

Unit 5: Trigonometry & The Unit Circle

5.1 Graphing Sine & Cosine Function

*web applets: <http://www.univie.ac.at/future.media/moe/galerie/fun2/fun2.html>
<http://www.ronblond.com/MathGlossary/Division04/TrigCircle/>

$$y = \sin \theta$$

$$* \sin \theta = \frac{y}{r}$$

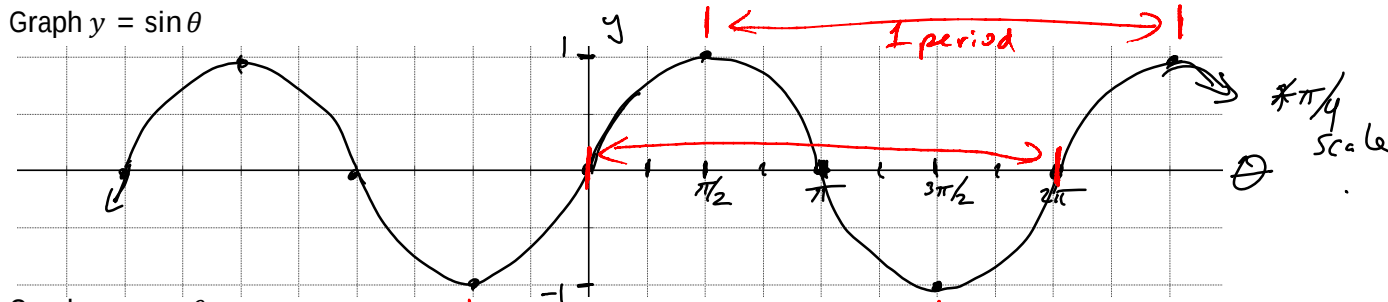
θ	0	$\pi/2$	π	$3\pi/2$	2π
$\sin \theta$	0	1	0	-1	0

$$y = \cos \theta$$

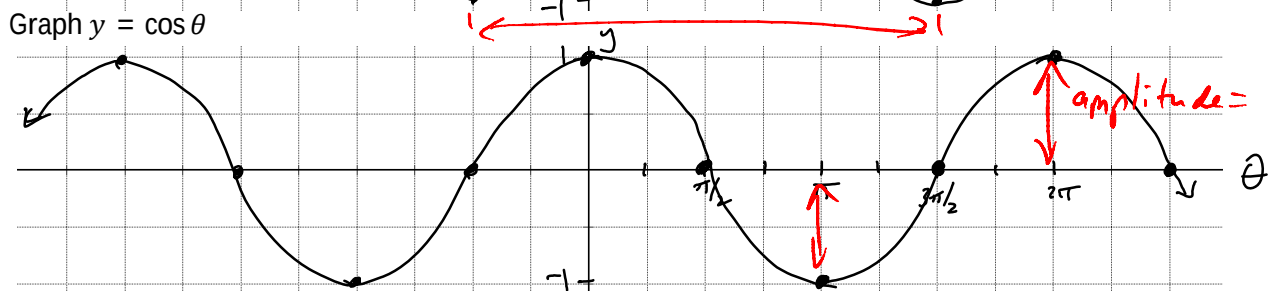
$$* \cos \theta = \frac{x}{r}$$

θ	0	$\pi/2$	π	$3\pi/2$	2π
$\cos \theta$	1	0	-1	0	1

Graph $y = \sin \theta$



Graph $y = \cos \theta$



Periodic Function: a function that repeats itself over regular intervals (cycles) of its domain

Period:

- the length of the interval of the domain over which a graph repeats itself
- the horizontal length of one cycle on a periodic graph

The period for both $y = \sin \theta$ & $y = \cos \theta$ is $2\pi / 360^\circ$.

Sinusoidal curve:

- the name given to a curve that fluctuates back and forth like a sine graph
- a curve that oscillates repeatedly up and down from a centre line

Amplitude (of a sinusoidal function): the maximum vertical distance the graph of a sinusoidal function varies above and below the horizontal central axis of the curve

$$* \text{amp} = \frac{|\text{max} - \text{min}|}{2}$$

The amplitude for both $y = \sin \theta$ & $y = \cos \theta$ is 1.

	$y = \sin \theta$	$y = \cos \theta$
Max/Min	1 / -1	1 / -1
Zeros of the function	$0, \pi, 2\pi, 3\pi \dots \theta = \pi n, n \in \mathbb{I}$	$\pi/2, 3\pi/2, 5\pi/2 \dots \theta = \pi/2 + \pi n, n \in \mathbb{I}$
y-intercept	$y = 0$	$y = 1$
Domain	$\{\theta \mid \theta \in \mathbb{R}\}$	$\{\theta \mid \theta \in \mathbb{R}\}$
Range	$\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$	$\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$

The amplitude can be changed by vertically expanding or compressing the graph.

$$y = a \sin \theta$$

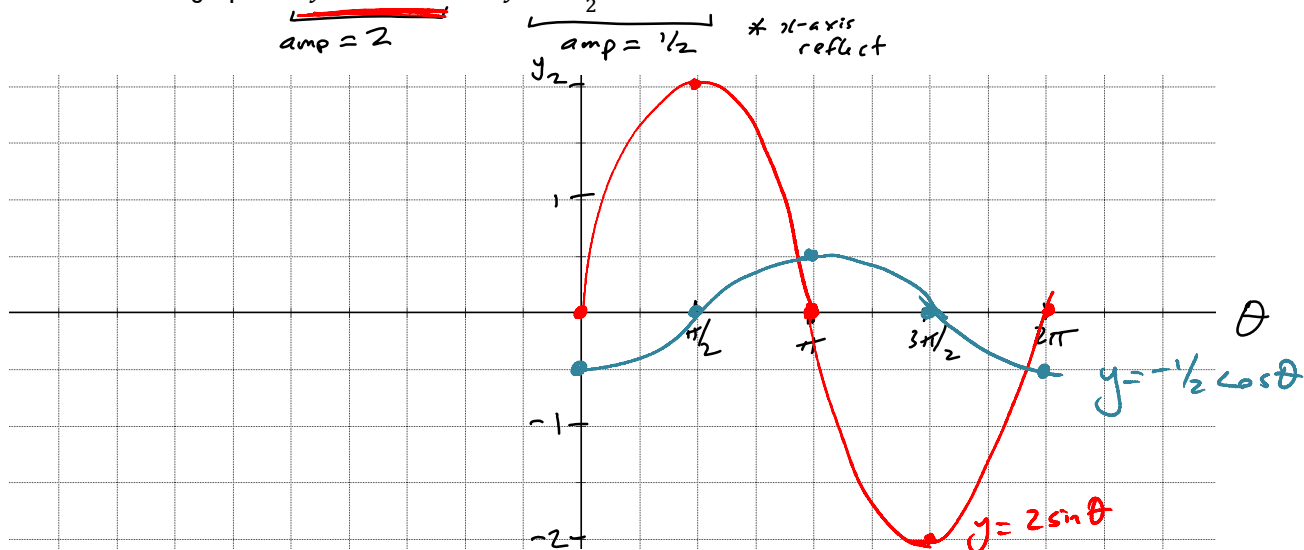
$$y = a \cos \theta$$

* $|a|$ = amplitude

Graph is expanded or compressed vertically $\times |a|$.

$0 < |a| < 1 \Rightarrow$ compression $|a| > 1 \Rightarrow$ expansion also: if $a < 0 \Rightarrow$ reflection in/over the x-axis

Ex. Sketch the graphs of $y = 2 \sin \theta$ and $y = -\frac{1}{2} \cos \theta$ on the axis below.



By expanding or compressing the graph horizontally the period is changed.

$$y = \sin b\theta$$

$$y = \cos b\theta$$

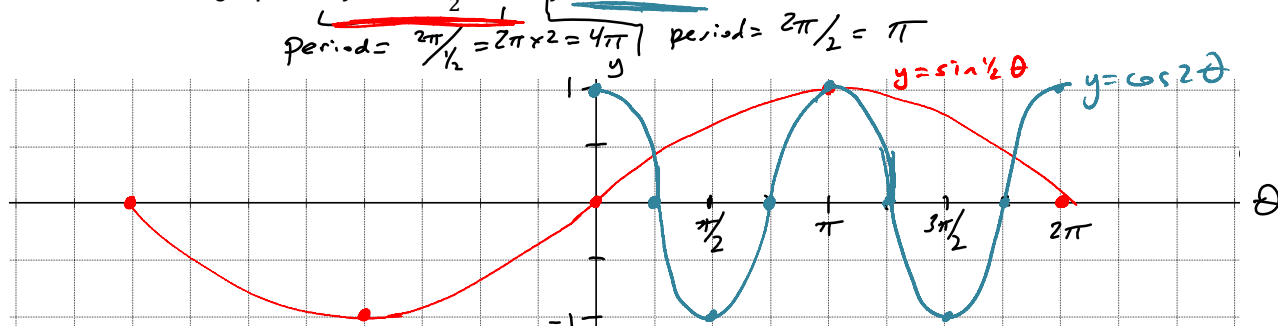
Graph is expanded or compressed horizontally $\times \frac{1}{|b|}$

$$\text{period} = \frac{\text{original period}}{|b|}$$

* $b \neq \text{period!!}$

$$\therefore \text{for sine \& cosine: period} = \frac{2\pi}{|b|}$$

Ex. Sketch the graphs of $y = \sin \frac{1}{2} \theta$ and $y = \cos 2\theta$ on the axis below.



Ex. Sketch the graph of $y = -4 \sin 3\theta$ on the axis below.

