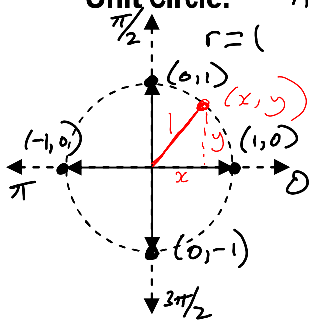
Unit 4: Trigonometry & The Unit Circle
4.2 The Unit Circle

Unit Circle: A circle of radius 1 unit centered at the origin.



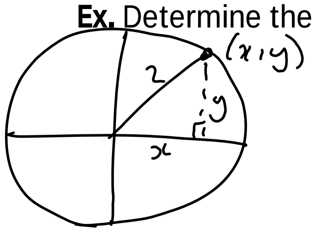
Equation of the unit circle

All points (x,y) on the unit circle are distance 1 unit from the origin

∴ by pythagoras: $x^2+y^2=1^2$

$\boxed{x^2+y^2=1}$

Ex. Determine the equation of the circle with center at the origin and radius 2 cm.



$x^2+y^2=2^2$

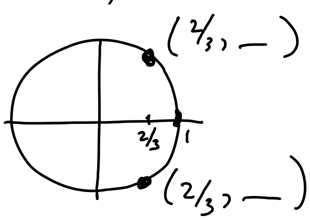
$\boxed{x^2+y^2=4}$

In general, circle with radius r centered at origin

$\boxed{x^2+y^2=r^2}$

Ex. Determine the coordinates for all points on the unit circle that satisfy the conditions given. Draw a diagram in each case.

a) x-coordinate is $\frac{2}{3}$



$x^2+y^2=1$

$(2/3)^2+y^2=1$

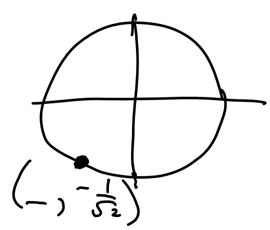
$y^2=1-4/9$

$y^2=5/9$

$y=\pm\frac{\sqrt{5}}{3}$

$\boxed{(2/3, \sqrt{5}/3)\ (2/3, -\sqrt{5}/3)}$

b) y-coordinate is $-\frac{1}{\sqrt{2}}$ & the point is in QIII



$x^2+y^2=1$

$x^2+\left(\frac{-1}{\sqrt{2}}\right)^2=1$

$x^2=1-1/2$

$x^2=1/2$

$x=\cancel{\pm}\ 1/\sqrt{2}$

$\boxed{(-1/\sqrt{2}, -1/\sqrt{2})}$ OR $\boxed{\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)}$ Rationalized

Recall (4.1): The formula for arc length given a central angle in radians is: $a=r\theta$

On the unit circle ($r=1$), this simplifies to: $a=\theta$

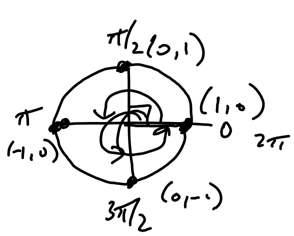
What does this mean? in the unit circle arclength is the same as the central angle

We can use the function $P(\theta)=(x,y)$ to link arc length, θ, of a central angle in the unit circle to coordinates, (x,y) of the point of intersection of the terminal arm and the unit circle.

Ex. Use $P(\theta)=(x,y)$ to find the coordinates of each $P(\theta)$.

a) $P(0)=(1,0)$ b) $P\left(\frac{\pi}{2}\right)=(0,1)$ c) $P(\pi)=(-1,0)$ d) $P\left(\frac{3\pi}{2}\right)=(0,-1)$ e) $P(2\pi)=(1,0)$

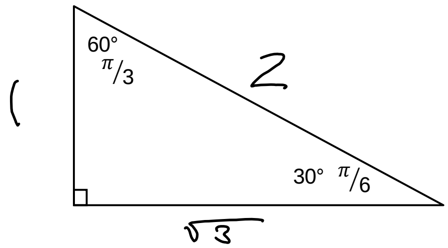
0 + 2π are coterminal

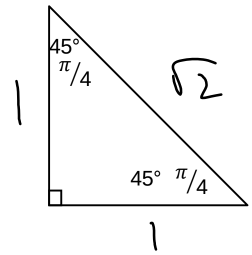


Recall Special Triangles:

30° – 60° – 90° $\quad\quad\quad\quad\quad$ 45° – 45° – 90°

$\pi/6 - \pi/3 - \pi/2$ $\quad\quad\quad\quad\quad$ $\pi/4 - \pi/4 - \pi/2$

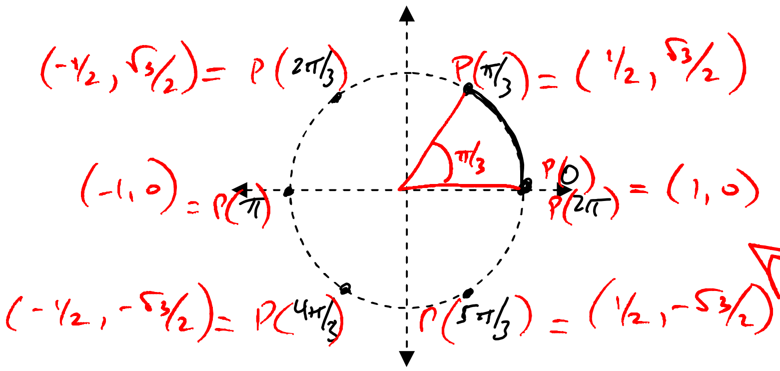




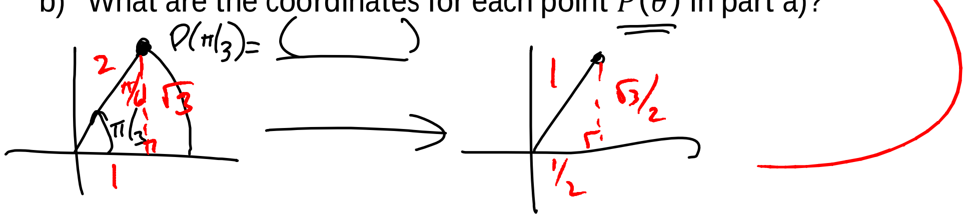
We can use this to help us find the coordinates of $P(\theta)$ for a number of angles θ.

Ex.

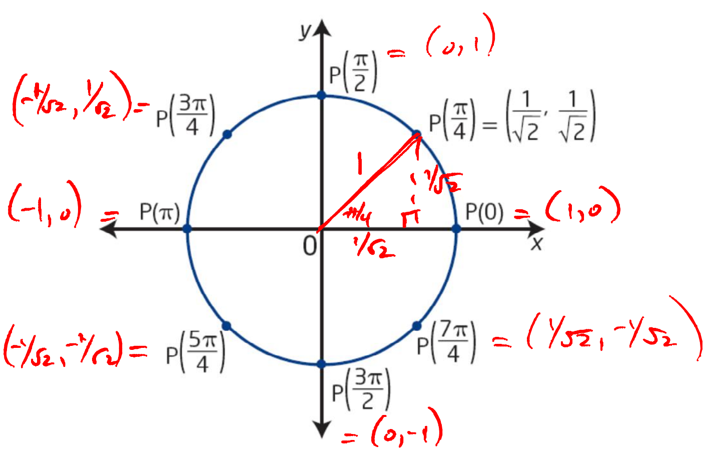
a) On a diagram of the unit circle, show the integral multiples of $\frac{\pi}{3}$ in the interval $0 \le \theta \le 2\pi$.

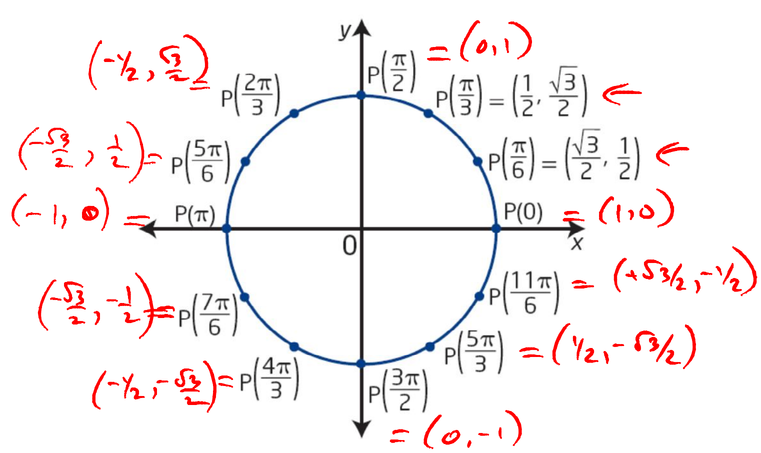


b) What are the coordinates for each point $P(\theta)$ in part a)?



c) Label the remaining points below:





Practice: pg. 186/# 1 – 7, 9, 12, 13, 17a, 20, C2