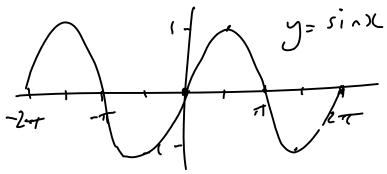
PreCalculus 12 Date ________________

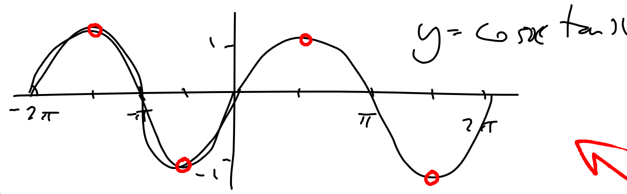
Unit 6: Trigonometric Identities

6.1 Reciprocal, Quotient, & Pythagorean Identities

Investigate

- Graph the curves $y = sinx$ and $y = cosxtanx$ over the domain $-2\pi \le x \le 2\pi$ using your graphing calculator. Sketch below on two separate grids.





- What do you notice about the graphs?

  appear to be the same

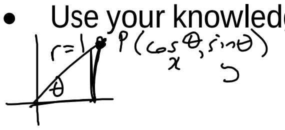
- Use your calculator to make and analyze a table for these functions using multiples of $\frac{\pi}{2}$.

	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\pi/4$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = sinx$	0	1	0	−1	0	.707	1	0	−1	0
$y = cosxtanx$	0	undefined ∞	0	∞	0	.707	∞	0	∞	0

- What do you notice about the two functions from the table?

  sine exists for all values of x
  but $cosxtanx$ is undefined at certain values of x

- Use your knowledge of $tanx$ to simplify $cosxtanx$.



  $\tan x = \frac{\sin x}{\cos x}$ $\quad = \cancel{\cos x}\left(\frac{\sin x}{\cancel{\cos x}}\right)$

  $= \sin x$

- Are the curves $y = sinx$ and $y = cosxtanx$ identical? Explain.

  Not everywhere, not at the undefined values of $cosxtanx$

  * on graph we circle (holes) the undefined values

- Why is it important to look at the graphs AND the table of values?

  Graph is deceiving, table showed us the differences

- What are the non-permissible values of x in the equation $sinx = cosxtanx$?

  $x \neq \pi/2 + \pi n, n \in I$

Trigonometric Identity: A trig statement that is true for all permissible values.

In Pre-Calculus 12, all the trigonometric identities we work with are on your formula sheet, however, in Calculus 12 (and beyond…) there is NO formula sheet. The more you know these identities, the easier it will be to use them now and in the future. You decide! ☺

Reciprocal Identities:

$\csc\theta = \frac{1}{\sin\theta}$ $\qquad$ $\sec\theta = \frac{1}{\cos\theta}$ $\qquad$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\qquad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

proof $\dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{o}{h}}{\frac{a}{h}} = \frac{o}{\cancel{h}} \times \frac{\cancel{h}}{a} = \frac{o}{a} = \tan\theta$

Ex.

a) Determine the non-permissible values, in degrees, for the equation $\sec\theta = \frac{\tan\theta}{\sin\theta}$.

$\frac{1}{\cos\theta}$ $\qquad$ $\frac{\sin\theta}{\cos\theta}$

$\boxed{\theta \neq 90^\circ n,\ n \in I}$

$\cos\theta \neq 0$ $\qquad$ $\sin\theta \neq 0$

$\theta \neq \pi/2 + \pi n,\ n \in I$ $\qquad$ $\theta \neq \pi n,\ n \in I$

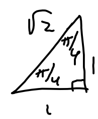
b) Numerically verify that $\theta = 60°$ and $\theta = \frac{\pi}{4}$ are solutions of the equation. Choose some other angle and verify that it is also a solution. $\qquad$ $\theta = 30°$?

$\sec 60° \overset{?}{=} \frac{\tan 60°}{\sin 60°}$

$2 = \frac{\sqrt{3}}{\sqrt{3}/2}$

$= \cancel{\sqrt{3}} \times \frac{2}{\cancel{\sqrt{3}}}$

$2 = 2$



$\sec \pi/4 \overset{?}{=} \frac{\tan \pi/4}{\sin \pi/4}$

$\sqrt{2} = \frac{1}{1/\sqrt{2}}$

$\sqrt{2} = \sqrt{2}$ ✓

$\sec 30° \overset{?}{=} \frac{\tan 30°}{\sin 30°}$

$\frac{2}{\sqrt{3}} = \frac{1/\sqrt{3}}{1/2}$

$= 1/\sqrt{3} \times 2/1$

$2/\sqrt{3} = 2/\sqrt{3}$ ✓

c) Use technology to graphically decide whether the equations could be an identity over the domain $-360° \leq \theta \leq 360°$.

$y_1 = \frac{1}{\cos\theta}$

$y_2 = \frac{\tan\theta}{\sin\theta}$

yes, could be an identity because they appear to be the same graph

Ex. Determine the non-permissible values, in radians, of the variable in each expression. Simplify the expression.

a) $\frac{\cot x}{\csc x \cos x}$ $\qquad$ $\frac{\cos x}{\sin x}$

$\cancel{\csc x \neq 0}$

$\cos x \neq 0$

$\sin x \neq 0$ $\quad$ same above

$\boxed{x \neq \pi/2\, n,\ n \in I}$

$\dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x} \cdot \frac{\cos x}{1}}$

$\dfrac{\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x}}$

$= \boxed{1}$

b) $\frac{\sec x}{\tan x}$ $= \dfrac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}}$

$= \frac{1}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\sin x}$

$= \frac{1}{\sin x}$

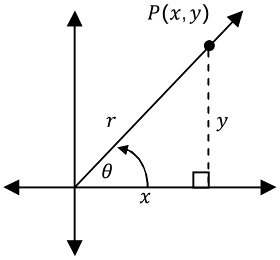
$= \boxed{\csc x}$

$\cos x \neq 0$

$\sin x \neq 0$

$\boxed{x \neq \pi/2\, n,\ n \in I}$

Pythagorean Identities:



$\sin\theta = y/r$ $\csc\theta = r/y$

$\cos\theta = x/r$ $\sec\theta = r/x$

$\tan\theta = y/x$ $\cot\theta = x/y$

By Pythagoras: $x^2 + y^2 = r^2$ $x^2 + y^2 = r^2$ $x^2 + y^2 = r^2$

divide by r^2

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$

$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

divide by y^2

$\left(\frac{x}{y}\right)^2 + 1 = \left(\frac{r}{y}\right)^2$

$\cot^2\theta + 1 = \csc^2\theta$

divide by x^2

$1 + \left(\frac{y}{x}\right)^2 = \left(\frac{r}{x}\right)^2$

$1 + \tan^2\theta = \sec^2\theta$

$$\boxed{\sin^2\theta + \cos^2\theta = 1 \qquad \cot^2\theta + 1 = \csc^2\theta \qquad 1 + \tan^2\theta = \sec^2\theta}$$

restrictions: $\cot\theta = \frac{\cos\theta}{\sin\theta}$ & $\csc\theta = \frac{1}{\sin\theta}$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ & $\sec\theta = \frac{1}{\cos\theta}$

$\therefore \sin\theta \neq 0$ $\therefore \cos\theta \neq 0$

$\theta \neq \pi n, n \in I$ $\theta \neq \pi/2 + \pi n, n \in I$

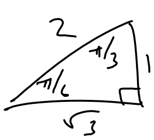
Pythagorean identities can be rearranged to form a difference of squares which can be factored into a product of two <u>*conjugate*</u> binomials: $x^2 - y^2 = (x - y)(x + y)$

ex. $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta = (1 - \cos\theta)(1 + \cos\theta)$

Ex. Verify that the equation $cot^2x + 1 = csc^2x$ is true when $x = \frac{\pi}{6}$.



$(\cot\pi/6)^2 + 1 = (\csc\pi/6)^2$

$(\sqrt{3})^2 + 1 = 2^2$

$3 + 1$

$4 = 4$ ✓

Ex. Use quotient identities to express the Pythagorean identity $cos^2x + sin^2x = 1$ as the equivalent identity $\underline{cot}^2x + 1 = \underline{csc}^2x$.

$\frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$\left(\frac{\cos x}{\sin x}\right)^2 + 1 = \csc^2 x$

$\boxed{\cot^2 x + 1 = \csc^2 x}$

Practice: pg. 296/ # 1, 3 – 8, 10 – 17, C2