PMA12 Date:_____
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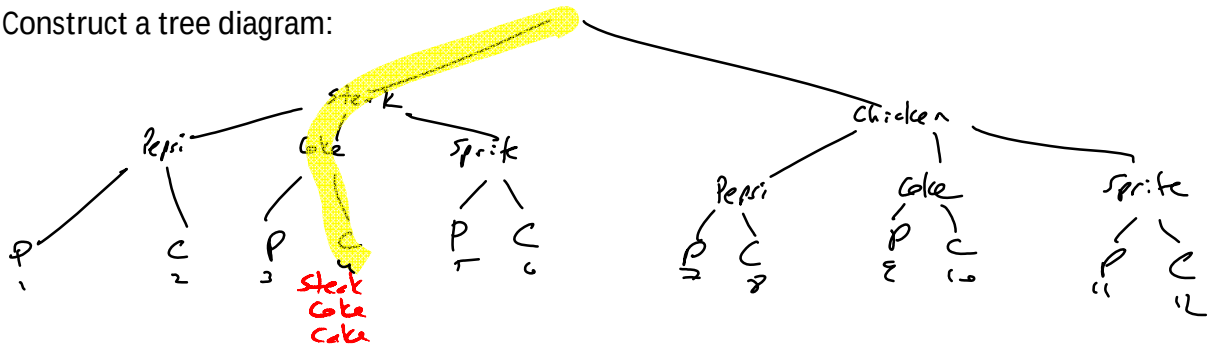
Unit 11: Combinatorics
11.1A Fundamental Counting Principle & Factorials

Ex. A restaurant special offers you a choice of steak or chicken for the main course; Pepsi, Coke or Sprite to drink; and pie or cake for dessert. How many meal combinations are possible?
Construct a tree diagram:



Each pathway illustrates one possible meal combination.
The total number is equal to the total number of end points on the bottom level.

Thus, there are 12 combinations.

Note: main course – 2 choices
drink – 3 choices
dessert – 2 choices

$2 \times 3 \times 2 = 12$

Fundamental Counting Principle

If one item can be selected m ways, a second item n ways, a third item p ways etc... then the total number of possible combinations is given by $m \times n \times p \times \ldots$

Ex. How many different possibilities are there for a four character license plate consisting of 2 letters followed by 2 digits if:

a) there are no restrictions?

4 levels of choices to be made (one per character)

work! → $26 \cdot 26 \cdot 10 \cdot 10$ *note: 10 digits from 0 to 9 !

$= 67{,}600$

b) there can be no repetitions?

4 levels of choices to be made (one per character)

$26 \cdot 25 \cdot 10 \cdot 9$

$= 58{,}500$

Ex. How many 3 digit numbers can be made (without repetition) from the digits 0, 3, 4, & 9?

*a number cannot begin with 0

$3 \cdot 3 \cdot 2 = 18$

How many such numbers are greater than 350?

0, 3, 4, 9

start with 4 or 9 OR start with 3 then 9

$2 \cdot 3 \cdot 2 + 1 \cdot 1 \cdot 2$

$12 + 2 = 14$

* AND = X
OR = +

Factorial Notation:

$n! = n \times (n-1) \times (n-2) \times \ldots \times 1$

Ex. $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Can also use the factorial function on your calculator: math, prb, #4

*many scientific calculators have factorial on it as well, with easier access

Ex. $10! = 3,628,800$

Ex. simplify: $\frac{7!}{3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!}$

$= 42 \cdot 20$

$= 840$

Ex. simplify: $\frac{(n-1)!}{n!}$

$= \frac{(n-1)!\,1}{n \cdot (n-1)!}$

$= \frac{1}{n}$

* $\frac{3!}{4!} = \frac{3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1}$

$= \frac{1}{4}$

Ex. solve: $\frac{n!}{(n-2)!} = 12$

$\frac{n(n-1)(n-2)!}{(n-2)!} = 12$

$n^2 - n - 12 = 0$

$(n-4)(n+3) = 0$

$n = 4$ $\quad n = -3$

*note: 0! = 1 by definition

Practice: pg. 524/ # 1, 3, 4, 6, 12 – 15, 19, 20, 25, 26, C3
Worksheets