Unit 8: Logarithmic Functions

8.1 Understanding Logarithms

Logarithmic Function: A function of the form $y = \log_c x$, where $c > 0$ and $c \neq 1$, that is the inverse of the exponential function $y = c^x$

Logarithm: * an exponent

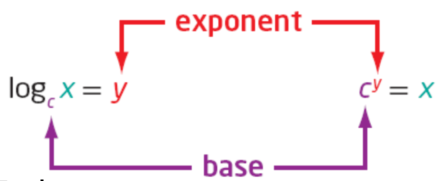
* in $x = c^y$, $\underbrace{y \text{ is called the "logarithm base } c \text{ of } x\text{"}}_{y = \log_c x}$

Common Logarithm:

* logarithm with base 10

ie. $\log_{10} x = \log x$

Logarithmic Form **Exponential Form**



ex. $2^5 = 32$

$\log_2 32 = 5$

ex. $(1/3)^{-3} = 27$

$\log_{1/3} 27 = -3$

Ex. Evaluate:

a) $\log_7 49 = 2$

What exponent on base 7 produces 49?

$\log_7 49 = x$

$7^x = 49$

$7^x = 7^2$

$\therefore x = 2$

b) $\log_6 1 = 0$

check

$6^0 = 1$ ✓

c) $\boxed{\log 0.001 = -3}$

$\log \frac{1}{1000} = x$

$10^x = \frac{1}{1000}$

$10^x = 10^{-3}$

$\therefore x = -3$

common log on calc!

d) $\log_2 \sqrt{8} = 3/2$

$\log_2 8^{1/2} = x$

$2^x = 8^{1/2}$

$2^x = 2^{3/2}$

$\therefore x = 3/2$

A few general rules ...

$\boxed{\log_c 1 = 0}$

$\boxed{\log_c c = 1}$

$\boxed{\log_c c^x = x}$

proof

$\log_c c^x = q$

$c^q = c^x$

$\therefore q = x$

$\boxed{c^{\log_c x} = x}$

proof

$c^{\log_c x} = b$

$\log_c b = \log_c x$

$\therefore b = x$

Ex. Determine the value of x:

a) $\log_5 x = -3$

$5^{-3} = x$

$\boxed{x = \frac{1}{125}}$

b) $\log_x 36 = 2$

$x^2 = 36$

$x = \pm\sqrt{36}$

$\boxed{x = 6}$ or $\cancel{-6}$

c) $\log_{64} x = \frac{2}{3}$

$64^{2/3} = x$

$(\sqrt[3]{64})^2 = x$

$4^2 = x$

$\boxed{x = 16}$

d) $\log(-100) = x$

$\cancel{10^x = -100}$

$\cancel{\log 0} = x$

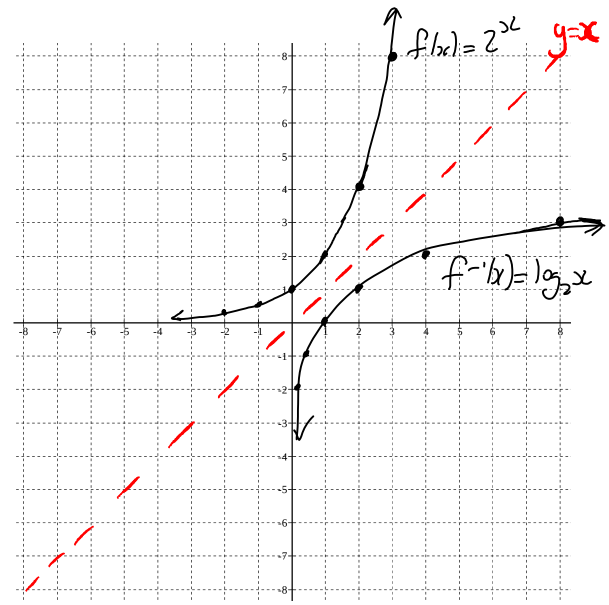
* The input of a logarithmic function must be positive

Ex. Given $f(x) = 2^x$:

a) State the inverse of $f(x)$.

$x = 2^y$

$y = \log_2 x$ $\quad \therefore \boxed{f^{-1}(x) = \log_2 x}$



b) Sketch $f(x) = 2^x$ and its inverse $f^{-1}(x)$. State the domain, range, intercepts, and asymptote equations for each.

$f(x) = 2^x$	$f^{-1}(x) = \log_2 x$
$\{x \mid x \in \mathbb{R}\}$	$\{x \mid x > 0, x \in \mathbb{R}\}$
$\{y \mid y > 0, y \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}\}$
horizontal asymptote $y = 0$	vertical asymptote $x = 0$
x-int: none	x-int $= 1$
y-int $= 1$	y-int: none

Did You Know?

A "standard" earthquake has amplitude of 1 micron, or 0.0001 cm, and magnitude 0. Each increase of 1 unit on the Richter scale is equivalent to a tenfold increase in the intensity of an earthquake.

Ex. The Richter magnitude, M, of an earthquake is defined as $M = \log\frac{A}{A_0}$, where A is the amplitude of the ground motion, usually in microns, measured by a sensitive seismometer, and A_0 is the amplitude, corrected for the distance to the actual earthquake, that would be expected for a "standard" earthquake.

a) In 1946, an earthquake struck Vancouver Island off the coast of British Columbia. It had an amplitude that was $10^{7.3}$ times A_0. What was the earthquake's magnitude on the Richter scale?

$M = \log \frac{10^{7.3} \cancel{A_0}}{\cancel{A_0}}$

$= \log 10^{7.3}$

$= \boxed{7.3}$

b) The strongest recorded earthquake in Canada struck Haida Gwaii, off the coast of British Columbia, in 1949. It had a Richter reading of 8.1. How many times as great as A_0 was its amplitude?

$8.1 = \log \frac{A}{A_0}$

$10^{8.1} = \frac{A}{A_0}$

$A = 10^{8.1} A_0$ $\qquad \boxed{10^{8.1} \text{ times as great}}$

c) Compare the seismic shaking of the 1949 Haida Gwaii earthquake with that of the earthquake that struck Vancouver Island in 1946.

Ratio of amplitudes $\quad \frac{\text{Haida Gwaii}}{\text{Vancouver Island}} = \frac{10^{8.1}}{10^{7.3}} = 10^{0.8}$

$\approx$ Haida Gwaii quake was about 6.3 times as intense as the Vanc. Isl. quake.

Practice: pg. 380/ # 1 – 7, 10 – 16, 18 – 24