PreCalculus 12 Date ___________________

Unit 7: Exponential Functions

7.1 Characteristics of Exponential Functions

When the creator of the game of chess showed his invention to the ruler of the country, the ruler was so pleased that he gave the inventor the right to name his prize for the invention. The man, who was very wise, asked the king that he be given one grain of rice for the first square of the chessboard, two for the second one, four for the third one, and so on. The ruler quickly accepted the inventor's offer, believing the man had made a mistake in not asking for more. * 64 squares

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096 . . .

geometric

$t_1 = 1$

$r = 2$

$n = 64$

rice on the 64th square $t_{64} = 1(2)^{63}$

$t_n = t_1 r^{n-1}$ $= 9,223,373,037,000,000,000$

All rice

$S_n = \frac{t_1(r^n - 1)}{r-1}$

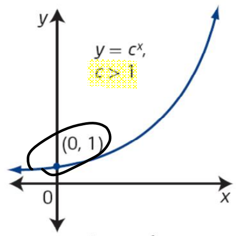
$S_{64} = \frac{1(2^{64} - 1)}{2-1}$

$= 18,446,744,070,000,000,000$

Exponential Function:

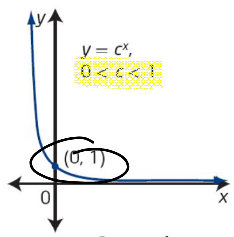
A function of the form $y = c^x$ where c is a constant ($c > 0$) and x is a variable

ex. $\underbrace{y = 4^x}_{\text{exponential}}$ vs. $\underbrace{y = x^4}_{\text{polynomial}}$



Increasing

Exponential Growth

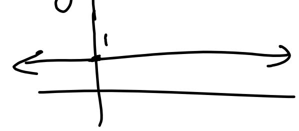


Decreasing

Exponential Decay

$c = 1$

Degenerate Case



Ex. Graph each exponential function. Then identify the following:

- the domain and range
- the *x*-intercept and *y*-intercept, if they exist
- whether the graph represents an increasing or a decreasing function
- the equation of the horizontal asymptote

a) $y = 4^x$

x	y
-2	1/16
-1	1/4
-1/2	1/2
0	1
1/2	2
1	4
2	16

$D\{x \mid x \in \mathbb{R}\}$

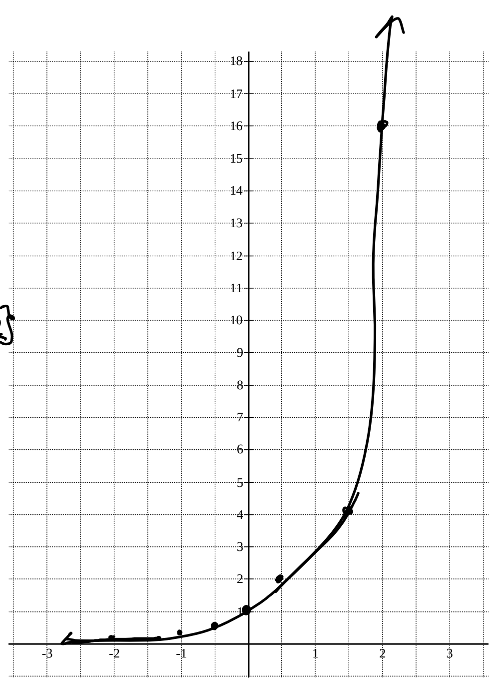
$R\{y \mid y > 0, y \in \mathbb{R}\}$

x-int: none

y-int: 1

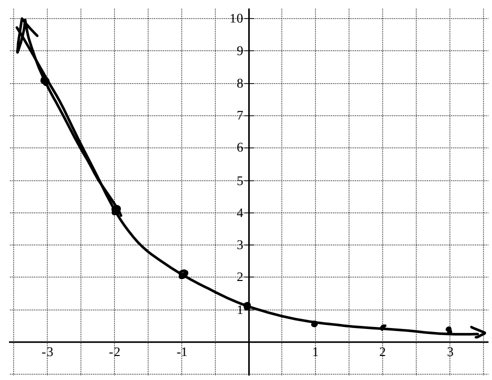
increasing

$y = 0$



b) $f(x) = \left(\frac{1}{2}\right)^x$

x	y
-3	8
-2	4
-1	2
0	1
1	1/2
2	1/4
3	1/8



$D\{x \mid x \in \mathbb{R}\}$

$R\{y \mid y > 0, y \in \mathbb{R}\}$

x-int: none

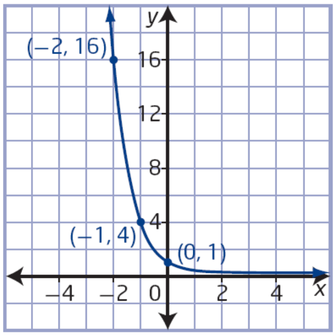
y-int: 1

decreasing

$y = 0$

Ex. What function of the form $y = c^x$ can be used to describe the graph shown?

a)



$y = c^x$

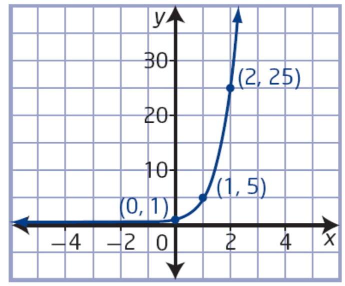
$4 = c^{-1}$

$4 = \frac{1}{c}$

$c = \frac{1}{4}$

$\boxed{y = (\frac{1}{4})^x}$

b)

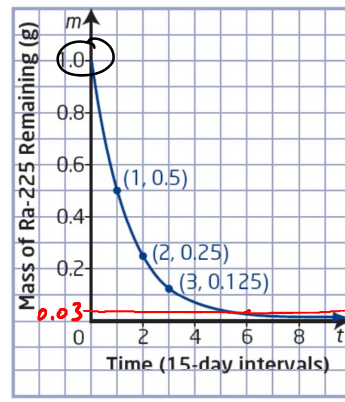


$y = c^x$

$5 = c^1$

$\boxed{y = 5^x}$

Ex. A radioactive sample of radium (Ra-225) has a **half-life** of 15 days. The mass, *m*, in grams, of Ra-225 remaining over time, *t*, in 15-day intervals, can be modeled using the exponential graph shown.



a) What is the initial mass of Ra-225 in the sample? What value does the mass of Ra-225 remaining approach as time passes?

$t = 0$

$m = \boxed{1.0g}$ remaining approaches 0g

b) What are the domain and range of this function?

$D\{t \mid t \geq 0, t \in \mathbb{R}\}$

$R\{m \mid 0 < m \leq 1, m \in \mathbb{R}\}$

c) Write the exponential decay model that relates the mass of Ra-225 remaining to time, in 15-day intervals. $m(t) = (\frac{1}{2})^t$

d) Estimate how many days it would take for Ra-225 to decay to $\frac{1}{30}$ of its original mass.

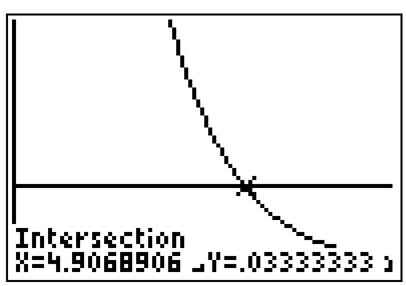
$\frac{1}{30} \approx 0.03$ Estimate

$t = 6 \Rightarrow 6(15) = 90$ days

$(\frac{1}{2})^t = \frac{1}{30}$

$y_1 = (\frac{1}{2})^x$

$y_2 = \frac{1}{30}$ intersect



$t \approx 4.9$

days $= 15(4.9) = 73.5$ days

Practice: pg. 342/ # 1 – 4, 5ac, 6 – 11 (use calculator for graphs), 13, 15, C1