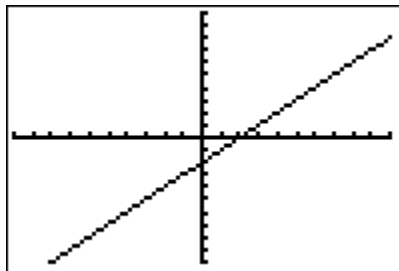


Unit 9: Rational Functions

9.2 Analysing Rational Functions

Graph the function $f(x) = \frac{x^2-5x+6}{x-3}$ with your graphing calculator & sketch the result. Is the graph what you expected? Explain.



Why no asymptote?

What about the non-permissible $x \neq 3$?

Simplify $f(x) = \frac{x^2-5x+6}{x-3}$ and use the simplified form to graph $f(x)$ by hand:

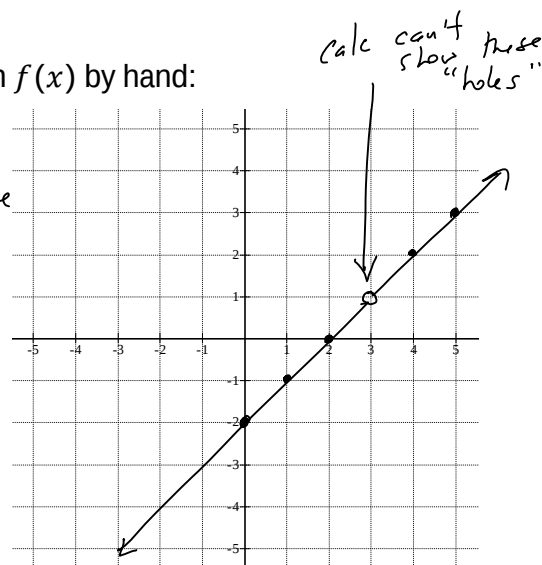
$$= \frac{(x-2)(x-3)}{x-3}$$

$$g(x) = \underline{\underline{x-2}}$$

$$f(3) = \text{undefined}$$

$$\underline{\underline{g(3) = 1}}$$

$f(x)$ & $g(x)$
are not the same
but they agree
at all but one
point (3,1)



A vertical asymptote of the graph of a corresponds to a non-permissible value in the equation of the function, but not all non-permissible values result in vertical asymptotes. Sometimes a non-permissible value instead results in a **point of discontinuity** in the graph.

Point of Discontinuity:

- a point at which the graph of a function is not continuous, a "hole" in the graph represented by an open circle
- occurs at non-permissible value(s) whose causing denominator factor can be cancelled with the numerator

Ex. Compare the functions $f(x) = \frac{x^2-2x}{4-2x}$ and $g(x) = \frac{x^2+2x}{4-2x}$. Explain any differences & verify by graphing with your calculator.

• $f(x)$ & $g(x)$ both have NPV $x \neq 2$

$$f(x) = \frac{x(x-2)}{2(2-x)}$$

$$g(x) = \frac{x(x+2)}{2(2-x)} \quad \left. \vphantom{\frac{x(x+2)}{2(2-x)}} \right\} \text{simplified}$$

$$= \frac{-x}{2}$$

Point of discontinuity at $(2, -1)$

$$\uparrow$$

$$-\frac{(2)}{2}$$

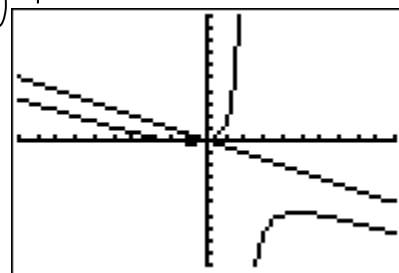
$x=2$ is a vertical asymptote

$$f(2) = \frac{0}{0} \quad g(2) = \frac{8}{0}$$

is indeterminate is undefined

↓ ↓

point of discontinuity vertical asymptote



Ex. Match the equation of each rational function with the most appropriate graph. Give reasons for each choice.

$$A(x) = \frac{x^2 + 2x}{x^2 - 4}$$

$$= \frac{x(x+2)}{(x-2)(x+2)}$$

vertical asymptote $x=2$

point of discontinuity at $(-2, \frac{1}{2})$

∴ A(x) Graph 2

$$B(x) = \frac{2x + 4}{x^2 + 1}$$

$$x^2 + 1 \neq 0$$

There are no NPVs

∴ Graph 3

$$C(x) = \frac{2x}{x^2 - 4}$$

$$= \frac{2x}{(x-2)(x+2)}$$

vertical asymptotes at $x=2$ and $x=-2$

∴ Graph 1

