PreCalculus 12 Date _____

Unit 11: Combinatorics
11.2 Combinations

Combination: A selection of objects without regard to order

Ex. All of the three-letter combinations of P, Q, R, and S are PQR, PQS, PRS, and QRS (arrangements such as PQR, RPQ, and QRP are the same combination)

Ex. 5 people compete in a race.

a) How many ways can medals for 1st, 2nd, & 3rd be awarded?

Permutation $_5P_3 = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2!}{2!} = 60$

b) If the top 3 times qualify for the relay team, how many such teams are possible?

Combination $60 = \#\text{ of 3 person groups} \cdot 3!$ $\therefore \#\text{ of teams} = \frac{60}{3!} = 10$.

The number of combinations of n distinct objects taken r at a time is:

$$_nC_r = \frac{_nP_r}{r!} = \frac{n!}{(n-r)!\,r!}$$

Calc: math, prb, #3

Ex. How many possible tickets are there for the Lotto 6/49 draw? (Choose 6 numbers from 1 to 49)

$_{49}C_6 = 13,983,816$

Ex. A group of 4 students is selected from 30 students. How many such groups are possible if:

a) there are no restrictions?

$_{30}C_4 = 27,405$

b) one person is selected president, another vice-president, the third treasurer, and the last secretary?

$= 27405 \cdot 4!$ OR $_{30}P_4 = 657,720$

c) John & Jane must be in the group?

$_2C_2 \cdot {}_{28}C_2 = 378$

d) John & Jane cannot both be in the group?

John or Jane + Neither

$_2C_1 \cdot {}_{28}C_3 + {}_2C_0 \cdot {}_{28}C_4 = 27,027$ * $_2C_0 = \frac{2!}{(2-0)!\,0!} = 1$

OR

All − John + Jane in

$27405 - 378 = 27,027$ $_2C_2 = \frac{2!}{(2-2)!\,2!} = 1$

Ex. How many 5 card hands are possible (from a regular 52 card deck) with:

4 suites (13 each) → Hearts, Diamonds (Red); Spades, Clubs (Black)

a) no restriction?

$$_{52}C_5 = 2,598,960$$

b) all hearts?

$$_{13}C_5 = 1287$$

c) 3 hearts & 2 spades?

$$_{13}C_3 \cdot {}_{13}C_2 = 22,308$$

d) exactly 3 hearts?

$$_{13}C_3 \cdot {}_{39}C_2 = 211,926$$

e) at least 3 hearts?

3H/5 or 4H/5 or 5H/5

$$_{13}C_3 \cdot {}_{39}C_2 + {}_{13}C_4 \cdot {}_{39}C_1 + {}_{13}C_5 \cdot {}_{39}C_0$$

$$= 211,926 + 27885 + 1287$$

$$= 241,098$$

* $_nC_1 = n$

Ex. Solve for n if $2({}_nC_2) = {}_{n+1}C_3$

$$2\left(\frac{n!}{(n-2)!\,2!}\right) = \frac{(n+1)!}{(n-2)!\,3!}$$

$$\frac{n!}{1} = \frac{(n-2)!\,(n+1)!}{(n-2)!\cdot 3!}$$

$$3! = \frac{(n+1)!}{n!}$$

$$6 = (n+1)$$

$$n = 5$$

check $2({}_5C_2) = {}_6C_3$

$20 = 20$ ✓

$_{13}C_3 \cdot {}_{49}C_2$

4H 6H 9H AH KS

4H 6H AH 9H KS

Practice: pg. 534/ # 1 – 6, 8 – 11, 13 – 20, 22, 23
Worksheet