PreCalculus 12 Date _____

Unit 8: Logarithmic Functions

8.2 Transformations of Logarithmic Functions

Vertically stretch by a factor of $|a|$ about the x-axis. Reflect in the x-axis if $a < 0$.

Vertically translate k units.

$$y = a \log_c (b(x - h)) + k$$

Horizontally stretch by a factor of $\left|\frac{1}{b}\right|$ about the y-axis. Reflect in the y-axis if $b < 0$.

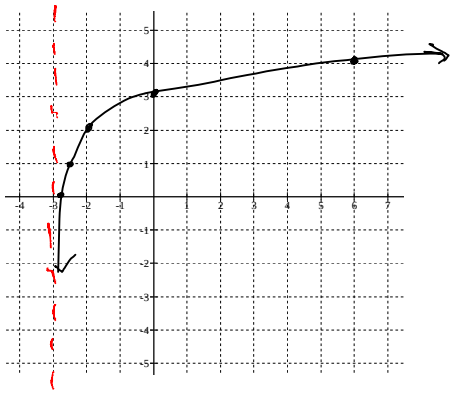
Horizontally translate h units.

Ex. Sketch the graph of $y = \log_3(x + 3) + 2$ and identify the requested characteristics.

$y = \log_3 x$

x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2

$x-3$	$y+2$
$-2\frac{8}{9}$	0
$-2\frac{2}{3}$	1
-2	2
0	3
6	4



Domain $\{x \mid x > -3, x \in \mathbb{R}\}$ Asymptote $x = -3$

$x + 3 > 0$

Range $\{y \mid y \in \mathbb{R}\}$ Intercepts $y\text{-int} = 3$ $x\text{-int} = -2\frac{6}{9}$

$y = \log_3(0+3) + 2$
$= 1 + 2$
$= 3$

$0 = \log_3(x+3) + 2$
$-2 = \log_3(x+3)$
$3^{-2} = x + 3$
$\frac{1}{9} - 3 = x$

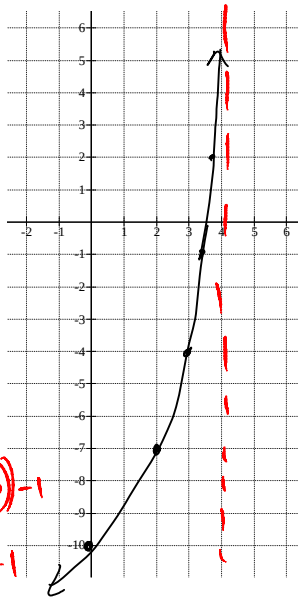
Ex. Sketch the graph of $y = 3\log_{\frac{1}{2}}(8 - 2x) - 1$ and identify the requested characteristics.

$y = \log_{\frac{1}{2}} x$

x	y
$\frac{1}{4}$	2
$\frac{1}{2}$	1
1	0
2	-1
4	-2
8	-3
16	-4

$(-2(x-4))$

$-\frac{1}{2}x +$	$3y - 1$
$3\frac{7}{8}$	5
$3\frac{3}{4}$	2
$3\frac{1}{2}$	-1
3	-4
2	-7
0	-10



Domain $\{x \mid x < 4, x \in \mathbb{R}\}$ Asymptote $x = 4$

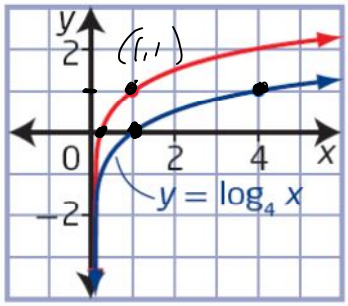
$-2x > 0$
$-2x > -8$

Range $\{y \mid y \in \mathbb{R}\}$ Intercepts $x\text{-int} = 3.6$ $y\text{-int} = -10$

$0 = 3\log_{\frac{1}{2}}(8 - 2x) - 1$
$\frac{1}{3} = \log_{\frac{1}{2}}(8 - 2x)$
$\left(\frac{1}{2}\right)^{\frac{1}{3}} = 8 - 2x$
$\frac{\left(\frac{1}{2}\right)^{\frac{1}{3}} - 8}{-2} = x$
$\frac{\frac{1}{\sqrt[3]{2}} - 8}{-2} = x$

$y = 3\log_{\frac{1}{2}}(8 - 2(0)) - 1$
$y = 3\log_{\frac{1}{2}}(8) - 1$
$= 3(-3) - 1$
$= -10$

Ex. The red graph can be generated by stretching the blue graph of $y = \log_4 x$. Write the equation that describes the red graph.



x-int changed
∴ horizontal stretch

$(4,1) \rightarrow (1,1)$

$\times 1/4$

$y = \log_4(4x)$

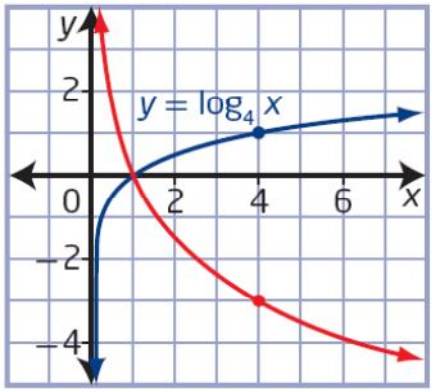
OR

$y = \log_4(bx)$

$1 = \log_4(b(1))$

$4^1 = b$

Ex. The red graph can be generated by stretching and reflecting the graph of $y = log_4 x$. Write the equation that describes the red graph.



x-int same
∴ vertical stretch

$(4,1) \rightarrow (4,-3)$

$\times -3$

$y = -3\log_4 x$

Ex. Welding is the most common way to permanently join metal parts together. Welders wear helmets fitted with a filter shade to protect their eyes from the intense light and radiation produced by a welding light. The filter shade number, *N*, is defined by the function $N = \frac{7(-\log T)}{3} + 1$, where *T* is the fraction of visible light that passes through the filter. Shade numbers range from 2 to 14, with a lens shade number of 14 allowing the least amount of light to pass through. The correct filter shade depends on the type of welding. A shade number 12 is suggested for arc welding. What fraction of visible light is passed through the filter to the welder, as a percent to the nearest ten thousandth?

$$12 = \frac{7(-\log T)}{3} + 1$$

$$11 = \frac{7(-\log T)}{3}$$

$$33 = -7\log T$$

$$\frac{33}{-7} = \log T$$

$$10^{-33/7} = T \approx 1.93 \times 10^{-5}$$

$$0.0000193 = 0.0019\%$$

Practice: pg. 389/ # 4 – 6, 8 – 10, 12 – 17, C3