

Unit 3: Polynomial Functions

3.1 Characteristics of Polynomial Functions

Polynomial Function: A function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a$$

- n whole number
- x variable
- coefficients a_n to a are real numbers

Ex. Determine if each function is polynomial. If it is state the degree, leading coefficient, and the constant term.

a) $g(x) = \sqrt{x} + 5$
 $\downarrow$
 $x^{1/2}$
 $1/2 \neq \text{whole}$
NOT
 Radical Function

c) $y = |x|$
NOT
 Absolute Value Function

e) $h(x) = \frac{1}{x}$
 x^{-1}
 $-1 \neq \text{whole}$
NOT
 Rational Function

b) $f(x) = 3x^4$
 YES ✓ degree = 4
 leading = 3
 constant = 0

d) $y = 2x^3 + 3x^2 - 4x - 1$
 YES ✓ degree = 3
 leading = 2
 constant = -1

f) $q(x) = 5x^2 - 3 - 6x^5 + \sqrt{3}x$
 YES ✓ degree = 5
 leading = -6
 constant = -3

Characteristics of Polynomial Functions

Degree 0: Constant

Degree 1: Linear

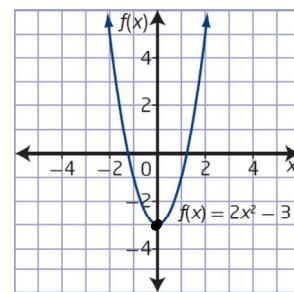
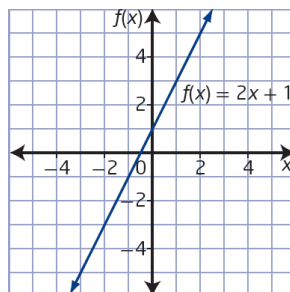
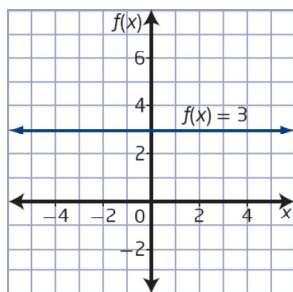
Degree 2: Quadratic

of x-intercepts: 0 (* except $f(x) = 0$)

1

0, 1, or 2

Ex.



End behaviour: extends horizontally

Domain: $\{x | x \in \mathbb{R}\}$

Range: $\{y | y = 3\}$

of x-intercepts: 0

of relative extrema: 0
 (maximum/minimum)

down in QIII & up in QI

$\{x | x \in \mathbb{R}\}$

$\{y | y \in \mathbb{R}\}$

1

0

up in QII & QI

$\{x | x \in \mathbb{R}\}$

$\{y | y \geq -3\}$

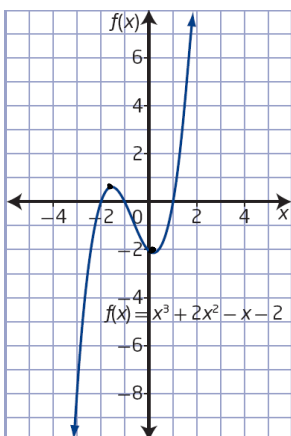
2

1

Degree 3: Cubic

of x-intercepts: 1, 2 or 3

Ex.



End behaviour: down in QIII & up in QI

Domain: $\{x | x \in \mathbb{R}\}$

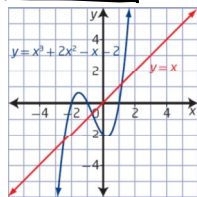
Range: $\{y | y \in \mathbb{R}\}$

of x-intercepts: 3

of relative extrema: 2

Summary: Even & Odd Polynomials → ALL Domain $\{x | x \in \mathbb{R}\}$; max # of x-int = n (degree); y-int = constant term

Odd degree ($a > 0$) * similar $y = x$



↓ QIII & ↑ QI

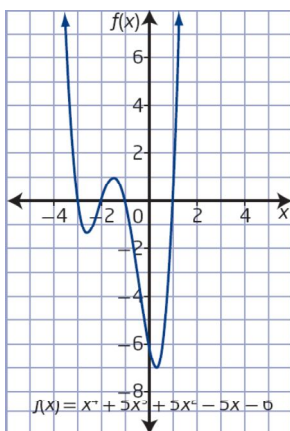
x-int 1 to n

Range $\{y | y \in \mathbb{R}\}$

no absolute max/min

Degree 4: Quartic

0, 1, 2, 3, or 4



up in QII & QI

$\{x | x \in \mathbb{R}\}$

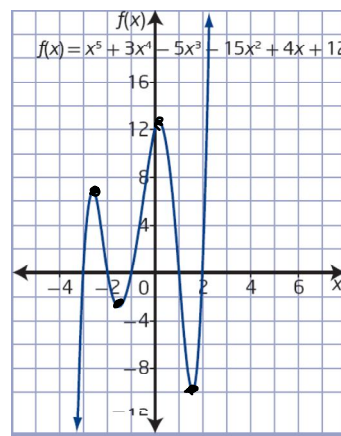
$\{y | y \geq -6.9, y \in \mathbb{R}\}$

4

3

Degree 5: Quintic

1, 2, 3, 4, or 5



down in QIII & up in QI

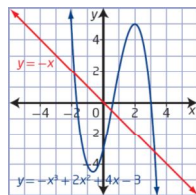
$\{x | x \in \mathbb{R}\}$

$\{y | y \in \mathbb{R}\}$

5

4

Odd degree ($a < 0$) max # of rel. extrema = $n - 1$



* similar to $y = -x$

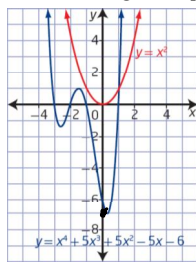
↑ QII & ↓ QIV

x-int 1 to n

Range $\{y | y \in \mathbb{R}\}$

no absolute max/min

Even degree ($a > 0$) * similar to $y = x^2$

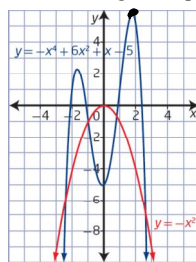


↑ in QII & I

x-int 0 to n

Range: $\{y | y \geq \text{abs. min}, y \in \mathbb{R}\}$
↑
guaranteed

Even degree ($a < 0$) * similar to $y = -x^2$



↓ in QIII & IV

x-int 0 to n

Range: $\{y | y \leq \text{abs. max}, y \in \mathbb{R}\}$
↑
guaranteed