PreCalculus 12 Date __________________

Unit 6: Trigonometric Identities
6.3 Proving Identities

So far we have **verified** identities graphically (showing two graphs appear the same) and numerically (showing an identity is true for certain values of the input angle).
However, to **prove** that an identity is true for all permissible values, it is necessary to express both sides of the identity in equivalent forms. One or both sides of the identity must be algebraically manipulated into an equivalent form to match the other side.
You cannot perform operations across the equal sign (ie. equation solving techniques) when proving a potential identity. Simplify the expressions on each side of the identity independently.

Ex. Prove the identity: $1 - \sin^2 x = \sin x \cos x \cot x$

$\cancel{\sin x} \cos x \cdot \frac{\cos x}{\cancel{\sin x}}$

$\cos^2 x = \cos^2 x$ ∎

QED

Tips!
- Use a vertical line to separate the two sides of the equation
- Start with the most complicated side
- Change to everything to $\sin x$ and/or $\cos x$
- Squared terms? - Check Pythagorean identities or factor
- Common denominators for simplifying fractions
- Will conjugates help you?

Ex. Prove each of the following is an identity for all permissible values x.

a) $\tan x = \frac{1-\cos 2x}{\sin 2x}$

$\frac{\sin x}{\cos x}$ | $\frac{1-(1-2\sin^2 x)}{2\sin x\cos x}$

$\frac{\cancel{2}\sin^{\cancel{2}} x}{\cancel{2}\cancel{\sin x}\cos x}$

$\frac{\sin x}{\cos x} = \frac{\sin x}{\cos x}$ ∎

b) $1 - \cos^2\theta = \cos^2\theta\tan^2\theta$

$\sin^2\theta$ | $\cancel{\cos^2\theta}\,\frac{\sin^2\theta}{\cancel{\cos^2\theta}}$

$\sin^2\theta = \sin^2\theta$ ∎

c) $\frac{1-\tan\theta}{\cot\theta-1} = \tan\theta$

$\frac{1-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}-1}$ | $\frac{\sin\theta}{\cos\theta}$

$\frac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\sin\theta}}$

$\frac{\cancel{\cos\theta-\sin\theta}}{\cos\theta}\cdot\frac{\sin\theta}{\cancel{\cos\theta-\sin\theta}}$

$\frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}$ ∎

d) $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$

$\frac{2\frac{\sin x}{\cos x}}{\sec^2 x}$ | $2\sin x\cos x$

$\frac{2\frac{\sin x}{\cos x}}{\frac{1}{\cos^2 x}}$

$\frac{2\sin x}{\cancel{\cos x}}\cdot\frac{\cos^{\cancel{2}} x}{1}$

$2\sin x\cos x = 2\sin x\cos x$ ∎

Ex. Prove:

a) $\frac{1-\cos x}{\sin x} = \frac{\sin x}{(1+\cos x)} \cdot \frac{(1-\cos x)}{(1-\cos x)}$

$\frac{\sin x(1-\cos x)}{1-\cos^2 x}$

$\frac{\cancel{\sin x}(1-\cos x)}{\sin^2 x}$

$\frac{1-\cos x}{\sin x} = \frac{1-\cos x}{\sin x}$

b) $\frac{1}{1+\sin x} = \frac{\sec x-\sin x\sec x}{\cos x}$

$\frac{\sec x(1-\sin x)}{\cos x}$

$\frac{1-\sin x}{\cos^2 x}$

$\frac{1-\sin x}{1-\sin^2 x}$

$\frac{\cancel{1-\sin x}\ 1}{(\cancel{1-\sin x})(1+\sin x)}$

$\frac{1}{1+\sin x} = \frac{1}{1+\sin x}$

c) $\cot x-\csc x = \frac{\cos 2x-\cos x}{\sin 2x+\sin x}$

$\frac{\cos x}{\sin x} - \frac{1}{\sin x}$

$\frac{\cos x-1}{\sin x}$

$\frac{(2\cos^2 x-1)-\cos x}{2\sin x\cos x+\sin x}$

$\frac{2\cos^2 x-\cos x-1}{\sin x(2\cos x+1)}$

$\frac{(\cancel{2\cos x+1})(\cos x-1)}{\sin x(\cancel{2\cos x+1})}$

$\frac{\cos x-1}{\sin x} = \frac{\cos x-1}{\sin x}$

d) $\frac{\sin 2x-\cos x}{4\sin^2 x-1} = \frac{\sin^2 x\cos x+\cos^3 x}{2\sin x+1}$

$\frac{2\sin x\cos x-\cos x}{(2\sin x+1)(2\sin x-1)}$

$\frac{\cos x(\cancel{2\sin x-1})}{(2\sin x+1)(\cancel{2\sin x-1})}$

$\frac{\cos x}{2\sin x+1}$

$\frac{\cos x\overset{1}{(\cancel{\sin^2 x+\cos^2 x})}}{2\sin x+1}$

$= \frac{\cos x}{2\sin x+1}$

* $2b^2-b-1$
$2b^2-2b+b-1$
$2b(b-1)+1(b-1)$
$(2b+1)(b-1)$

Practice: pg. 314/# 1 – 16, C2