PMA12 Date:________________

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# *Unit 8: Logarithmic Functions*
## *8.4 Logarithms & Exponential Equations*

**Part I: Use logarithms to solve for exponents**

**If $L = R$ then $\log_c L = \log_c R$**

**Ex.** Solve to the nearest hundredth:

a) $2^x = 5$

$\log 2^x = \log 5$ ← base 10 for calc.

$x \log 2 = \log 5$

$x = \dfrac{\log 5}{\log 2}$ ← no calc until end.

$\boxed{x = 2.32}$

b) $8(3^{2x}) = 568$

$3^{2x} = 71$

$\log 3^{2x} = \log 71$

$2x \log 3 = \log 71$

$x = \dfrac{\log 71}{2 \log 3}$ or $x = \dfrac{\log 71}{\log 9}$

$\boxed{x = 1.94}$

c) $3^{x+2} = 5^{2x-3}$

$\log 3^{x+2} = \log 5^{2x-3}$

$(x+2)\log 3 = (2x-3)\log 5$ Brackets!

$x\log 3 + 2\log 3 = 2x\log 5 - 3\log 5$ Distribute

$x\log 3 - 2x\log 5 = -3\log 5 - 2\log 3$ Gather $x$ terms

$x(\log 3 - 2\log 5) = -3\log 5 - 2\log 3$

$x = \dfrac{(-3\log 5 - 2\log 3)}{(\log 3 - 2\log 5)}$

$\boxed{x = 3.31}$

**Ex.** A city's population doubles every 8 years. How long, to the nearest year, will it take for the population to grow from 25,000 to 1 million people?

$F = Ir^{t/n}$

$1\,000\,000 = 25000(2)^{t/8}$

$40 = 2^{t/8}$

$\log 40 = t/8 \log 2$

$t = 8\log 40 / \log 2 = \boxed{43 \text{ years}}$

also remember

$*\ F = I(1 \pm r)^n$

$4^{2x} = 8^{x+1}$

$\log_2 4^{2x} = \log_2 8^{x+1}$

$2x \log_2 4 = (x+1)\log_2 8$

$2x(2) = (x+1)\,3$

$4x = 3x + 3$

$\boxed{x = 3}$

**Ex.** Evaluate to the nearest hundredth:

$\log_5 782 = x$

$\therefore\ 5^x = 782$

$x \log 5 = \log 782$

$x = \dfrac{\log 782}{\log 5}$

$\boxed{x = 4.14}$

$\log_c a = x$

$c^x = a$

$\log_b c^x = \log_b a$

$x \log_b c = \log_b a$

$x = \dfrac{\log_b a}{\log_b c}$

The previous example demonstrates the Change of Base Rule:

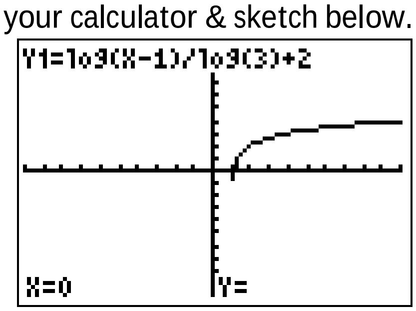
$$\boxed{\log_c a = \frac{\log_x a}{\log_x c}}$$

$x$ is a base of your choosing

*also use to graph non base 10 logs with calc

**Ex.** Graph $y = \log_3(x-1) + 2$ with your calculator & sketch below.

$Y_1 = \dfrac{\log(x-1)}{\log 3} + 2$



**Ex.** Evaluate to the nearest tenth:

$\log_3 16$

$= \dfrac{\log 16}{\log 3}$

$= \boxed{2.5}$

**Ex.** Evaluate without a calculator:

$\log_4 32$

$= \dfrac{\log_2 32}{\log_2 4}$

$= \boxed{\dfrac{5}{2}}$

## Part II: Solving logarithmic equations

$\sqrt{x} = \sqrt{y}$

$x = y$

~~$\sqrt{x} + \sqrt{y} = \sqrt{z}$~~

**Case I** Equations with all log terms If $\log_c x = \log_c y$ then $x = y$

single! log statement on each side

**Ex.** Solve:

a) $\log_3(3x + 1) = \log_3 2 + \log_3 8$

$\log_3(3x+1) = \log_3 16$ $\qquad 3x+1>0$

$x > -\frac{1}{3}$

$\therefore\ 3x+1 = 16$

$3x = 15$

$\boxed{x = 5}$ ✓

b) $\log x = \log 4 - \log(x - 3)$ $\qquad x>0 \qquad \boxed{x>3}$

$\log x = \log\left(\frac{4}{x-3}\right)$

$\log x + \log(x-3) = \log 4$

$\log(x^2 - 3x) = \log 4$

$\therefore\ \frac{x}{1} = \frac{4}{x-3}$

$x^2 - 3x = 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\boxed{x = 4}$, $\cancel{-1}$

**Case II** Equations with log terms and constant terms If [illegible]

* gather + collapse all log terms to one side

**Ex.** Solve:

a) $\log_8(x + 3) - 1 = -\log_8(x + 1)$ $\qquad x>-3 \qquad \boxed{x>-1}$

$\log_8(x+3) + \log_8(x+1) = 1$

$\log_8[(x+3)(x+1)] = 1$ $\qquad$ OR $\log_8 8$

$\log_8(x+3)(x+1) = \log_8 8$

$8^1 = x^2 + 4x + 3$

$0 = x^2 + 4x - 5$

$0 = (x+5)(x-1)$

$x = \cancel{5}, \boxed{1}$

b) $\log_2(3 - x) + \log_2(1 - x) = 3$

$\log_2(3 - 4x + x^2) = 3$

$2^3 = 3 - 4x + x^2$

$8 = 3 - 4x + x^2$

$0 = x^2 - 4x - 5$

$0 = (x-5)(x+1)$

$x = \cancel{5}, \boxed{-1}$

Practice: pg. 412/ # 1 – 13, 15 – 18, 20 – 22