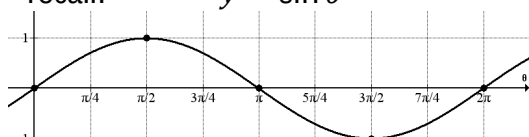


Unit 5: Trigonometry & The Unit Circle

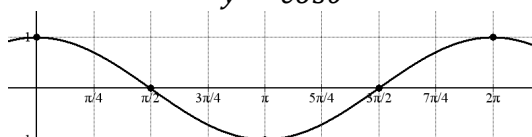
5.2 Transformations of Sinusoidal Functions

recall: $y = \sin \theta$



*starts middle $\rightarrow$ max $\rightarrow$ middle $\rightarrow$ min $\rightarrow$ middle

$y = \cos \theta$



*starts max $\rightarrow$ middle $\rightarrow$ min $\rightarrow$ middle $\rightarrow$ max

Vertical displacement (translation): moves the graph up or down.

$$y = \sin \theta + q$$

$$y = \cos \theta + q$$

* q = vertical displacement

Shifts the graph q units vertically: $q > 0$ translate up

$q < 0$ translate down

Ex. Sketch each function on the grid below for two periods:

a) $y = \sin \theta + 2$

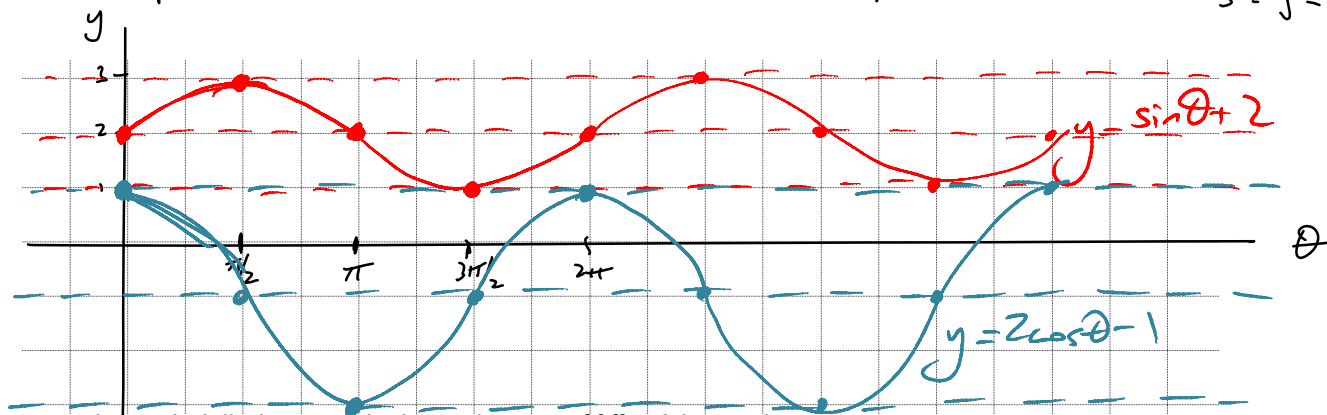
Range $-1 \leq y \leq 1$
 $+2$ $+2$
 $1 \leq y \leq 3$

amp = 1

b) $y = 2 \cos \theta - 1$

Range $-1 \leq y \leq 1$
 $\times 2$ $\times 2$
 -1 -1
 $-3 \leq y \leq 1$

amplitude = 2



Note: the vertical displacement is always the new **middle** of the graph

Phase shift (horizontal translation): moves the graph left or right

$$y = \sin(\theta - p)$$

$$y = \cos(\theta - p)$$

Shifts the graph p units horizontally: $p > 0$ translate right

$p < 0$ translate left

Ex. Sketch each function for the stated number of cycles on the grid below:

a) $y = -\cos\left(\theta + \frac{\pi}{3}\right)$; 2 cycles

x-axis reflection

phase shift

$\frac{\pi}{3} = 2\pi/6$

period = $2\pi = 12\pi/6$

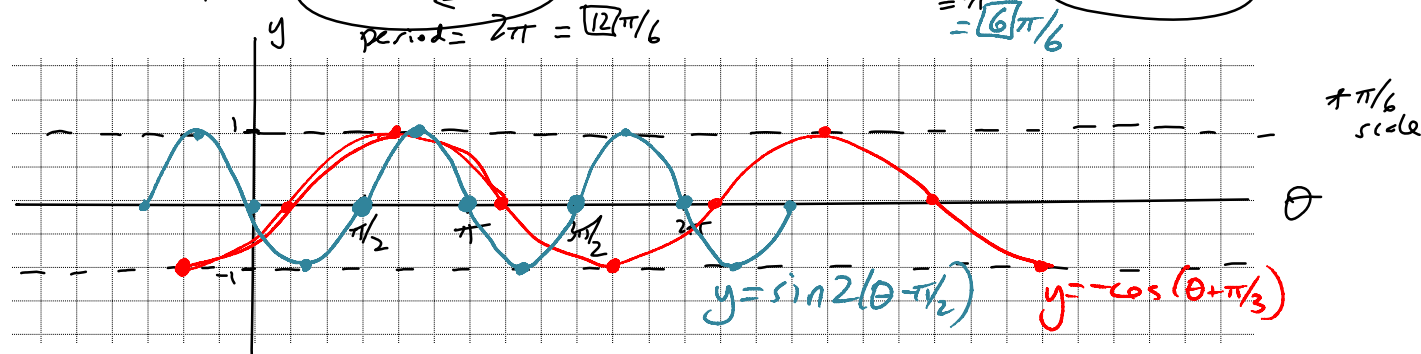
b) $y = \sin 2\left(\theta - \frac{\pi}{2}\right)$; 3 cycles

period

$2\pi \times 1/2$

$= \pi = 6\pi/6$

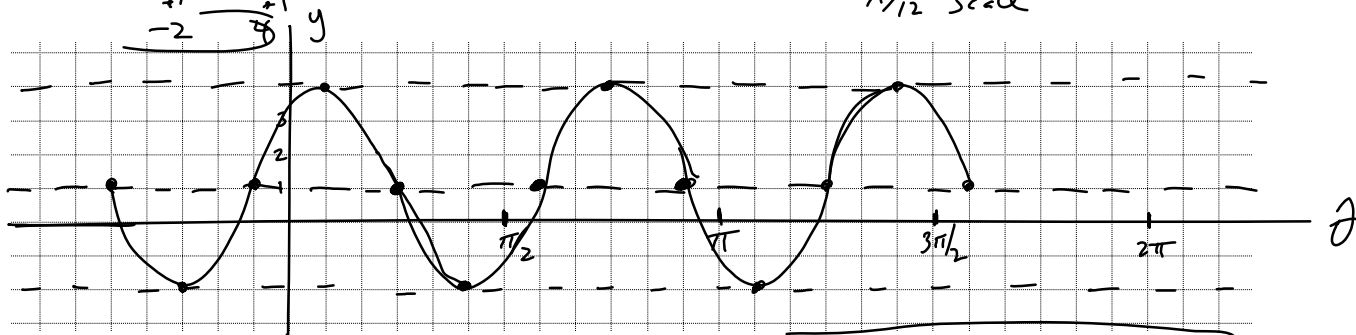
p.s. $\pi/2$



Ex. Sketch the graph of $y = -3 \sin 3\left(\theta - \frac{\pi}{4}\right) + 1$ for 3 cycles.

amp = 3 period = $\frac{2\pi}{3} = \frac{8\pi}{12}$ phase shift = $\frac{\pi}{4} = \frac{3\pi}{12}$ vertical displacement = 1 ↑

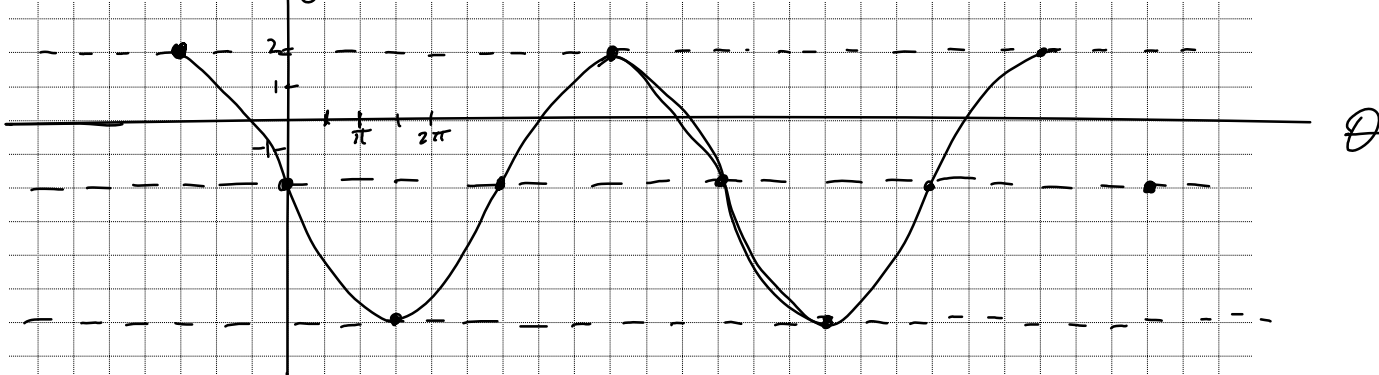
∴ range: $\begin{matrix} -1 \\ \times 3 \\ -3 \\ +1 \\ -2 \end{matrix}$ $\begin{matrix} 1 \\ \times 3 \\ 3 \\ +1 \\ 4 \end{matrix}$ * x-axis reflection * use a scale that has a common denominator with the period and the phase shift $\frac{\pi}{12}$ scale



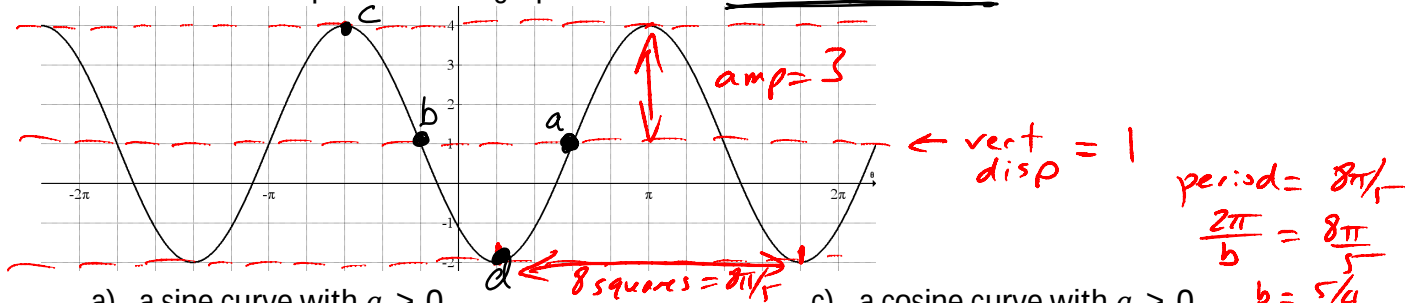
Ex. Sketch the graph of $y = 4 \cos\left(\frac{\theta}{3} + \frac{\pi}{2}\right) - 2$ for 2 periods.

amp = 4 period = $\frac{2\pi}{1/3} = 6\pi$ phase shift = $\frac{3\pi}{2}$ vertical displacement = 2 ↓

∴ range: $\begin{matrix} -1 \\ \times 4 \\ -4 \\ -2 \\ -6 \end{matrix}$ $\begin{matrix} 1 \\ \times 4 \\ 4 \\ -2 \\ 2 \end{matrix}$ * $\pi/2$ scale



Ex. Determine the equation of the graph shown with minimal phase shift if the function is:



a) a sine curve with $a > 0$

$$y = 3 \sin \frac{5}{4}(\theta - \frac{3\pi}{5}) + 1$$

b) a sine curve with $a < 0$

$$y = -3 \sin \frac{5}{4}(\theta + \frac{\pi}{5}) + 1$$

c) a cosine curve with $a > 0$

$$y = 3 \cos \frac{5}{4}(\theta + \frac{3\pi}{5}) + 1$$

d) a cosine curve with $a < 0$

$$y = -3 \cos \frac{5}{4}(\theta - \frac{\pi}{5}) + 1$$