Section 4.1 Extra Practice

1. Draw each angle in standard position. In what quadrant does each angle lie?

a) 215°

b) –70°

c) 110°

d) –300°

2. Draw each angle in standard position. Name the quadrant in which the angle lies.

a) $\frac{2\pi}{3}$

b) $\frac{3\pi}{4}$

c) $\frac{\pi}{6}$

d) π

3. Change the degree measures to radians. Give answers as both exact and approximate measures to the nearest hundredth of a unit.

a) 150°

b) 240°

c) 45°

d) 310°

4. Change the radian measures to degrees. Round to two decimal places if necessary.

a) $\frac{4\pi}{5}$

b) $\frac{5\pi}{6}$

c) $\frac{11\pi}{16}$

d) $-\frac{7\pi}{4}$

5. Change the radian measures to degrees, rounding to the nearest whole degree.

a) 3.2

b) 4

c) 6

d) –2.5

6. Determine the two subsequent positive angles that are coterminal with the given angle. Round approximate measures to the nearest hundredth of a unit.

a) 450°

b) $\frac{\pi}{5}$

c) 1.7

7. Explain how you would find the subsequent negative angle that is coterminal with each given angle.

a) 40°

b) $\frac{9\pi}{4}$

c) 0.8

8. Write an expression for all the angles that are coterminal with each given angle.

a) 75°

b) $\frac{\pi}{3}$

c) 1

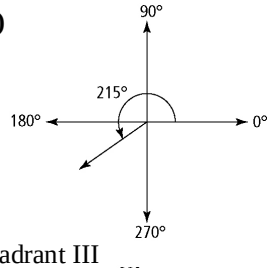
9. A circle with a radius of 16.2 cm is drawn on a large piece of cardboard. A central angle of 74° is drawn. What is the length of the arc subtended by this angle, rounded to the nearest tenth of a cm?

10. The radius of a circle is 7 cm, and the length of an arc on the circle is 10 cm. In radians, what is the central angle that subtends this arc length? Give your answer to the nearest hundredth.

PREC12

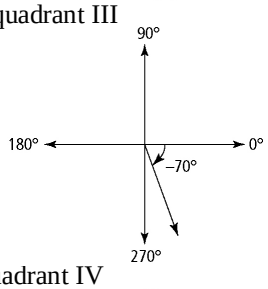
4.1 Extra Practice KEY

1. a)



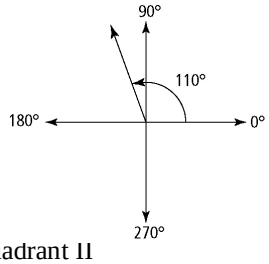
quadrant III

b)



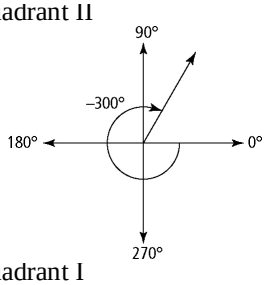
quadrant IV

c)



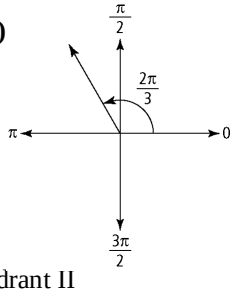
quadrant II

d)



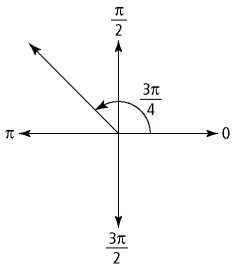
quadrant I

2. a)



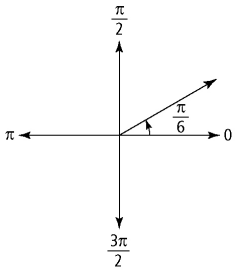
quadrant II

b)



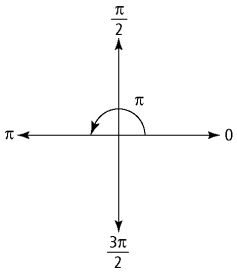
quadrant II

c)



quadrant I

d)



no quadrant

3. a) $\frac{5\pi}{6}$, 2.62 **b)** $\frac{4\pi}{3}$, 4.19

c) $\frac{\pi}{4}$, 0.79 **d)** $\frac{31\pi}{18}$, 5.41

4. a) 144° **b)** 150° **c)** 123.75° **d)** –315°

5. a) 183° **b)** 229° **c)** 344° **d)** –143°

6. a) 810°, 1170° **b)** $\frac{11\pi}{5}$, $\frac{21\pi}{5}$ **c)** 7.98, 14.27

7. a) subtract 360°

b) subtract 2π, and use fractions to determine the exact value

c) subtract 2π using your calculator, and then round your answer to the required accuracy

8. a) $75° \pm (360°)n$, where n is a natural number

b) $\left(\frac{\pi}{3} \pm 2\pi n\right)$ radians, where n is a natural number

c) $(1 \pm 2\pi n)$ radians, where n is a natural number

9. 20.9 cm

10. 1.43 radians

Section 4.2 Extra Practice

1. Write the equation of a circle with the given radius, and its centre at (0, 0).

a) 4 units **b)** $\sqrt{5}$ units

c) 9.1 units **d)** 11 units

2. Which point(s) lies on the unit circle? Explain how you know.

$\left(-\frac{5}{13}, \frac{12}{13}\right)$ $\left(\frac{5}{6}, \frac{1}{2}\right)$ $\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$

3. Each of the following points lies on the unit circle. Find the missing coordinate satisfying the given conditions.

a) $\left(-\frac{2}{3}, y\right)$ in quadrant III

b) $\left(x, \frac{4}{5}\right)$ in quadrant II

c) $\left(\frac{5}{6}, y\right)$ in quadrant IV

d) $\left(x, \frac{1}{7}\right)$ in quadrant I

4. The point P(x, y) is located where the terminal arm of angle θ and the unit circle intersect. Determine the coordinates of point P for the given angle.

a) $\theta = 45°$ **b)** $\theta = 270°$

c) $\theta = -60°$ **d)** $\theta = 210°$

5. The point P(x, y) is the point at the intersection of angle θ. If P(θ) is the point at the intersection of the terminal arm of angle θ and the unit circle, determine the exact coordinates of each.

a) $P\left(\frac{3\pi}{4}\right)$ **b)** $P\left(-\frac{2\pi}{3}\right)$

c) $P(2\pi)$ **d)** $P\left(\frac{11\pi}{6}\right)$

6. Identify a measure for θ in the interval $0 \le \theta < 360°$ such that P(θ) is the given point.

a) $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ **b)** (–1, 0)

c) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ **d)** $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

7. Identify a measure for θ in the interval $0 \le \theta < 2\pi$ such that P(θ) is the given point.

a) (1, 0) **b)** $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

c) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **d)** (–1, 0)

8. On a diagram of the unit circle, show all the integral multiples of $\frac{\pi}{4}$ in the interval $0 \le \theta < \pi$. On your diagram, label the coordinates for each point P(θ).

9. Consider a point where $P(\theta) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

a) Determine the coordinates of $P\left(\theta + \frac{\pi}{2}\right)$.

b) Determine the coordinates of $P\left(\theta - \frac{\pi}{2}\right)$.

10. If $P(\theta) = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ determine the following.

a) the coordinates of $P\left(\theta + \frac{\pi}{2}\right)$

b) the coordinates of $P\left(\theta - \frac{\pi}{2}\right)$

4.2 Extra Practice KEY

1. a) $x^2 + y^2 = 16$ **b)** $x^2 + y^2 = 5$
c) $x^2 + y^2 = 82.81$ **d)** $x^2 + y^2 = 121$

2. $\left(-\frac{5}{13}, \frac{12}{13}\right)$ and $\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$; When the coordinates are substituted into $x^2 + y^2 = 1$, the LHS equals the RHS.

3. a) $\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$ **b)** $\left(-\frac{3}{5}, \frac{4}{5}\right)$

c) $\left(\frac{5}{6}, -\frac{\sqrt{11}}{6}\right)$ **d)** $\left(\frac{4\sqrt{3}}{7}, \frac{1}{7}\right)$

4. a) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ **b)** $(0, -1)$

c) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ **d)** $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

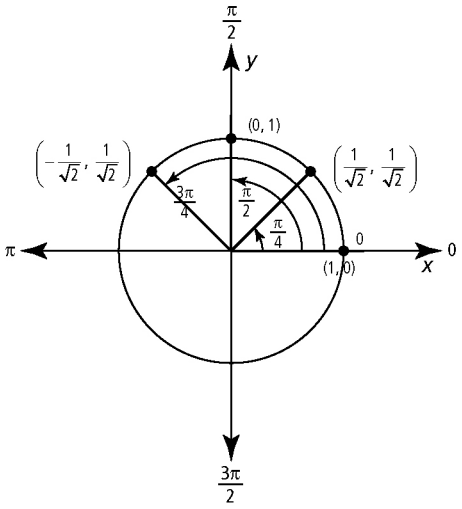
5. a) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ **b)** $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

c) $(1, 0)$ **d)** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

6. a) 225° **b)** 180° **c)** 315° **d)** 240°

7. a) 0π **b)** $\frac{5\pi}{3}$ **c)** $\frac{5\pi}{6}$ **d)** π

8.



9. a) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ **b)** $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

10. a) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ **b)** $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Section 4.3 Extra Practice

1. What is the exact value of each trigonometric ratio?

a) $\sin 30°$ **b)** $\cos 240°$ **c)** $\tan 315°$

d) $\sin 270°$ **e)** $\csc 60°$ **f)** $\sec 180°$

2. Determine the exact value of each of the following.

a) $\cot \frac{\pi}{3}$ **b)** $\sin \frac{7\pi}{6}$ **c)** $\sec \pi$

d) $\cos \pi$ **e)** $\tan \frac{\pi}{3}$ **f)** $\csc \frac{3\pi}{4}$

3. Determine the approximate value for each trigonometric ratio to the nearest hundredth of a unit.

a) $\sin 40°$ **b)** $\cos 215°$

c) $\cot 337°$ **d)** $\tan (-50°)$

4. Determine the approximate value for each. Give answers to the nearest hundredth of a unit.

a) $\sec 2.5$ **b)** $\tan\left(-\frac{\pi}{5}\right)$

c) $\csc \frac{3\pi}{7}$ **d)** $\sin 0.75$

5. In which quadrant will θ terminate if angle θ is in standard position with the given conditions?

a) $\cos \theta < 0$

b) $\sin \theta > 0$

c) $\cot \theta > 0$

d) $\cos \theta > 0$ and $\cot \theta < 0$

e) $\sin \theta < 0$ and $\sec \theta > 0$

f) $\sec \theta < 0$ and $\tan \theta < 0$

6. Express each quantity as the same trigonometric ratio using its reference angle. For example, $\cos 160° = -\cos 20°$.

a) $\sin 230°$ **b)** $\cos 310°$ **c)** $\tan 100°$

d) $\csc 260°$ **e)** $\cot 200°$ **f)** $\sec 290°$

7. Determine the exact measure of all angles that satisfy the given conditions.

a) $\tan \theta = -1$, domain $0° \le \theta < 360°$

b) $\cos\theta = \frac{\sqrt{3}}{2}$, domain $-180° \le \theta < 180°$

c) $\csc \theta = 2$, domain $-180° \le \theta < 90°$

d) $\sin \theta = 1$, domain $-360° \le \theta < 360°$

8. Determine the exact measure of each angle.

a) $\sin\theta = \frac{\sqrt{3}}{2}$, domain $0 \le \theta < 2\pi$

b) $\sec \theta = -1$, domain $-\pi \le \theta < 2\pi$

c) $\cos\theta = -\frac{1}{2}$, domain $0 \le \theta < 2\pi$

d) $\cot \theta = -1$, domain $-\pi \le \theta < 2\pi$

9. Determine the approximate measure of each angle. Use diagrams to show the number of possible solutions and the quadrants in which they lie. Then, give answers to the nearest hundredth of a unit, where possible.

a) $\sin \theta = 0.42$, domain $-\pi \le \theta \le \pi$

b) $\cot \theta = -4.87$, domain $-\frac{\pi}{2} \le \theta \le \pi$

c) $\sec \theta = 4.87$, domain $-360° \le \theta < 180°$

d) $\tan \theta = 1.5$, domain $-180° \le \theta < 360°$

10. The point D(5, –12) lies on the terminal arm of an angle θ in standard position. What is the exact value of each trigonometric ratio for θ?

PREC12

4.3 Extra Practice KEY

1. a) $\frac{1}{2}$ **b)** $-\frac{1}{2}$ **c)** -1 **d)** -1 **e)** $\frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$ **f)** -1

2. a) $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ **b)** $-\frac{1}{2}$ **c)** -1 **d)** -1

e) $\sqrt{3}$ **f)** $\sqrt{2}$

3. a) 0.64 **b)** –0.82 **c)** –2.36 **d)** –1.19

4. a) –1.25 **b)** –0.73 **c)** 1.03 **d)** 0.68

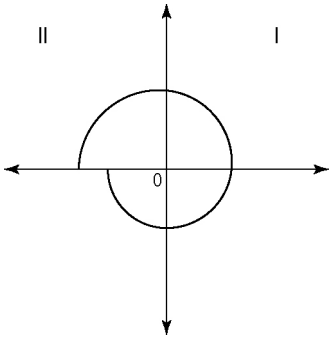
5. a) II or III **b)** I or II **c)** I or III
d) IV **e)** IV **f)** II

6. a) –sin 50° **b)** cos 50° **c)** –tan 80°
d) –csc 80° **e)** cot 20° **f)** sec 70°

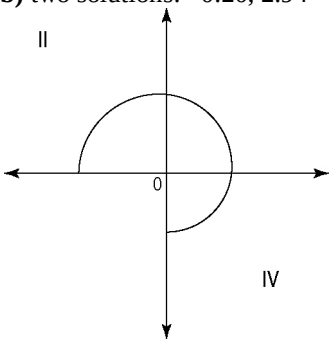
7. a) 135°, 315° **b)** –30°, 30°
c) 30° **d)** –270°, 90°

8. a) $\frac{\pi}{3}, \frac{2\pi}{3}$ **b)** $-\pi, \pi$ **c)** $\frac{2\pi}{3}, \frac{4\pi}{3}$ **d)** $-\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$

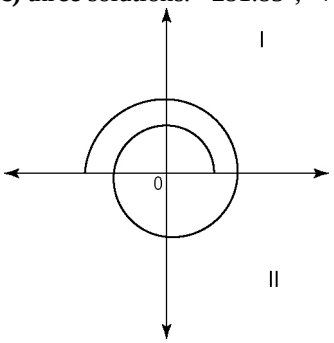
9. a) two solutions; 0.43, 2.71



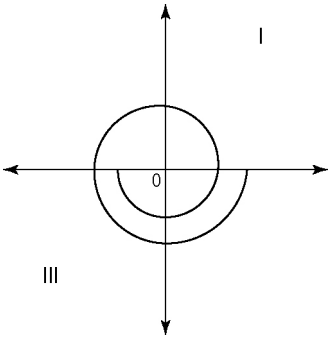
b) two solutions: –0.20, 2.94



c) three solutions: –281.85°, –78.15°, 78.15°



d) three solutions: –123.69°, 56.31°, 236.31°



10. $\sin\theta = -\frac{12}{13}$ $\csc\theta = -\frac{13}{12}$

$\cos\theta = \frac{5}{13}$ $\sec\theta = \frac{13}{5}$

$\tan\theta = -\frac{12}{5}$ $\cot\theta = -\frac{5}{12}$

PREC12

Section 4.4 Extra Practice

1. Solve for θ, where $0 \le \theta \le 360°$.

a) $\cos\theta - 0.5 = 0$

b) $\tan\theta + \sqrt{3} = 0$

c) $2\sin\theta + 1 = 2$

d) $\sec\theta = -2$

2. Solve for x, where $0 \le x \le 2\pi$.

a) $\cos^2 x - 0.25 = 0$

b) $4\sin^2 x - 3 = 0$

c) $(\sin x - 1)(\tan x - 1) = 0$

d) $2\cos^2 x - 5\cos x + 2 = 0$

3. Determine the exact roots for each trigonometric equation in the specified domain.

a) $\sin^2 x + \sin x - 2 = 0$, $-180° \le x < 180°$

b) $2\cos^2 x - 3\cos x + 1 = 0$, $0 \le x < 2\pi$

c) $\cos x - 2\sin x \cos x = 0$, $-\pi \le x < \pi$

4. Solve each equation for $0 \le \theta < 2\pi$. Give solutions to the nearest hundredth of a radian.

a) $\tan\theta = 4.36$

b) $\cos\theta = -0.19$

c) $\sin\theta = 0.91$

d) $\cot\theta = 12.3$

5. Verify that $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ are solutions to the equation $\sin^2\theta - 1 = 0$.

6. Does $\cos\theta = 2$ have a solution? Explain.

7. Solve each equation for $0 \le x \le 2\pi$, rounding solutions to four decimal places.

a) $5\tan^2 x + 2\tan x - 7 = 0$

b) $\tan^2 x - 5\tan x + 6 = 0$

c) $\tan^2 x - 4\tan x = 0$

8. The solution to $\cos\theta = 1$ in the domain $0 \le \theta < 2\pi$ is $\theta = 0$. Write the general solution for the equation in which the domain is real numbers.

9. Write the general solution for the equation $\sin x(\sin x + 1) = 0$.

10. Write the general solution for the equation $4\sin^2 x + 2\sin x - 2 = 0$.

BLM 4–6 Section 4.4 Extra Practice

1. a) 60°, 300° **b)** 120°, 300°

c) 30°, 150° **d)** 120°, 240°

2. a) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **b)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

c) $\frac{\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4}$ **d)** $\frac{\pi}{3}, \frac{5\pi}{3}$

3. a) $\frac{\pi}{2}$ **b)** $0, \frac{\pi}{3}, \frac{5\pi}{3}$ **c)** $-\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

4. a) 1.35, 4.49 **b)** 1.76, 4.90

c) 1.14, 2.00 **d)** 0.08, 3.22

5.

LS	RS
$\sin^2\theta - 1$	0
$= \left(\sin\frac{\pi}{2}\right)^2 - 1$	
$= (1)^2 - 1$	
$= 0$	

LS	RS
$\sin^2\theta - 1$	0
$= \left(\sin\frac{3\pi}{2}\right)^2 - 1$	
$= (-1)^2 - 1$	
$= 0$	

6. No. Example: The range of the cosine function is [–1, 1]. Cosine is undefined for values that are outside of this range.

7. a) 0.7854, 2.1910, 3.9270, 5.3326

b) 1.1071, 1.240, 4.2487, 4.3906

c) 0, 1.3258, 4.4674

8. $2\pi n,\ n \in \mathrm{I}$

9. $x = \pi n, -\frac{\pi}{2} + 2\pi n$

10. $(1 + 4n)\frac{\pi}{6},\ n \in \mathrm{I}$