Unit 8: Logarithmic Functions

8.3 Laws of Logarithms

Since logarithms are exponents, the laws of logarithms are related to the laws of powers.

Let $M = c^x$ and $N = c^y$:

$MN = c^x \cdot c^y$

$MN = c^{x+y}$

$\log_c MN = x + y$

$\log_c MN = \log_c M + \log_c N$

$* \log_c M = x$

$\log_c N = y$

$\frac{M}{N} = \frac{c^x}{c^y}$

$\frac{M}{N} = c^{x-y}$

$\log_c \frac{M}{N} = x - y$

$\log_c \frac{M}{N} = \log_c M - \log_c N$

$M^P = (c^x)^P$

$M^P = c^{Px}$

$\log_c M^P = Px$

$\log_c M^P = P \log_c M$

Product Law of Logarithms	Quotient Law of Logarithms	Power Law of Logarithms
$\log_c MN = \log_c M + \log_c N$	$\log_c \frac{M}{N} = \log_c M - \log_c N$	$\log_c M^P = P \log_c M$

Ex. Write each expression in terms of individual logarithms of x, y, and z.

a) $\log_5 \frac{xy}{z}$

$= \boxed{\log_5 x + \log_5 y - \log_5 z}$

b) $\log_7 \sqrt[3]{x}$

$= \log_7 x^{1/3}$

$= \boxed{\frac{1}{3}\log_7 x}$

c) $\log_6 \frac{1}{x^2}$

$= \log_6 x^{-2}$

$= \boxed{-2\log_6 x}$

OR

$= \log_6 1 - \log_6 x^2$

$= 0 - 2\log_6 x$

$= \boxed{-2\log_6 x}$

d) $\log \frac{x^3}{y\sqrt{z}}$

$= \log x^3 - (\log y + \log z^{1/2})$

$= \log x^3 - \log y - \log z^{1/2}$

$= \boxed{3\log x - \log y - \frac{1}{2}\log z}$

Ex. Use the laws of logarithms to simplify and evaluate each expression.

a) $\log_6 8 + \log_6 9 - \log_6 2$

$= \log_6 \left(\frac{8 \cdot 9}{2}\right)$

$= \log_6 36$

$= \boxed{2}$

b) $\log_7 7\sqrt{7}$

$= \log_7 7 + \log_7 7^{1/2}$

$= 1 + \frac{1}{2}(1)$

$= \boxed{3/2}$

OR

$\log_7 (7^1 \cdot 7^{1/2})$

$= \log_7 7^{3/2}$

$= \boxed{3/2}$

c) $\cancel{2}\log_2 12^2 - (\log_2 6 + \cancel{\tfrac{1}{3}}\log_2 27^{1/3})$

$= \log_2 \left(\frac{144}{6 \cdot 3}\right)$

$= \log_2 8$

$= \boxed{3}$

d) $\log_9 36 + \log_{27} 8 - \log_3 4$

$\log_9 36 = x$

$9^x = 36$

$3^{2x} = 36$

$\frac{1}{2}\log_3 36 = \cancel{2}x$

$\log_3 36^{1/2} = x$

$\log_3 6 = x$

$\log_{27} 8 = y$

$27^y = 8$

$3^{3y} = 8$

$\frac{1}{3}\log_3 8 = \cancel{3}y$

$\log_3 8^{1/3} = y$

$\log_3 2 = y$

$* \log_3 4$

$= \log_9 16$

$= \log_3 6 + \log_3 2 - \log_3 4$

$= \log_3 \left(\frac{6 \cdot 2}{4}\right)$

$= \log_3 3 = \boxed{1}$

Ex. If $\boxed{\log_2 5 = x}$ & $\log_2 3 = y$ express each statement in terms of x and/or y:

a) $\log_2 125$

$= \log_2 5^3$

$= 3\log_2 5$

$= \boxed{3x}$

b) $\log_2 75$

$= \log_2 (25 \cdot 3)$

$= 2\log_2 5 + \log_2 3$

$= \boxed{2x + y}$

c) $\log_2 36$

$= \log_2 (9 \cdot 4)$

$= 2\log_2 3 + \log_2 4$

$= \boxed{2y + 2}$

Ex. Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

a) $\log_7 x^2 + \log_7 x - 5\frac{\log_7 x}{2}$ $\;\frac{5}{2}$

$x \neq 0$ and $x > 0$

$= \log_7 \left(\frac{x^3}{x^{5/2}}\right)$

$= \log_7 x^{1/2}$

$= \boxed{\frac{1}{2}\log_7 x}$ $\quad \boxed{x > 0}$

b) $\log_5(2x - 2) - \log_5(x^2 + 2x - 3)$

$2x - 2 > 0$

$x > 1$

$x^2 + 2x - 3 > 0$

$(x+3)(x-1) > 0$

$x < -3$ or $x > 1$

$= \log_5 \left(\frac{2\cancel{(x-1)}}{(x+3)\cancel{(x-1)}}\right)$

$= \boxed{\log_5 \frac{2}{x+3}}$

$\boxed{x > 1}$

Ex. The pH (acidity) of a solution is defined by:

pH $= -\log[H+]$ where [H+] is the hydrogen ion concentration in moles per litre.

a) Determine the pH of potato soil, to the nearest tenth, if the soil has a hydrogen ion concentration of $5 \times 10^{-8}\ mol/L$.

$pH = -\log(5 \times 10^{-8})$

$= \boxed{7.3}$

b) What is the hydrogen ion concentration of a solution with pH $= 3.1$?

$-3.1 = \cancel{-}\log[H^+]$

$10^{-3.1} = [H^+]$

$[H^+] = \boxed{7.9 \times 10^{-4}\ mol/L}$

Practice: pg. 400/ # 1 – 5, 7 – 13, 16, 18, **19**, 20, C3