

Unit 11: Combinatorics

11.2 Combinations

Combination: A selection of objects without regard to order

Ex. All of the three-letter combinations of P, Q, R, and S are PQR, PQS, PRS, and QRS (arrangements such as PQR, RPQ, and QRP are the same combination)

Ex. 5 people compete in a race.

a) How many ways can medals for 1st, 2nd, & 3rd be awarded?

Permutation

$${}_5P_3 = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2!}{2!} = \boxed{60}$$

b) If the top 3 times qualify for the relay team, how many such teams are possible?

Combination

$$60 = \frac{\# \text{ of } 3 \text{ person groups}}{3!} \cdot 3! \quad \therefore \frac{\# \text{ of teams}}{3!} = \frac{60}{3!} = \boxed{10}$$

The number of combinations of n distinct objects taken r at a time is:

$$\begin{aligned} {}_nC_r &= \frac{{}_nP_r}{r!} \\ &= \frac{n!}{(n-r)!r!} \end{aligned}$$

Calc: math, prb, #3

Ex. How many possible tickets are there for the Lotto 6/49 draw? (Choose 6 numbers from 1 to 49)

$$49C_6 = \boxed{13,983,816}$$

Ex. A group of 4 students is selected from 30 students. How many such groups are possible if:

a) there are no restrictions?

$$30C_4 = \boxed{27,405}$$

b) one person is selected president, another vice-president, the third treasurer, and the last secretary?

$$= 27405 \cdot 4! \quad \text{OR} \quad {}_{30}P_4 = \boxed{657,720}$$

c) John & Jane must be in the group?

$${}^1_2C_2 \cdot {}_{28}C_2 = \boxed{378}$$

d) John & Jane cannot both be in the group?

$$\overset{\text{John or Jane}}{2C_1} \cdot {}_{28}C_3 + \overset{\text{Neither}}{2C_0} \cdot {}_{28}C_4 = \boxed{27,027}$$

OR

$$\text{All} - \text{John \& Jane in} = 27405 - 378 = \boxed{27,027}$$

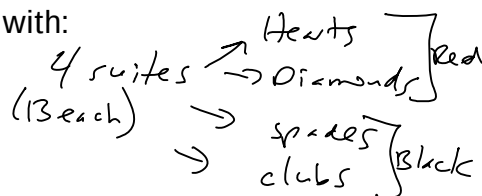
$$* {}_2C_0 = \frac{2!}{(2-0)!0!} = 1$$

$${}_2C_2 = \frac{2!}{(2-2)!2!} = 1$$

Ex. How many 5 card hands are possible (from a regular 52 card deck) with:

a) no restriction?

$$52C_5 = 2,598,960$$



b) all hearts?

$$13C_5 = 1287$$

c) 3 hearts & 2 spades?

$$13C_3 \cdot 13C_2 = 22,308$$

d) exactly 3 hearts?

$$13C_3 \cdot 39C_2 = 211,926$$

e) at least 3 hearts?

$$34C_5 \text{ or } 44C_5 \text{ or } 54C_5$$

$$13C_3 \cdot 39C_2 + 13C_4 \cdot 39C_1 + 13C_5 \cdot 39C_0$$

$$= 211,926 + 27,885 + 1287$$

$$= 241,098$$

$$* nC_1 = n$$

Ex. Solve for n if $2(nC_2) = {}_{n+1}C_3$

$$2 \left(\frac{n!}{(n-2)!2!} \right) = \frac{(n+1)!}{(n+1-3)!3!}$$

$$\frac{n!}{1} = \frac{(n+1)!}{(n-2)! \cdot 3!}$$

$$3! = \frac{(n+1)!}{n!}$$

$$6 = \frac{(n+1)n!}{n!}$$

$$n = 5$$

check $2(5C_2) = 6C_3$
 $20 = 20 \checkmark$

~~$$13C_3 \cdot 49C_2$$~~
~~$$4H \ 6H \ 9H \ 10H \ 13S$$~~
~~$$4H \ 6H \ 10H \ 9H \ 13S$$~~