PreCalculus 12 Date __________________

Unit 3: Polynomial Functions
3.2 The Remainder Theorem

Ex. Evaluate 327 ÷ 12 using long division and write two statements summarizing the result.

dividend divisor

$$\begin{array}{r} 27 \\ 12\overline{)327} \\ -24 \\ \hline 87 \\ -84 \\ \hline 3 \end{array}$$

27 ← quotient

3 ← remainder

$\frac{327}{12} = 27 + \frac{3}{12}$

OR

$327 = 27(12) + 3$

The result of the division of a polynomial in x, $P(x)$, by a binomial of the form $x - a, a \in I$, is $\boxed{\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}}$, where $Q(x)$ is the quotient and R is the remainder.
This division statement can also be rewritten into an alternate form $\boxed{P(x) = (x - a)Q(x) + R}$.

Ex. Divide the polynomial $P(x) = 5x^3 + 10x - 13x^2 - 9$ by $x - 2$. Express the result in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$. Identify and restrictions on the variable. Verify your answer using the alternate form of the division statement.

$x \neq 2$

$$\begin{array}{r} 5x^2 - 3x + 4 \\ x-2\overline{)5x^3 - 13x^2 + 10x - 9} \\ -(5x^3 - 10x^2) \\ \hline -3x^2 + 10x \\ -(-3x^2 + 6x) \\ \hline 4x - 9 \\ -(4x - 8) \\ \hline -1 \end{array}$$

← descending order

$$\frac{5x^3 - 13x^2 + 10x - 9}{x-2} = 5x^2 - 3x + 4 + \frac{-1}{x-2}$$

$5x^3 - 13x^2 + 10x - 9 = (x-2)(5x^2 - 3x + 4) + (-1)$
$= 5x^3 - 3x^2 + 4x - 10x^2 + 6x - 8 - 1$
$= 5x^3 - 13x^2 + 10 - 9$ ✓

*evaluate $P(2) = 5(2)^3 - 13(2)^2 + 10(2) - 9 = -1$

$P(2) = -1$

Ex. The volume of a rectangular prism is given by $V(x) = x^3 + 3x^2 - 36x + 32$. Determine the possible measures for w and h in terms of x if the length, l, is $x - 4$.

$V = lwh$

$\frac{V}{l} = wh$

$$\begin{array}{r} x^2 + 7x - 8 \\ x-4\overline{)x^3 + 3x^2 - 36x + 32} \\ -(x^3 - 4x^2) \\ \hline 7x^2 - 36x \\ -(7x^2 - 28x) \\ \hline -8x + 32 \\ -(-8x + 32) \\ \hline 0 \end{array}$$

$x^2 + 7x - 8 = wh$

$(x+8)(x-1) = wh$

The possible measures for w & h are $x+8$ and $x-1$

*evaluate $V(4)$

$= (4)^3 + 3(4)^2 - 36(4) + 32$

$V(4) = 0$

Synthetic Division – quicker way to solve polynomial divisions with a divisor of the form $x - a$

Ex. Perform each division using synthetic division.

a) $\frac{2x^3+3x^2+15-4x}{x+3}$ $x-(-3)$ $a=-3$

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -4 & 15 \\ + & \downarrow & -6 & 9 & -15 \\ \hline & 2 & -3 & 5 & \boxed{0} \end{array}$$

$\underbrace{2 \quad -3 \quad 5}_{\text{quotient}}$

$2x^2-3x+5$ $R=0$

b) $\frac{4+x^3-3x+7x^2}{x-2}$

$$\begin{array}{r|rrrr} 2 & 1 & 7 & -3 & 4 \\ + & \downarrow & 2 & 18 & 30 \\ \hline & 1 & 9 & 15 & 34 \end{array}$$

$x^2+9x+15$ $R=34$

* if $f(x) = 2x^3 + 3x^2 + 15 - 4x$ evaluate $f(-3)$

$f(-3) = 2(-3)^3+3(-3)^2+15-4(-3)$
$= 0$

* if $g(x) = x^3 + 7x^2 - 3x + 4$ evaluate $g(2)$

$g(2) = 34$

The Remainder Theorem

When a polynomial in x, $P(x)$, is divided by $x-a$, the remainder is $P(a)$

Ex. Determine the remainder when $P(x) = x^3 - 10x + 6$ is divided by $x + 4$ using:

a) the remainder theorem.

$R = P(-4)$
$= (-4)^3-10(-4)+6$
$R = -18$

b) verify using synthetic division.

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -10 & 6 \\ + & \downarrow & -4 & 16 & -24 \\ \hline & 1 & -4 & 6 & -18 \end{array}$$ ✓

Ex. When the polynomial $\underbrace{x^3 - kx^2 + 17x + 6}_{P(x)}$ is divided by $x - 3$, the remainder is 12. What is the value of k.

by the remainder theorem $P(3) = 12$

$(3)^3-k(3)^2+17(3)+6=12$
$27-9k+51+6=12$
$-9k+84=12$
$-9k=-72$
$\boxed{k=8}$

Practice: pg. 124/# 2 – 5, 7 – 10, 14 – 16, 17ab