Unit 6: Trigonometric Identities

6.2 Sum, Difference, and Double-Angle Identities

Sum & Difference Identities

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ $\quad\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$ $\quad\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ $\quad\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Double Angle Identities

$\sin 2\theta = 2\sin\theta\cos\theta$ $\quad\cos 2\theta=\cos^2\theta-\sin^2\theta$ $\quad\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 1-2\sin^2\theta$

+pythagorean identity

Ex. Write each expression as a single trigonometric function:

a) $\underbrace{\sin 48°}_{\alpha}\underbrace{\cos 17°}_{\beta}-\underbrace{\cos 48°}_{\alpha}\underbrace{\sin 17°}_{\beta}$.

$\sin(\alpha-\beta)=\sin(48°-17°)=\sin 31°$

c) $\cos^2\left(\frac{\pi}{3}\right)-\sin^2\left(\frac{\pi}{3}\right)$

$\cos 2\theta = \cos\frac{2\pi}{3}$

b) $\cos\frac{\pi}{2}\cos\frac{\pi}{3}-\sin\frac{\pi}{2}\sin\frac{\pi}{3}$

$\cos(\alpha+\beta)=\cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right)=\cos\frac{5\pi}{6}$

d) $4\sin 6\theta\cos 6\theta$

$=2\left[\underline{2\sin 6\theta\cos 6\theta}\right]$

$=2\sin 12\theta$

Ex. Consider the expression $\frac{\sin 2x}{\cos 2x+1}$.

a) What are the non-permissible values?

$\cos 2x+1\neq 0$

$\cos 2x\neq -1$

$2\cos^2 x-1\neq -1$

$2\cos^2 x\neq 0$

$\cos^2 x\neq 0$

$\cos x\neq 0$

$\therefore \boxed{x\neq\frac{\pi}{2}+\pi n,\ n\in I}$

b) Simplify the expression to one of the three primary trigonometric functions.

$\frac{2\sin x\cos x}{(2\cos^2x-1)+1}$

$\frac{\cancel{2}\sin x\cancel{\cos x}}{\cancel{2}\cos^{\cancel{2}}x}$

$\frac{\sin x}{\cos x}=\tan x$

new possible identity?

$\frac{\sin 2x}{\cos 2x+1}=\tan x$

c) Verify your answer from b) in the interval $[-2\pi, 2\pi]$ using technology.

Graph $y_1=\frac{\sin 2x}{\cos 2x+1}$

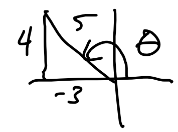
$y_2=\tan x$

appear the same $\therefore$ verified

Ex. If $\cos\theta = -\frac{3}{5}$, θ in quadrant II, determine the exact value of:

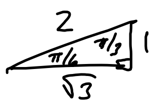
a) $\cos\left(\underset{\alpha}{\theta} + \underset{\beta}{\frac{\pi}{6}}\right)$

$= \cos\theta\cos\pi/6 - \sin\theta\sin\pi/6$



$= \left(-3/5\right)\left(\sqrt{3}/2\right) - \left(4/5\right)\left(1/2\right)$

$= -\frac{3\sqrt{3}}{10} - \frac{4}{10}$



$= \boxed{\frac{-3\sqrt{3}-4}{10}}$

b) $\sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(4/5\right)\left(-3/5\right)$

$= \boxed{-24/25}$

Ex. Find the exact value of:

a) $\sin\frac{\pi}{12}$

$* \pi/12 = 15°$

$45° - 30°$ or $60° - 45°$

$= \sin\left(\pi/4 - \pi/6\right)$

$= \sin\pi/4\cos\pi/6 - \cos\pi/4\sin\pi/6$

$= \left(1/\sqrt{2}\right)\left(\sqrt{3}/2\right) - \left(1/\sqrt{2}\right)\left(1/2\right)$

$= \sqrt{3}/2\sqrt{2} - 1/2\sqrt{2}$

$= \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

b) $\tan 105°$

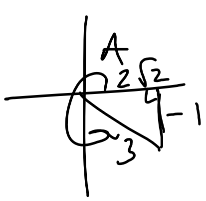
$= \tan(60° + 45°)$

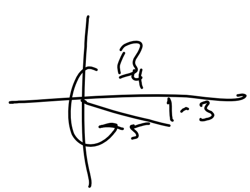
$= \frac{\tan 60° + \tan 45°}{1 - \tan 60 \tan 45}$

$= \boxed{\frac{\sqrt{3}+1}{1-\sqrt{3}}}$

Ex. If $\sin A = -\frac{1}{3}$ and $\cos B = \frac{4}{5}$, A & B in same quadrant, determine the exact value of $\tan(A - B)$.

$\sin A$: QIII or IV $\cos B$: QI or IV





$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \frac{\left(-1/2\sqrt{2}\right) - \left(-3/4\right)}{1 + \left(-1/2\sqrt{2}\right)\left(-3/4\right)}$

rationalize

$* \frac{-1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{-\sqrt{2}}{4}$

$= \frac{\frac{-\sqrt{2}}{4} + \frac{3}{4}}{1 + \left(\frac{-\sqrt{2}}{4}\right)\left(\frac{-3}{4}\right)}$

$= \frac{\frac{3-\sqrt{2}}{4}}{\frac{16 + 3\sqrt{2}}{16}}$

$\frac{3-\sqrt{2}}{\cancel{4}} \times \frac{\cancel{16}\,4}{16+3\sqrt{2}} = \boxed{\frac{4(3-\sqrt{2})}{16+3\sqrt{2}}}$ or $\frac{12-4\sqrt{2}}{16+3\sqrt{2}}$

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