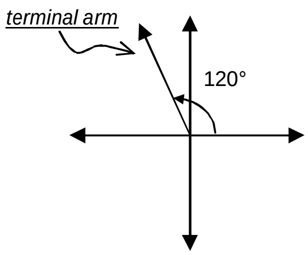
<u>Standard Position Trigonometry Review</u>

<u>Standard position angle</u> – vertex at the origin and initial arm on the positive *x*-axis.

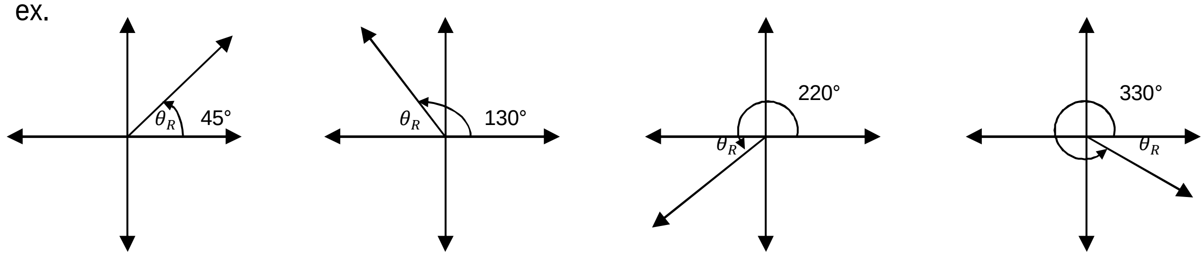
ex.



Counter clockwise rotation: positive angles

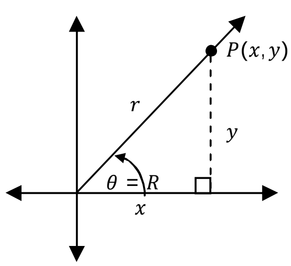
<u>Reference angle</u> (θ_R) – acute <u>positive</u> angle between the terminal arm and the <u>*x*-axis</u>.

ex.



QI: $\theta_R = \theta$	QII: $\theta_R = 180° - \theta$	QIII: $\theta_R = \theta - 180°$	QIV: $\theta_R = 360° - \theta$
$\theta_R = 45°$	$\theta_R = 50°$	$\theta_R = 40°$	$\theta_R = 30°$

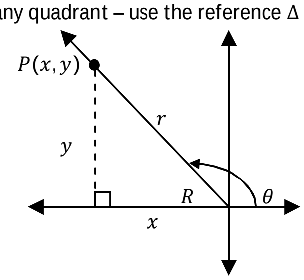
To determine trigonometric ratio values (sine, cosine, tangent) for standard position angle θ we use a point $P(x, y)$ on the terminal arm and the right triangle containing the reference angle (reference triangle).



$r = radius$

$r = \sqrt{x^2 + y^2}$

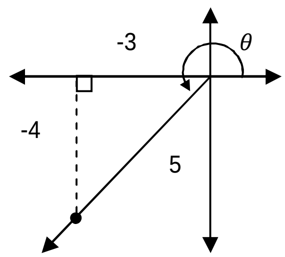
Can be done in any quadrant – use the reference Δ to state the ratios



$$\therefore \quad \boxed{sin\theta = \frac{y}{r} \qquad cos\theta = \frac{x}{r} \qquad tan\theta = \frac{y}{x}}$$

ex. The point $(-3, -4)$ lies on the terminal arm of θ in standard position. Determine the exact values (not decimal) of $sin\theta$, $cos\theta$, and $tan\theta$. Note: **NOT** finding θ!

Draw a sketch (not to scale):



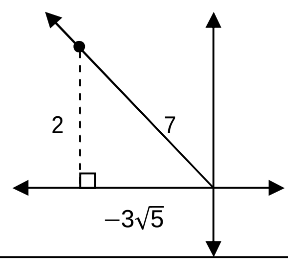
*label sides as negative when labelling sides on the negative axis but r is <u>always</u> positive

$r = \sqrt{(-3)^2 + (-4)^2}$
$= \sqrt{9 + 16}$
$= \sqrt{25} = 5$

$\therefore$ $\boxed{sin\theta = \frac{-4}{5} \qquad cos\theta = \frac{-3}{5} \qquad tan\theta = \frac{4}{3}}$

ex. If θ is in quadrant II and $sin\theta = \frac{2}{7}$ determine the exact values of $cos\theta$, and $tan\theta$.

Draw a sketch (not to scale):



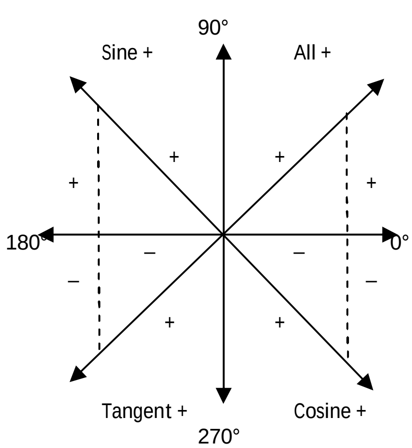
since $sin\theta = \frac{2}{7}$ we know that $y = 2$ and $r = 7$

$x = -\sqrt{(7)^2 - (2)^2}$ *will be negative due to its position
$= -\sqrt{45}$
$= -\sqrt{9}\sqrt{5} = -3\sqrt{5}$

$\therefore$ $\boxed{cos\theta = \frac{-3\sqrt{5}}{7} \qquad tan\theta = \frac{-2}{3\sqrt{5}}}$ *don't place negatives in the denominator

Due to the positive and negative values that occur in the reference triangles, trig ratio values can be positive or negative depending on what quadrant you are in.

Identify which ratios are positive in which quadrants: (consider all 4 possible reference triangles, one per quadrant)



<u>A</u>ll **<u>S</u>**oup **<u>T</u>**urns **<u>C</u>**old or **<u>A</u>**ll **<u>S</u>**tudents **<u>T</u>**ake **<u>C</u>**alculus

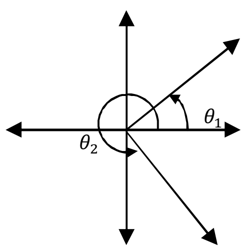
Solving for angles in standard position.

ex. Solve to the nearest degree: $\cos\theta = 0.3156$; $\underbrace{0 \le \theta < 360°}$

*common restriction – without restriction there would be infinite solutions

since cosine is positive the solutions are in quadrants I and IV (recall All Soup Turns Cold)

sketch to remind us where the solutions are help us find them



The two solutions share the same reference angle.

Find θ_R: $\theta_R = \cos^{-1} 0.3156$
$\theta_R = 72°$

$\therefore\ \theta_1 = 72°$ $\qquad \theta_2 = 360 - 72 = 288°$

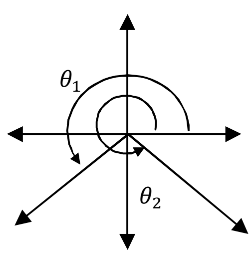
$\boxed{\theta = 72° \text{ or } 288°}$

In general:

QI $\quad \theta = \theta_R$
QII $\quad \theta = 180° - \theta_R$
QIII $\quad \theta = 180° + \theta_R$
QIV $\quad \theta = 360° - \theta_R$

ex. solve to the nearest tenth: $\sin\theta = -0.8971$; $0 \le \theta < 360°$

↑
sine negative
QIII & IV



$\theta_R = \sin^{-1} 0.8971$

↑
do **<u>NOT</u>** put any negatives in when identifying the reference angle!

$\theta_R = 63.8°$

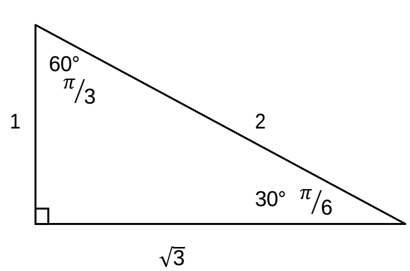
$\therefore\ \theta_1 = 180 + 63.8$ $\qquad \theta_2 = 360 - 63.8$

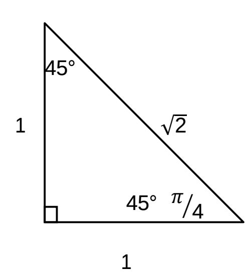
$\boxed{\theta = 243.8° \text{ or } 296.2°}$

Special triangles (for exact value ratios):

$30° - 60° - 90°$ $\qquad$ $45° - 45° - 90°$

$\pi/6 - \pi/3 - \pi/2$ $\qquad$ $\pi/4 - \pi/4 - \pi/2$



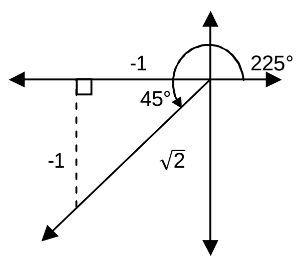


These triangles allow us to produce exact values for all the trig ratios involving30°, 60° or 45°:

	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Can also state the exact trig ratios of angles whose reference angle is one of these special angles.

ex. Determine $\cos 225°$



Reference: $\theta_R = 225 - 180 = 45°$

$\therefore \boxed{\cos 225° = \frac{-1}{\sqrt{2}}}$

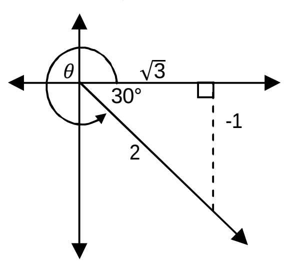
OR

cosine – in QIII

$\cos 225° = -\cos 45° = \boxed{-\frac{1}{\sqrt{2}}}$

ex. If $\sin\theta = -\frac{1}{2}$ and $\sec\theta = \frac{2}{\sqrt{3}}$ determine θ.

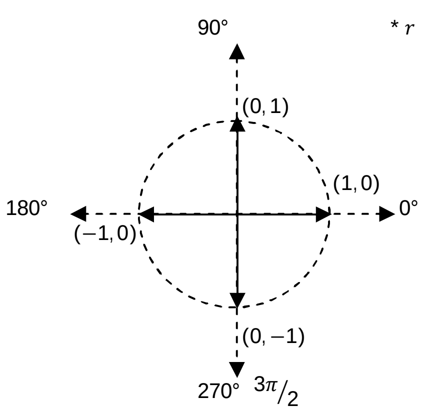
Quads III & <u>IV</u> Quads I & <u>IV</u> $\therefore$ θ is in Quad IV



$\theta_R = 30°$ $\therefore \theta = 360 - 30°$

$= \boxed{330°}$

We can also determine exact values for standard position angles whose terminal arms lie on the axis (ie. 0°, 90°, 180°, 270°, 360°, etc ...) by using a ***unit circle*** (terminal arm length $r = 1$).



* $r = 1$

recall: $sin\theta = \frac{y}{r}$ $cos\theta = \frac{x}{r}$ $tan\theta = \frac{y}{x}$

	0°	90°	180°	270°
$\sin\theta$	$\frac{0}{1} = 0$	$\frac{1}{1} = 1$	$\frac{0}{1} = 0$	$\frac{-1}{1} = -1$
$\cos\theta$	$\frac{1}{1} = 1$	$\frac{0}{1} = 0$	$\frac{-1}{1} = -1$	$\frac{0}{1} = 0$
$\tan\theta$	$\frac{0}{1} = 0$	$\frac{1}{0} = \infty$	$\frac{0}{-1} = 0$	$\frac{-1}{0} = \infty$

Note: ∞ = unde ined