PreCalculus 12 Date _____

Unit 11: Combinatorics

11.1B Permutations

Permutation: An ordered arrangement or sequence of all or part of a set

Ex. The possible permutation of the letters ABC are: ABC, ACB, BAC, BCA, CAB, and CBA

A ***permutation*** is an arrangement of objects in a definite order (as opposed to a ***combination*** where order is not important – to be covered in 11.2)

Ex. How many ways can five people be arranged in a line?

FCP $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120$

The number of permutations of n distinct object is equal to $n!$

ex. How many ways can 9 people be arranged in a line:

a) without restrictions?

$9! = 362,880$

b) if Bob & Doug insist on standing next to each other?

$8! \cdot 2! = 80,640$

c) if Jill, Jane, & Jenn insist on standing together?

$7! \cdot 3! = 30,240$

d) if Clark & Lex refuse to stand next to each other?

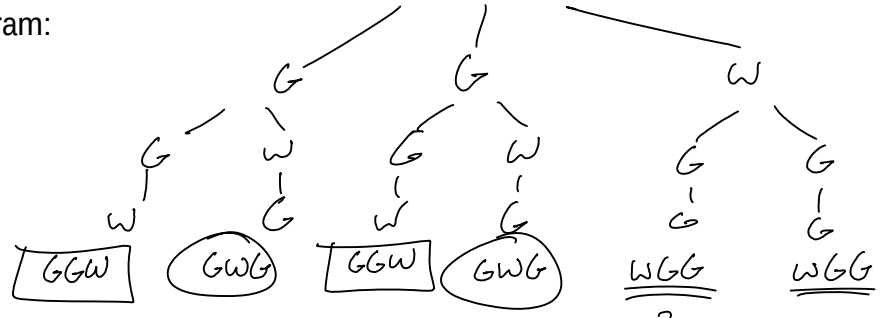
All – two are together

$= 9! - 8!2! = 362,880 - 80,640 = 282,240$

What if some of the items being arranged are not distinct (ie. cannot be told apart)?

Ex. How many ways can two identical green chairs and one white chair be arranged in a line?

Tree diagram:



*3! = 6 outcomes, but not all are unique. There are only 3 unique arrangements

*ie. divide by the factorial of the # of repeated objects

$\frac{3!}{2!1!}$

The number of permutations of n objects of which a objects are identical, another b objects are identical, etc... is given by

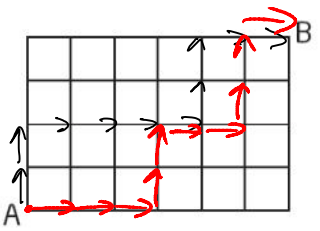
$$\frac{n!}{a!\,b!\ldots}$$

Ex. How many different arrangements can you make using the letters of *aardvark*?

$\frac{8!}{(3!2!)} = 3360$

8 letters repeating?
3 a's
2 r's

Ex. How many paths can you follow from A to B in a four by six rectangular grid if you move only up or to the right?



UURRRRUURR
RRRUURRUUR

$\frac{10!}{6!4!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6!}{6!4!} = 210$

10 moves total
6 right & 4 up

Often we only take some of the available items and arrange them.

Ex. How many ways can you arrange 3 people chosen from a group of 10?

FCP

$\underline{10}\ \underline{9}\ \underline{8} = 720$

$= \frac{10\cdot 9\cdot 8\cdot 7!}{7!}$

$= \frac{10!}{7!} = \frac{10!}{(10-3)!}$

$_{10}P_3$ "10 Permutate 3"

The number of permutations of n objects taken r at a time is given by:

$$_nP_r = \frac{n!}{(n-r)!}$$

Ex. How many different 4 digit numbers can be made using the digits 1, 2, 3, 4, 5, & 6 without repetition?

$_6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 360$

Ex. One hundred people each bought one ticket for a prize draw which has different 1st, 2nd & 3rd prizes. How many ways could the prizes be distributed?

$_{100}P_3 = 970{,}200$

acceptable work ☺

$_nP_r$ function in calc: math, prb, #2 ($_nP_r$ also found on most scientific calculators)

Do **NOT** forget about the Fundamental Counting Principle!!!

Practice: pg. 524/ # 2, 5, 7, 8, 10, 11, 16 – 18, 22 – 24, 32
Worksheet