PREC12
Quiz6.4(S)

Name KEY

Find the exact roots of each equation for $0 \le x < 2\pi$:

1) $2\cos x + 1 = \sec x$

$\cos x\left(2\cos x + 1 = \frac{1}{\cos x}\right)\cos x$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = 1/2 \quad \cos x = -1$

$x = \pi/3, 5\pi/3 \quad x = \pi$

2) $\sin x + \sqrt{3}\cos x = 0$

$\sin x = -\sqrt{3}\cos x$

$\frac{\sin x}{\cos x} = -\sqrt{3}$

$\tan x = -\sqrt{3}$

$x = 2\pi/3, 5\pi/3$

3) $2\cos^2 x = 3\sin x$

$2(1 - \sin^2 x) = 3\sin x$

$2 - 2\sin^2 x = 3\sin x$

$0 = 2\sin^2 x + 3\sin x - 2$

$0 = (2\sin x - 1)(\sin x + 2)$

$\sin x = 1/2 \quad \sin x = -2$ reject

$x = \pi/6, 5\pi/6$

Find the exact general radian solutions of each equation:

4) $\cos 2x = 1 - 2\sin x$

$1 - 2\sin^2 x = 1 - 2\sin x$

$0 = 2\sin^2 x - 2\sin x$

$0 = 2\sin x(\sin x - 1)$

$\sin x = 0 \quad \sin x = 1$

$x = \pi n$

$x = \pi/2 + 2\pi n \quad ; n \in I$

5) $\sqrt{2}\sin 2x - 2\sin x = 0$

$2\sqrt{2}\sin x\cos x - 2\sin x = 0$

$2\sin x(\sqrt{2}\cos x - 1) = 0$

$\sin x = 0 \quad \cos x = 1/\sqrt{2}$

$x = \pi n$

$x = \pi/4 + 2\pi n \quad ; n \in I$

$x = 7\pi/4 + 2\pi n$

6) $3\sin^2 x - 3\cos^2 x + 1 = \sin^2 2x$

$3(\sin^2 x - \cos^2 x) + 1 = (\sin 2x)^2$

$3(2\sin^2 x - 1) + 1 = (2\sin x \cos x)^2$

$6\sin^2 x - 3 + 1 = 4\sin^2 x \cos^2 x$

$\cdot\ 6\sin^2 x - 2 = 4\sin^2 x(1 - \sin^2 x)$

$6\sin^2 x - 2 = 4\sin^2 x - 4\sin^4 x$

$4\sin^4 x + 2\sin^2 x - 2 = 0$

$2\sin^4 x + \sin^2 x - 1 = 0$

$(2\sin^2 x - 1)(\sin^2 x + 1) = 0$

$\sin^2 x = 1/2 \qquad \sin^2 x = -1$

$\sin x = \pm 1/\sqrt{2}$

$x = \pi/4 + \pi n/2,\ n \in I$

OR

$3(1 - \cos^2 x) - 3\cos^2 x + 1 = \_\_$

$3 - 3\cos^2 x - 3\cos^2 x + 1 = \_\_$

$* \ 4 - 6\cos^2 x = \_\_$

OR

$3\sin^2 x - 3(1 - \sin^2 x) + 1 = \_\_$

$3\sin^2 x - 3 + 3\sin^2 x + 1 = \_\_$

$\cdot\ 6\sin^2 x - 2 = \_\_$

OR

$3(1 - 2\cos^2 x) + 1 = 4\sin^2 x \cos^2 x$

$3 - 6\cos^2 x + 1 = 4(1 - \cos^2 x)\cos^2 x$

$* \ 4 - 6\cos^2 x = 4\cos^2 x - 4\cos^4 x$

$4\cos^4 x - 10\cos^2 x + 4 = 0$

$2\cos^4 x - 5\cos^2 x + 2 = 0$

$(2\cos^2 x - 1)(\cos^2 x - 2) = 0$

$\cos^2 x = 1/2 \qquad \cos^2 x = 2$

$\cos x = \pm 1/\sqrt{2}$

$x = \pi/4 + \pi n/2,\ n \in I$