Section 6.1 Extra Practice

1. Determine the non-permissible values of x, in radians, for each expression.

a) $\frac{\sin x}{\cos x}$

b) $\frac{\sec x}{\sin x}$

c) $\frac{\tan x}{1 - \cos x}$

d) $\frac{\cot x}{\sin x + 1}$

2. Determine the non-permissible values, in radians, for the following equation.

$$\frac{1 + \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 - \cos\theta}$$

3. Simplify each expression to one of the three primary trigonometric functions, $\sin x$, $\cos x$, or $\tan x$.

a) $\frac{\cot x}{\csc x}$

b) $\cot x \sin x$

c) $\frac{1}{\cot x \sec x}$

d) $\frac{1 - \tan x}{\cot x - 1}$

4. Verify graphically, using technology, that the expression in #3b) is equivalent to its simplified form.

5. Simplify each expression.

a) $2(\csc^2 x - \cot^2 x)$

b) $\cot^2 x (\sec^2 x - 1)$

c) $\frac{\sin^2 x}{\cos^2 x} + \sin x \csc x$

d) $\frac{\cos x}{\sin x \cot x}$

e) $\tan x \cos^2 x$

f) $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x}$

6. Use a graphing calculator to determine whether each equation might be an identity.

a) $\sin^2 x \sec^2 x = \sec^2 x - 1$

b) $\frac{1}{\sec x} + \frac{1}{\csc x} = 1$

c) $\cot x + \tan x = \csc x \cot x$

7. Simplify each expression, then rewrite the expression as one of the three reciprocal trigonometric functions, $\csc x$, $\sec x$, or $\cot x$.

a) $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x}$

b) $\cos x + \tan x \sin x$

c) $\sin x + \cos x \cot x$

8. Verify the following equation is true for $x = \frac{\pi}{6}$.

$$\sin^4 x - \cos^4 x = 2\sin^2 x - 1$$

9. Consider the following equation. $\sec x + \sec x \cos x = 1 + \sec x$.

Show that the equation is true for $x = \frac{\pi}{4}$.

10. Consider the equation $\frac{\cos x}{\sec x - 1} + \frac{\cos x}{\sec x + 1} = 2\cot^2 x$.

a) Verify the equation is true for $x = \frac{\pi}{6}$.

b) What are the non-permissible values of the equation in the domain $0° \le x < 360°$.

11. Algebraically transform the Pythagorean identity $\cos^2 x + \sin^2 x = 1$ into the equivalent identity $\cot^2 x + 1 = \csc^2 x$

KEY 6.1 Extra Practice

1. a) $x \neq \frac{\pi}{2} + \pi n;\ n \in \mathrm{I}$

b) $x \neq \pi n;\ n \in \mathrm{I}$ and $x \neq \frac{\pi}{2} + \pi n;\ n \in \mathrm{I}$

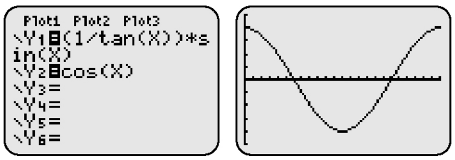
c) $x \neq 2\pi n;\ n \in \mathrm{I}$ and $x \neq \frac{\pi}{2} + \pi n;\ n \in \mathrm{I}$

d) $x \neq \pi n$ and $x \neq \frac{3\pi}{2} + 2\pi n; n \in \mathrm{I}$

2. $\theta \neq \pi n;\ n \in \mathrm{I}$

3. a) $\cos x$ **b)** $\cos x$ **c)** $\sin x$ **d)** $\tan x$

4.



5. a) 2 **b)** 1 **c)** $\sec^2 x$ **d)** 1 **e)** $\sin x \cos x$ **f)** 1

6. a) may be an identity
b) not an identity **c)** not an identity

7. a) $\cot x$ **b)** $\sec x$ **c)** $\csc x$

8.
$$\begin{aligned}
\text{Left side} &= \sin^4\left(\frac{\pi}{6}\right) - \cos^4\left(\frac{\pi}{6}\right) \\
&= \frac{1}{16} - \frac{9}{16} \\
&= -\frac{1}{2}
\end{aligned}$$
$$\begin{aligned}
\text{Right side} &= 2\sin^2\left(\frac{\pi}{6}\right) - 1 \\
&= -\frac{1}{2} \\
&= \text{Left side}
\end{aligned}$$

9.
$$\begin{aligned}
\text{Left side} &= \sec\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) \\
&= \frac{1}{\cos\left(\frac{\pi}{4}\right)} + \frac{\cos\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} \\
&= \frac{2}{\sqrt{2}} + 1
\end{aligned}$$
$$\begin{aligned}
\text{Right side} &= 1 + \sec\left(\frac{\pi}{4}\right) \\
&= 1 + \frac{1}{\cos\left(\frac{\pi}{4}\right)} \\
&= 1 + \frac{2}{\sqrt{2}} \\
&= \text{Left side}
\end{aligned}$$

10. a)
$$\begin{aligned}
\text{LS} &= \cos\left(\frac{\pi}{6}\right) \div \left(\sec\left(\frac{\pi}{6}\right) - 1\right) + \cos\left(\frac{\pi}{6}\right) \div \left(\sec\left(\frac{\pi}{6}\right) + 1\right) \\
&= \frac{\sqrt{3}}{2} \div \left(\frac{2}{\sqrt{3}} - 1\right) + \frac{\sqrt{3}}{2} \div \left(\frac{2}{\sqrt{3}} + 1\right) \\
&= \frac{\sqrt{3}}{2} \div \left(\frac{2 - \sqrt{3}}{\sqrt{3}}\right) + \frac{\sqrt{3}}{2} \div \left(\frac{2 + \sqrt{3}}{\sqrt{3}}\right) \\
&= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2 - \sqrt{3}} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2 + \sqrt{3}} \\
&= \frac{3}{4 - 2\sqrt{3}} + \frac{3}{4 + 2\sqrt{3}} \\
&= \frac{12 + 6\sqrt{3} + 12 - 6\sqrt{3}}{16 - 12} \\
&= 6
\end{aligned}$$
$$\begin{aligned}
\text{RS} &= 2\cot^2\left(\frac{\pi}{6}\right) \\
&= \frac{2}{\tan^2\left(\frac{\pi}{6}\right)} \\
&= 2 \div \frac{1}{3} \\
&= 6 \\
&= \text{LS}
\end{aligned}$$

b) $x \neq 0°, 180°$

11.
$$\begin{aligned}
\cos^2 x + \sin^2 x &= 1 \\
\frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x} &= \frac{1}{\sin^2 x} \\
\cot^2 x + 1 &= \csc^2 x
\end{aligned}$$

Section 6.2 Extra Practice

1. Write each expression as a single trigonometric function.

a) $\sin 28° \cos 35° + \cos 28° \sin 35°$
b) $\cos 10° \cos 7° - \sin 10° \sin 7°$
c) $\cos \frac{\pi}{12} \cos \frac{\pi}{4} + \sin \frac{\pi}{12} \sin \frac{\pi}{4}$
d) $\sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4}$

2. Simplify and then give an exact value for each expression.

a) $\cos 25° \cos 5° - \sin 25° \sin 5°$
b) $\sin 40° \cos 20° + \cos 40° \sin 20°$
c) $\sin \frac{\pi}{3} \cos \frac{\pi}{6} + \cos \frac{\pi}{3} \sin \frac{\pi}{6}$
d) $\cos \frac{7\pi}{12} \cos \frac{\pi}{3} + \sin \frac{7\pi}{12} \sin \frac{\pi}{3}$

3. Write each expression as a single trigonometric function.

a) $2\sin \frac{\pi}{6} \cos \frac{\pi}{6}$
b) $\cos^2 \frac{\pi}{3} - \sin^2 \frac{\pi}{3}$
c) $1 - 2\sin^2 15°$
d) $\dfrac{2\tan \frac{\pi}{6}}{1 - \tan^2 \frac{\pi}{6}}$

4. Simplify each expression using a sum identity.

a) $\sin (90° + A)$ **b)** $\cos (90° + A)$
c) $\sin (\pi + A)$ **d)** $\cos (2\pi + A)$

5. Simplify each expression using a difference identity.

a) $\sin (90° - A)$ **b)** $\sin (270° - A)$
c) $\sin \left(\frac{\pi}{2} - A\right)$ **d)** $\cos \left(\frac{3\pi}{2} - A\right)$

6. Simplify each expression to a single primary trigonometric function.

a) $\dfrac{\sin 2\theta}{2\sin \theta}$
b) $\cos 3x \cos x - \sin 3x \sin x$
c) $\dfrac{\cos 2\theta - 1}{2\sin \theta}$
d) $\dfrac{\sin^3 x}{\cos 2x - \cos^2 x}$

7. Determine the exact value of each trigonometric expression.

a) $\cos \frac{2\pi}{3}$
b) $\tan 15°$
c) $\sin 105°$
d) $\cos \frac{5\pi}{6}$

8. Determine whether each equation is true.

a) $\cos 80° = \cos 75° \cos 5° - \sin 75° \sin 5°$
b) $\cos (-24°) = \cos 16° - \cos 40°$
c) $\tan 70° = \dfrac{2\tan 30°}{1 - \tan^2 70°}$

9. If $\angle A$ and $\angle B$ are both in quadrant I, and $\sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$, evaluate each of the following.

a) $\cos (A - B)$
b) $\sin (A + B)$
c) $\cos 2B$
d) $\sin 2A$

10. If $\cos A = \frac{12}{13}$, and $\angle A$ is in quadrant IV, find the exact value of $\sin 2A$.

KEY 6.2 Extra Practice

1. a) sin 63° **b)** cos 17°

c) $\cos\left(-\frac{\pi}{6}\right)$ **d)** $\sin\left(\frac{\pi}{12}\right)$

2. a) $\frac{\sqrt{3}}{2}$ **b)** $\frac{\sqrt{3}}{2}$ **c)** 1 **d)** $\frac{1}{\sqrt{2}}$

3. a) $\sin\frac{\pi}{3}$ **b)** $\cos\frac{2\pi}{3}$

c) cos 30° **d)** $\tan\frac{\pi}{3}$

4. a) cos A **b)** –sin A **c)** –sin A **d)** cos A

5. a) cos A **b)** –cos A **c)** cos A **d)** –sin A

6. a) cos θ **b)** cos (4*x*) **c)** –sin θ **d)** –sin θ

7. a) $-\frac{1}{2}$ **b)** $2 - \sqrt{3}$ **c)** $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ **d)** $-\frac{\sqrt{3}}{2}$

8. a) true **b)** false **c)** false

9. a) $\frac{56}{65}$ **b)** $\frac{63}{65}$ **c)** $\frac{-119}{169}$ **d)** $\frac{24}{25}$

10. $-\frac{120}{169}$

Section 6.3 Extra Practice

1. Rewrite each expression in terms of sine and cosine only. Then simplify.

a) $\dfrac{\sec x}{\tan x}$

b) $\dfrac{\cot^2 x}{1 - \sin^2 x}$

c) $\dfrac{\csc x - \sin x}{\cot x}$

2. Factor and simplify each rational trigonometric expression.

a) $\dfrac{\tan x - \tan x \sin^2 x}{\cos^2 x}$

b) $\dfrac{\sin^2 x + \sin x - 6}{5\sin x + 15}$

c) $\dfrac{\cos^2 x - 4}{7\cos x - 14}$

d) $\dfrac{\sin^2 x \tan x - \tan x}{\sin x \tan x + \tan x}$

3. Use the Pythagorean identities to prove each identity for all permissible values of x.

a) $\csc^2 x(1 - \cos^2 x) = 1$

b) $(\tan x - 1)^2 = \sec^2 x - 2 \tan x$

c) $\dfrac{\sin^2 x + \cos^2 x}{\sec x} = \cos x$

4. Prove each identity. Use a common denominator to express two terms as one term, when necessary.

a) $\dfrac{1 + \tan x}{1 + \cot x} = \tan x$

b) $\dfrac{\sec x}{\sin x} - \dfrac{\sin x}{\cos x} = \cot x$

c) $\dfrac{\cot x + \tan x}{\sec x} = \csc x$

5. Prove each identity, using factoring.

a) $\dfrac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$

b) $\dfrac{\sin x + \tan x}{\cos x + 1} = \tan x$

c) $\dfrac{\cos x + 1}{\sin x + \tan x} = \cot x$

6. Verify each potential identity, then prove each identity.

a) $\dfrac{\cos x}{1 - \sin x} = \dfrac{1 + \sin x}{\cos x}$

b) $\dfrac{1 + \cos x}{\sin x} = \dfrac{\sin x}{1 - \cos x}$

c) $\dfrac{\cos x}{\sec x - 1} + \dfrac{\cos x}{\sec x + 1} = 2\cot^2 x$

7. Prove the following algebraically.

a) $\cos (x + y) \cos (x - y) = \cos^2 x - \sin^2 y$

b) $\dfrac{1 + \cos 2x}{\sin 2x} = \cot x$

c) $1 + \sin 2x = (\sin x + \cos x)^2$

d) $\sec^2 x = \dfrac{2}{1 + \cos 2x}$

8. Verify each equation is true for $x = 30°$. Then prove each equation is an identity.

a) $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

b) $\cos x + \cos x \tan^2 x = \sec x$

9. Consider the equation $\dfrac{\cos^2 x}{1 + 2\sin x - 3\sin^2 x} = \dfrac{1 + \sin x}{1 + 3\sin x}$.

a) Show that the equation is true for $x = 3.2$ radians.

b) Use a graph to show that the equation may be an identity.

10. a) Prove that $\tan\theta = \dfrac{1 - \cos 2\theta}{\sin 2\theta}$.

b) State any non-permissible values.

11. Prove the following identity.
$1 + \sin 2x = (\sin x + \cos x)^2$

12. Prove the following identity.
$\cos 3x + 1 = 4\cos^3 x - 3\cos x + 1$

KEY 6.3 Extra Practice

1. a) $\dfrac{1}{\sin x}$ **b)** $\dfrac{1}{\sin^2 x}$ **c)** $\cos x$

2. a) $\tan x$ **b)** $\dfrac{\sin x - 2}{5}$ **c)** $\dfrac{\cos x + 2}{7}$

d) $\sin x - 1$

3. a) Example:

$$\begin{aligned}\text{Left side} &= \csc^2 x\left(1 - \cos^2 x\right)\\ &= \frac{1}{\sin^2 x}(\sin^2 x)\\ &= 1\\ &= \text{Right side}\end{aligned}$$

b) Example:

$$\begin{aligned}\text{Left side} &= (\tan x - 1)^2\\ &= \tan^2 x - 2\tan x + 1\\ &= \frac{\sin^2 x - 2\sin x\cos x + \cos^2 x}{\cos^2 x}\\ \text{Right side} &= \frac{1}{\cos^2 x} - \frac{2\sin x}{\cos x}\\ &= \frac{1 - 2\sin x\cos x}{\cos^2 x}\\ &= \frac{\sin^2 x - 2\sin x\cos x + \cos^2 x}{\cos^2 x}\\ &= \text{Left side}\end{aligned}$$

c) Example:

$$\begin{aligned}\text{Right side} &= \cos x\\ \text{Left side} &= \frac{\sin^2 x + \cos^2 x}{\sec x}\\ &= 1 \div \frac{1}{\cos x}\\ &= \cos x\\ &= \text{Right side}\end{aligned}$$

4. a) Example:

$$\begin{aligned}\text{Right side} &= \tan x\\ \text{Left side} &= \frac{1 + \tan x}{1 + \cot x}\\ &= 1 + \frac{\sin x}{\cos x} \div \left(1 + \frac{\cos x}{\sin x}\right)\\ &= \frac{\cos x + \sin x}{\cos x} \times \frac{\sin x}{\sin x + \cos x}\\ &= \frac{\sin x}{\cos x}\\ &= \tan x\\ &= \text{Right side}\end{aligned}$$

b) Example:

$$\begin{aligned}\text{Right side} &= \cot x\\ \text{Left side} &= \frac{\sec x}{\sin x} - \frac{\sin x}{\cos x}\\ &= \frac{1}{\sin x\cos x} - \frac{\sin^2 x}{\sin x\cos x}\\ &= \frac{\cos^2 x}{\sin x\cos x}\\ &= \cot x\\ &= \text{Right side}\end{aligned}$$

c) Example:

$$\begin{aligned}\text{Right side} &= \csc x\\ &= \frac{1}{\sin x}\\ \text{Left side} &= \frac{\cot x + \tan x}{\sec x}\\ &= \cos x\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)\\ &= \frac{\cos^2 x + \sin^2 x}{\sin x}\\ &= \frac{1}{\sin x}\\ &= \text{Right side}\end{aligned}$$

5. a) Example:

$$\begin{aligned}\text{Left side} &= \frac{\csc x + \cot x}{\tan x + \sin x}\\ &= \frac{1 + \cos x}{\sin x} \div \frac{\sin x + \sin x\cos x}{\cos x}\\ &= \frac{1 + \cos x}{\sin x} \times \frac{\cos x}{\sin x(1 + \cos x)}\\ &= \frac{\cos x}{\sin^2 x}\\ \text{Right side} &= \cot x\csc x\\ &= \frac{\cos x}{\sin^2 x}\\ &= \text{Left side}\end{aligned}$$

b) Example:

$$\begin{aligned}\text{Right side} &= \tan x\\ \text{Left side} &= \frac{\sin x + \tan x}{\cos x + 1}\\ &= \frac{\sin x\cos x + \sin x}{\cos x} \times \frac{1}{\cos x + 1}\\ &= \frac{\sin x(\cos x + 1)}{\cos x} \times \frac{1}{\cos x + 1}\\ &= \tan x\\ &= \text{Right side}\end{aligned}$$

c) Example:

$$\begin{aligned}\text{Right side} &= \cot x\\ \text{Left side} &= \frac{\cos x + 1}{\sin x + \tan x}\\ &= (\cos x + 1) \times \frac{\cos x}{\sin x(\cos x + 1)}\\ &= \cot x\\ &= \text{Right side}\end{aligned}$$

6. a) Example:

$$\begin{aligned}\text{Right side} &= \frac{1+\sin x}{\cos x}\\ \text{Left side} &= \frac{\cos x}{1 - \sin x}\\ &= \frac{\cos x(1 + \sin x)}{(1 - \sin x)(1 + \sin x)}\\ &= \frac{\cos x(1 + \sin x)}{1 - \sin^2 x}\\ &= \frac{\cos x(1 + \sin x)}{\cos^2 x}\\ &= \frac{1 + \sin x}{\cos x}\\ &= \text{Right side}\end{aligned}$$

b) Example:

$$\begin{aligned}\text{Left side} &= \frac{1 + \cos x}{\sin x}\\ \text{Right side} &= \frac{\sin x}{1 - \cos x} \times \frac{1 + \cos x}{1 + \cos x}\\ &= \frac{\sin x(1 + \cos x)}{\sin^2 x}\\ &= \frac{1 + \cos x}{\sin x}\\ &= \text{Left side}\end{aligned}$$

c) Example:

$$\begin{aligned}\text{Right side} &= 2\cot^2 x\\ &= \frac{2\cos^2 x}{\sin^2 x}\\ \text{Left side} &= \frac{\cos x}{\sec x - 1} + \frac{\cos x}{\sec x + 1}\\ &= \cos x \div \frac{1 - \cos x}{\cos x} + \cos x \div \frac{1 + \cos x}{\cos x}\\ &= \frac{\cos^2 x}{1 - \cos x} + \frac{\cos^2 x}{1 + \cos x}\\ &= \frac{2\cos^2 x}{1 - \cos^2 x}\\ &= \frac{2\cos^2 x}{\sin^2 x}\\ &= \text{Right side}\end{aligned}$$

7. a) Example:

$$\begin{aligned}\text{Right side} &= \cos^2 x - \sin^2 y\\ \text{Left side} &= \cos(x + y)\cos(x - y)\\ &= (\cos x\cos y - \sin x\sin y)(\cos x\cos y + \sin x\sin y)\\ &= \cos^2 x\cos^2 y - \sin^2 x\sin^2 y\\ &= \cos^2 x(1 - \sin^2 y) - \sin^2 y(1 - \cos^2 x)\\ &= \cos^2 x - \sin^2 y\cos^2 x - \sin^2 y + \sin^2 y\cos^2 x\\ &= \cos^2 x - \sin^2 y\\ &= \text{Right side}\end{aligned}$$

b) Example:

$$\begin{aligned}\text{Right side} &= \cot x\\ &= \frac{\cos x}{\sin x}\\ \text{Left side} &= \frac{1 + \cos 2x}{\sin 2x}\\ &= \frac{1 + 2\cos^2 x - 1}{2\sin x\cos x}\\ &= \frac{\cos x}{\sin x}\\ &= \text{Right side}\end{aligned}$$

c) Example:

$$\begin{aligned}\text{Right side} &= (\sin x + \cos x)^2\\ &= \sin^2 x + 2\sin x\cos x + \cos^2 x\\ &= 1 + 2\sin x\cos x\\ \text{Left side} &= 1 + \sin 2x\\ &= 1 + 2\sin x\cos x\\ &= \text{Right side}\end{aligned}$$

d) Example:

$$\begin{aligned}\text{Right side} &= \frac{2}{1+\cos 2x}\\ &= \frac{2}{1+2\cos^2 x - 1}\\ &= \frac{1}{\cos^2 x}\\ \text{Left side} &= \sec^2 x\\ &= \frac{1}{\cos^2 x}\\ &= \text{Right side}\end{aligned}$$

8. a) Verify for $x = 30°$:

$$\begin{aligned}\text{Left side} &= \sec^4 30° - \sec^2 30°\\ &= \frac{16}{9} - \frac{4}{3}\\ &= \frac{4}{9}\end{aligned}$$

$$\begin{aligned}\text{Right side} &= \tan^4 30° + \tan^2 30° \\ &= \frac{1}{9} - \frac{1}{3} \\ &= \frac{4}{9} \\ &= \text{Left side}\end{aligned}$$

Example:

$$\begin{aligned}\text{Left side} &= \sec^4 x - \sec^2 x \\ &= \frac{1}{\cos^2 x}\left(\frac{1 - \cos^2 x}{\cos^2 x}\right) \\ &= \frac{\sin^2 x}{\cos^4 x}\end{aligned}$$

$$\begin{aligned}\text{Right side} &= \tan^4 x + \tan^2 x \\ &= \tan^2 x(\tan^2 x + 1) \\ &= \frac{\sin^2 x}{\cos^2 x}\left(\frac{1}{\cos^2 x}\right) \\ &= \frac{\sin^2 x}{\cos^4 x} \\ &= \text{Left side}\end{aligned}$$

b) Verify for $x = 30°$:

$$\begin{aligned}\text{Left side} &= \cos 30° + \cos 30° \tan^2 30° \\ &= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{6} \\ &= \frac{2\sqrt{3}}{3}\end{aligned}$$

$$\begin{aligned}\text{Right side} &= \sec 30° \\ &= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{2\sqrt{3}}{3} \\ &= \text{Left side}\end{aligned}$$

Example:

$$\begin{aligned}\text{Left side} &= \cos x + \cos x \tan^2 x \\ &= \frac{\cos^3 x + \cos x \sin^2 x}{\cos^2 x} \\ &= \frac{\cos x(\cos^2 x + \sin^2 x)}{\cos^2 x}\end{aligned}$$

$$\frac{1}{\cos x}$$

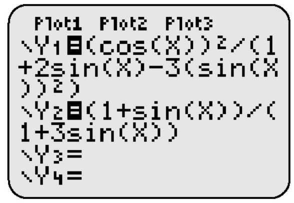
$$\begin{aligned}\text{Right side} &= \sec x \\ &= \frac{1}{\cos x} \\ &= \text{Left side}\end{aligned}$$

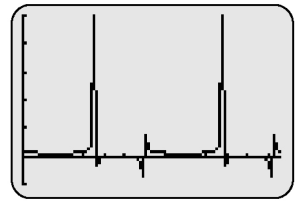
9. a) Verify for $x = 3.2$:

$$\begin{aligned}\text{Left side} &= \frac{\cos^2 3.2}{1 + 2\sin 3.2 - 3\sin^2 3.2}\cdot \\ &\approx 1.14153\end{aligned}$$

$$\begin{aligned}\text{Right side} &= \frac{1 + \sin 3.2}{1 + 3\sin 3.2} \\ &\approx 1.14153\end{aligned}$$

b)





From the graph, the equation appears to be an identity.

10. a) Example:

$$\text{Left side} = \tan\theta$$

$$\begin{aligned}\text{Right side} &= \frac{1 - \cos 2\theta}{\sin 2\theta} \\ &= \frac{1 - (1 - 2\sin^2\theta)}{2\sin\theta\cos\theta} \\ &= \frac{\sin\theta}{\cos\theta} \\ &= \tan\theta \\ &= \text{Left side}\end{aligned}$$

b) $\theta \neq \frac{n\pi}{2}; n \in \mathrm{I}$

11. Example:

$$\begin{aligned}\text{Left side} &= 1 + \sin 2x \\ &= 1 + 2\sin x\cos x\end{aligned}$$

$$\begin{aligned}\text{Right side} &= (\sin x + \cos x)^2 \\ &= \sin^2 x + 2\sin x\cos x + \cos^2 x \\ &= 1 + 2\sin x\cos x \\ &= \text{Left side}\end{aligned}$$

12. Example:

$$\begin{aligned}\text{Left side} &= \cos 3x + 1 \\ &= \cos(2x + x) + 1 \\ &= \cos 2x\cos x - \sin 2x\sin x + 1 \\ &= \cos x(2\cos^2 x - 1) - 2\sin x(\sin x\cos x) + 1 \\ &= 2\cos^3 x - \cos x - 2\sin^2 x\cos x + 1 \\ &= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x) + 1 \\ &= 4\cos^3 x - 3\cos x + 1\end{aligned}$$

$$\begin{aligned}\text{Right side} &= 4\cos^3 x - 3\cos x + 1 \\ &= \text{Left side}\end{aligned}$$

Section 6.4 Extra Practice

1. Solve each equation algebraically over the domain $0 \le x < 2\pi$.

a) $\sin 2x - \cos x = 0$
b) $\cos 2x = 0$
c) $2\cos^2 x - 1 = 0$
d) $\cos^2 x - 2 = \cos x$

2. Solve each equation algebraically over the domain $0° \le x < 360°$.

a) $\cos 2x = \cos 3x$
b) $2 \cos^2 x - 5 \sin x - 5 = 0$
c) $\cot^2 x = 0$

3. Rewrite each equation in terms of cosine only. Then, solve algebraically for $0 \le x < 2\pi$.

a) $\cos 2x - 5 \cos x = 2$
b) $\cot^2 x + 2 = 0$
c) $1 + \cos x = 2 \sin^2 x$

4. Solve $2 \cos^2 x = 1$ algebraically over the domain $-180° \le x \le 180°$.

5. Solve $\tan^2 x + 2 \tan x + 1 = 0$ algebraically over the domain $0 \le x < 2\pi$.

6. Determine the mistake the student made in the following work. Then, complete a correct solution.

$$\sin 2x = 1$$
$$\sin x = \frac{1}{2}$$
$$x = 60° \text{ and } 120°$$

7. A student is asked to write the equation of the general solution of the following equation: $\sin 2x = 1$

The solutions for this equation in the domain $0 \le x < 2\pi$ are $\frac{\pi}{4}, \frac{5\pi}{4}$.

The student writes the general solution as $\frac{\pi}{4} + 2\pi n, \frac{5\pi}{4} + 2\pi n;\ n \in \mathrm{I}$.

a) What error did the student make?
b) Write the correct general solution for this equation.

8. **a)** Explain how to solve the equation $\cos x - 2 \sin x \cos x = 0$ graphically, using the intersection feature of the graphing calculator.
b) Solve the equation from part a) algebraically over the domain $0 \le x < 2\pi$.

9. Solve $(\sin x - 1)(\tan x - 1) = 0$ algebraically for all values of x.

10. Solve the following equation for x. Give the general solution in degrees.

$2 \cos 2x + 1 = 0$

KEY 6.4 Extra Practice

1. a) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

b) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$

c) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$ **d)** π

2. a) 0°, 72°, 144°, 216°, 288° **b)** 270°

c) 90°, 270°

3. a) $\frac{2\pi}{3}, \frac{4\pi}{3}$

b) no solution

c) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

4. –135°, –45°, 45°, 135°

5. $\frac{3\pi}{4}, \frac{7\pi}{4}$

6. The error was in dividing 1 by 2.

$$\sin 2x = 1$$
$$2x = 90°$$
$$x = 45°$$

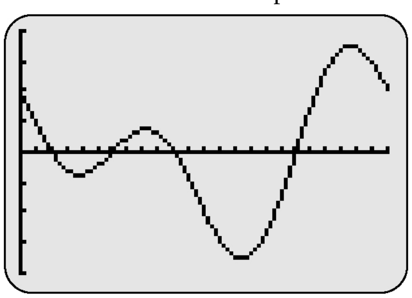
7. a) The student used 2π rather than π; because the equation is $\sin 2x = 1$, the period of the function is π.

b) $\frac{\pi}{4} + \pi n, \frac{5\pi}{4} + \pi n; n \in \mathrm{I}$

8. a) Graph the function

$Y_1 = \cos x - 2 \sin x \cos x$ using Xmin = 0 and

Xmax = 2π. The x-intercepts are the solutions.



b) $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

9. $\frac{\pi}{2} + 2\pi n, \frac{\pi}{4} + \pi n; n \in \mathrm{I}$

10. 60°, 120°, 240°, 300°

General Solution: 60° + 180°n, 120° + 180°n; $n \in \mathrm{I}$