PreCalculus 12 Date _____

Unit 6: Trigonometric Identities

6.4 Solving Trig Equations Using Identities

Sometimes, solving a trigonometric equation involves making substitutions using trig identities.

Ex. Solve each equation algebraically without a calculator over the domain $0 \le \theta \le 2\pi$.

a) $\cos 2x + 1 - \cos x = 0$

$\cos^2 x - \sin^2 x$

$1 - 2\sin^2 x$

$2\cos^2 x + 1 - \cos x = 0$

$\cos x(2\cos x - 1) = 0$

$\cos x = 0 \qquad \cos x = \frac{1}{2}$

$\frac{x}{r} = 0$

$\therefore x = 0$

(0,1)

(0,-1)

2, $\sqrt{3}$, $\pi/3$, 1

$2\pi - \pi/3$

$x = \pi/2, 3\pi/2, \pi/3, 5\pi/3$

b) $1 - \cos^2 x = 3\sin x - 2$

$\sin^2 x = 3\sin x - 2$

$\sin^2 x - 3\sin x + 2 = 0$ $\qquad b^2 - 3b + 2 = 0$

$(\sin x - 2)(\sin x - 1) = 0$

$\sin x = 2$ no solutions $\qquad \sin x = 1$

$\frac{y}{r} = 1$

(0,1) $\therefore y = 1$

$x = \pi/2$

Ex. Solve the equation $\cos^2 x = \cot x \sin x$ algebraically without a calculator in the domain $0° \le x < 360°$. Verify graphically.

$\cos^2 x = \frac{\cos x}{\sin x} \sin x$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

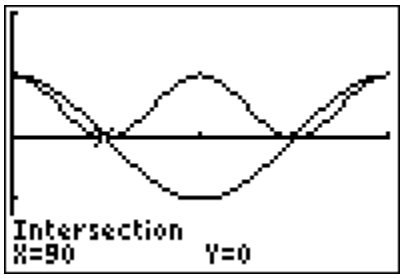
$\cos x = 0 \qquad \cos x = 1$

$\frac{x}{r} = 1$

(1, 0)

$\therefore x = 1$

$x = 90°, 270°, 0°$



$x = 0°, 90°, 270°$ ✓

Ex. Algebraically without a calculator, find the general solution to each equation expressed in radians.

a) $\sin 2x = \sqrt{2}\cos x$

$2\sin x\cos x = \sqrt{2}\cos x$

$2\sin x\cos x - \sqrt{2}\cos x = 0$

$\cos x(2\sin x - \sqrt{2}) = 0$

$\cos x = 0$ $\quad \sin x = \sqrt{2}/2$

$x = \pi/2, 3\pi/2$ $\quad x = \pi/4, 3\pi/4$

$x = \pi/2 + \pi n, \ n \in I$

$x = \pi/4 + 2\pi n$

$x = 3\pi/4 + 2\pi n$

b) $2\sin x = 7 - 3\csc x$

$\sin x(2\sin x) = \left(7 - \frac{3}{\sin x}\right)\sin x$

$2\sin^2 x = 7\sin x - 3$

$2\sin^2 x - 7\sin x + 3 = 0$

$2\sin^2 x - 6\sin x - \sin x + 3$

$2\sin x(\sin x - 3) - 1(\sin x - 3)$

$(\sin x - 3)(2\sin x - 1) = 0$

$\sin x = 3$ $\quad \sin x = 1/2$

$x = \pi/6, 5\pi/6$

$x = \pi/6 + 2\pi n, \ n \in I$

$x = 5\pi/6 + 2\pi n$

Practice: pg. 320/# 1 – 6, 8 – 11, 13, C1, C2