Unit 7: Exponential Functions
7.3 Solving Exponential Equations

Ex. Rewrite each expression as a power with a base of 3.

a) 27

$= \boxed{3^3}$

b) 9^2

$= (3^2)^2$

$= \boxed{3^4}$

c) $27^{\frac{1}{3}}(\sqrt[3]{81})^2$

$= (3^3)^{1/3}(3^4)^{2/3}$

$= 3^1 \cdot 3^{8/3}$

$= \boxed{3^{11/3}}$

Ex. Solve each equation:

a) $8^{3x-2} = 16^{x+1}$

$(2^3)^{3x-2} = (2^4)^{x+1}$

$2^{9x-6} = 2^{4x+4}$

$\therefore 9x-6 = 4x+4$

$5x = 10$

$\boxed{x=2}$

check

$8^{3(2)-2} \stackrel{?}{=} 16^{(2)+1}$

$8^4 = 16^3$

$4096 = 4096$ ✓

b) $27^{x+3} = \left(\frac{1}{9}\right)^{2x-5}$

$(3^3)^{x+3} = (3^{-2})^{2x-5}$

$3^{3x+9} = 3^{-4x+10}$

$\therefore 3x+9 = -4x+10$

$7x = 1$

$\boxed{x = 1/7}$

Problems involving exponential growth/decay can generally be modelled using one of two formulas.

Case 1: <u>percentage</u> growth/decay

$F = I(1 \pm r)^n$	F = final amount
	I = initial amount
+ growth	r = rate of growth/decay (in decimal form)
- decay	n = time (in same period as r; ie. if r is monthly then n in months)

Ex. John invests \$5000 at 8%/a, compounded <u>quarterly</u>.

Compound interest

r = rate per compounding period

n = total # of compounds

a) How much will his investment be worth in 5 years?

$r = \frac{.08}{4} = .02$

$n = 4 \cdot 5 = 20$

$F = 5000(1.02)^{20}$

$= \boxed{\$7429.74}$

b) How long will it take for his investment to grow to \$8880?

$\frac{8880}{5000} = \frac{\cancel{5000}}{\cancel{5000}}(1.02)^n$

$\underbrace{1.776 = 1.02^n}_{\text{on calc} \rightarrow \text{intersect}}$

$n = 29$ compounds

$\div 4 = \boxed{7.25 \text{ years}}$

Ex. The intensity of light below the surface of a lake decreases by 4% for every metre below the surface. What percentage of the original light intensity remains at a depth of 10 m?

$F = I(\underbrace{\cancel{1-0.04}}_{0.96})^{10}$

$= I(0.665)$

$\to \boxed{66.5\% \text{ remaining}}$

Case 2: factor growth/decay

$F = Ir^{t/n}$	F = final amount
	I = initial amount
$r > 1 \rightarrow$ growth	r = growth/decay factor (multiplier)
$0 < r < 1 \rightarrow$ decay	n = time to achieve r
	t = total time (same units as r)

Ex. An initial sample of 1000 bacteria triples every 25 hours.

a) Approximately how many bacteria will there be in 4 days?

$r = 3$ $\quad n = 25 \quad \to 96 \text{ hours}$

$F = 1000(3)^{96/25}$

$= \boxed{67943 \text{ bacteria}}$

b) How long will it take the number of bacteria to reach 59 049 000?

$59\,049\,\cancel{000} = \cancel{1000}(3)^{t/25}$

$3^{10} = 3^{t/25}$

$\therefore \quad 10 = t/25$

$\boxed{t = 250 \text{ hours}}$

Ex. A 100 g sample of the extremely rare element Sdoutzium-72 decays to 6.25 g in 40 seconds. What is the half life of Sdoutzium-72?

$r = 1/2 \qquad n = ?$

$\frac{6.25}{100} = \cancel{100}(1/2)^{40/n}$

$1/16 = (1/2)^{40/n}$

$(1/2)^4 = (1/2)^{40/n}$

$\therefore \quad 4 = 40/n \qquad n = \boxed{10 \text{ seconds}}$

Practice: pg. 364/ #4, 5, 7cdeh, 8cd, 9 – 14