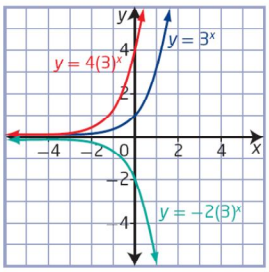
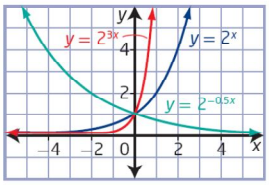
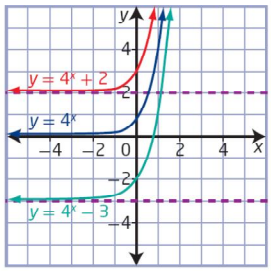
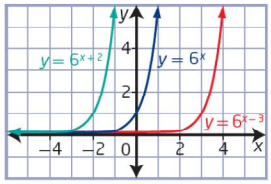
PreCalculus 12 Date _____

Unit 7: Exponential Functions

7.2 Transformations of Exponential Functions

The general form of the exponential function $f(x) = c^x$ $(c > 0)$ is $f(x) = a(c)^{b(x-h)} + k$.
Transformations applied the same as in unit 1.

Parameter	Transformation	Example
a	• Vertical stretch about the x-axis by a factor of $\lvert a\rvert$ • For $a < 0$, reflection in the x-axis • $(x, y) \rightarrow (x, ay)$	
b	• Horizontal stretch about the y-axis by a factor of $\frac{1}{\lvert b\rvert}$ • For $b < 0$, reflection in the y-axis • $(x, y) \rightarrow \left(\frac{x}{b}, y\right)$	
k	• Vertical translation up or down • $(x, y) \rightarrow (x, y + k)$ Key $y = k$ is the horizontal asymptote	
h	• Horizontal translation left or right • $(x, y) \rightarrow (x + h, y)$	

Ex. Sketch each graph, state domain, range, intercepts, and the equation of the horizontal asymptote:

a) $y = 3(2)^{x-4}$

$x+4$	$3y$
1	3/8
2	3/4
3	3/2
4	3
5	6
6	12
7	24
8	48

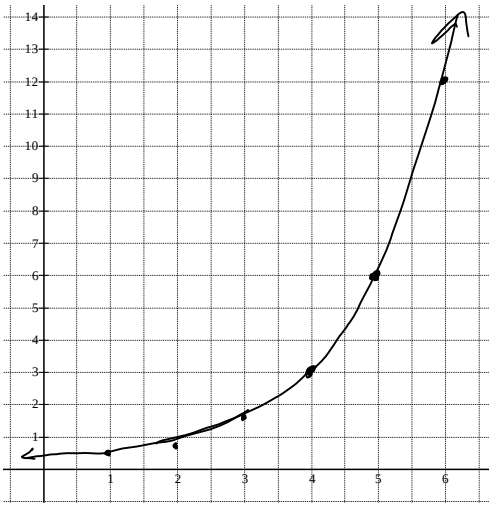
$y = 2^x$

x	y
−3	1/8
−2	1/4
−1	1/2
0	1
1	2
2	4
3	8
4	16

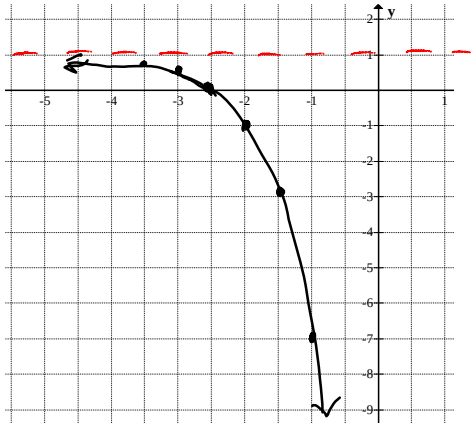
b) $y = -\frac{1}{2}(2)^{2x+6} + 1$

$2(x+3)$

$\frac{1}{2}x - 3$	$-\frac{1}{2}y + 1$
−4.5	15/16
−4	7/8
−3.5	3/4
−3	1/2
−2.5	0
−2	−1
−1.5	−3
−1	−7

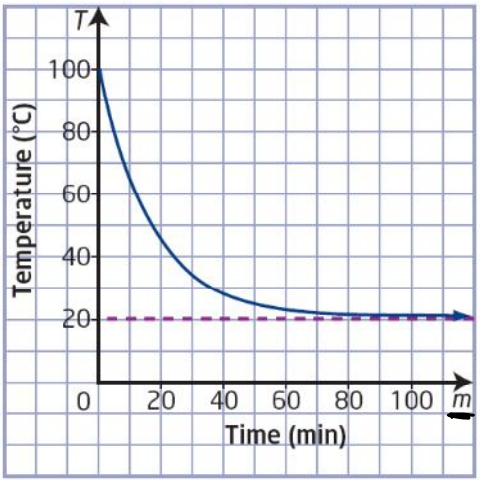


D $\{x \mid x \in \mathbb{R}\}$
R $\{y \mid y > 0, y \in \mathbb{R}\}$
asymptote $y = 0$
x-int: none
y-int: $3(2)^{0-4}$
$3(1/16) = 3/16$



D $\{x \mid x \in \mathbb{R}\}$
R $\{y \mid y < 1, y \in \mathbb{R}\}$
asymptote $y = 1$
x-int: −2.5
y-int: $-\frac{1}{2}(2)^{2(0)+6} + 1$
$-\frac{1}{2}(64) + 1 = -31$

Ex. A cup of water is heated to 100 °C and then allowed to cool in a room with an air temperature of 20°C. The temperature, *T*, in degrees Celsius, is measured every minute as a function of time, *m*, in minutes, and these points are plotted on a coordinate grid. It is found that the temperature of the water decreases exponentially at a rate of 25% every 5 min. A smooth curve is drawn through the points, resulting in the graph shown.



a) What is the transformed exponential function in the form $y = a(c)^{b(x-h)} + k$ that can be used to represent this situation?

25% decrease → 75% retention → base 3/4

base function $T = (3/4)^t$ ← t 5 minute intervals

$T = (3/4)^{m/5}$

asymptote $T = 20$ ↑ vertical displacement

no horizontal translation - start at time 0

$T = a(3/4)^{m/5} + 20$

$100 = a(3/4)^{0/5} + 20$

$100 = a + 20$

$a = 80$

sub in a point (0,100)

$$T(m) = 80(3/4)^{m/5} + 20$$

b) Describe how each of the parameters in the transformed function relates to the information provided.

a, b, h, k

$a = 80$ Temp difference between initial cup temp & room temp

$b = 1/5$ The time interval (5 min.) between 25% temp drops

$h = 0$ starting cooling process at time 0

$k = 20$ The air temp of room

Practice: pg. 354/ # 1 – 4, 6, 7, (8 IF want more practice sketching graphs), 9 – 12 (use calc to sketch)