\left[\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta\frac{\cos^2\theta}{\cos^2\theta}\right.$	
$\left[\frac{\sin^2\theta-\sin^2\theta\cos^2\theta}{\cos^2\theta}\right.$	
$\left[\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}\right.$	
$\left[\frac{\sin^2\theta\sin^2\theta}{\cos^2\theta}\right.$	
$\tan^2\theta\sin^2\theta$	

5.

Left side	
$\sin\theta + \cos\theta\cot\theta$	$\csc\theta$
$\sin\theta + \cos\theta\dfrac{\cos\theta}{\sin\theta}$	
$\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta}$	
$\dfrac{1}{\sin\theta}$	
$\csc\theta$	

6.

Left Side	Right Side
$\dfrac{1-\cos\theta}{\sin^2\theta}$	$\dfrac{1}{1+\cos\theta}$
$= \dfrac{1-\cos\theta}{1-\cos^2\theta}$	
$= \dfrac{1-\cos\theta}{(1+\cos\theta)(1-\cos\theta)}$	
$= \dfrac{1}{1+\cos\theta}$	

7.

Left Side	
$\dfrac{\csc\theta}{\tan\theta + \cot\theta}$	$\cos\theta$
$\dfrac{\frac{1}{\sin\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$	
$\dfrac{\frac{1}{\sin\theta}}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}}$	
$\dfrac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta\cos\theta}}$	
$\frac{1}{\sin\theta} \cdot \frac{\sin\theta\cos\theta}{1}$	

8.

LEFT SIDE	RIGHT SIDE
$= \dfrac{\left(\frac{\cos\theta}{\sin\theta}\right)\sin\theta}{\left(\frac{1}{\sin\theta} - 1\right)\sin\theta}$	$= \dfrac{\left(\frac{1}{\sin\theta} + 1\right)\sin\theta}{\left(\frac{\cos\theta}{\sin\theta}\right)\sin\theta}$
$= \dfrac{\cos\theta}{1-\sin\theta}$	$= \dfrac{1+\sin\theta}{\cos\theta}\dfrac{(1-\sin\theta)}{(1-\sin\theta)}$
	$= \dfrac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}$
	$= \dfrac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$
	$= \dfrac{\cos\theta}{1-\sin\theta}$

9.

LEFT SIDE	RIGHT :
$\dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x}$ ← ½ **mark**	$\dfrac{1}{\sin^2 x} \cdot \dfrac{1}{\cos^2 x}$
$\dfrac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$	$\dfrac{1}{\sin^2 x \cos^2 x}$
$\dfrac{1}{\sin^2 x \cos^2 x}$	

10.

LEFT SIDE
$= \dfrac{(\quad 1 \quad)}{\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)}\dfrac{\cos\theta}{\cos\theta}$
$= \dfrac{\cos\theta}{(1+\sin\theta)}\dfrac{(1-\sin\theta)}{(1-\sin\theta)}$
$= \dfrac{\cos(1-\sin\theta)}{1-\sin^2\theta}$
$= \dfrac{\cos(1-\sin\theta)}{\cos^2\theta}$
$= \dfrac{1-\sin\theta}{\cos\theta}$

11.

LEFT SIDE	RIGHT
$\dfrac{\cot\theta - 1}{1 - \tan\theta}$	$\dfrac{\csc\theta}{\sec\theta}$
$= \dfrac{\frac{\cos\theta}{\sin\theta} - 1}{1 - \frac{\sin\theta}{\cos\theta}}$	$= \dfrac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}}$
$= \dfrac{\left(\frac{\cos\theta}{\sin\theta} - 1\right)\sin\theta\cos\theta}{\left(1 - \frac{\sin\theta}{\cos\theta}\right)\sin\theta\cos\theta}$	$= \dfrac{\cos\theta}{\sin\theta}$
$= \dfrac{\cos^2\theta - \sin\theta\cos\theta}{\sin\theta\cos\theta - \sin^2\theta}$	
$= \dfrac{\cos\theta(\cos\theta - \sin\theta)}{\sin\theta(\cos\theta - \sin\theta)}$	
$= \dfrac{\cos\theta}{\sin\theta}$	

12.

LEFT SIDE	
$(1-\sin\theta)(\sec\theta + \tan\theta)$	$\dfrac{1}{\sec\theta}$
$= (1-\sin\theta)\left(\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}\right)$	$= \dfrac{1}{\frac{1}{\cos\theta}}$
$= (1-\sin\theta)\left(\dfrac{1+\sin\theta}{\cos\theta}\right)$	$= \cos\theta$
$= \dfrac{1-\sin^2\theta}{\cos\theta}$	
$= \dfrac{\cos^2\theta}{\cos\theta}$	
$= \cos\theta$	

13.

LEFT SIDE	RIGHT SIDE
$\frac{\sin\theta\cos\theta}{1+\cos\theta}$	$\frac{1-\cos\theta}{\tan\theta}$
$=\frac{\sin\theta\cos\theta}{1+\cos\theta}\cdot\frac{(1-\cos\theta)}{(1-\cos\theta)}$	
$=\frac{\sin\theta\cos\theta(1-\cos\theta)}{1-\cos^2\theta}$	
$=\frac{\sin\theta\cos\theta(1-\cos\theta)}{\sin^2\theta}$	
$=\frac{\cos\theta(1-\cos\theta)}{\sin\theta}$	
$=\cot\theta(1-\cos\theta)$	
$=\frac{1-\cos\theta}{\tan\theta}$	

14.

Left side	Right side
$\frac{\sin 2\theta}{2-2\cos^2\theta}$	$\cot\theta$
$=\frac{2\sin\theta\cos\theta}{2(1-\cos^2\theta)}$	
$=\frac{2\sin\theta\cos\theta}{2\sin^2\theta}$	
$=\frac{\cos\theta}{\sin\theta}$	
$=\cot\theta$	

15.

LEFT SIDE	RIGHT SIDE
$=\frac{\sin\theta+\frac{\sin\theta}{\cos\theta}}{1+\cos\theta}$	$=\frac{2\sin\theta\cos\theta}{2\cos^2\theta}$
$=\frac{\frac{\sin\theta\cos\theta+\sin\theta}{\cos\theta}}{1+\cos\theta}$	$=\frac{\sin\theta}{\cos\theta}$
$=\frac{\sin\theta(\cos\theta+1)}{\cos\theta}\cdot\frac{1}{1+\cos\theta}$	
$=\frac{\sin\theta}{\cos\theta}$	

16.

LEFT SIDE
$\frac{2\sin\theta\cos\theta}{\cos\theta}+\frac{1-2\sin^2\theta}{\sin\theta}$
$2\sin\theta+\frac{1}{\sin\theta}-2\sin\theta$
$\frac{1}{\sin\theta}$
$\csc\theta$

17.

RIGHT SIDE
$=\frac{\frac{\cos^2\theta}{\sin^2\theta}-\frac{\sin^2\theta}{\sin^2\theta}}{\frac{1}{\sin\theta}}$
$=\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}\div\frac{1}{\sin\theta}$
$=\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}\cdot\frac{\sin\theta}{1}$
$=\frac{\cos 2\theta}{\sin\theta}$