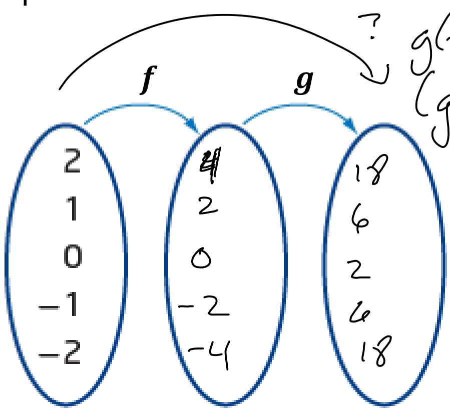
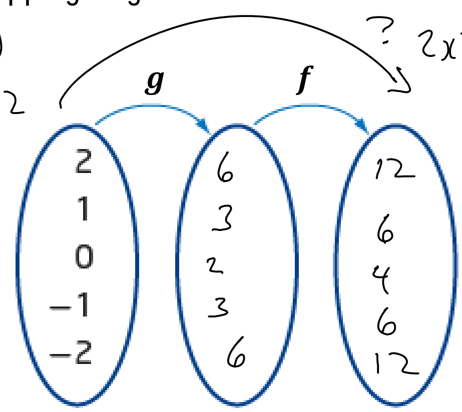
Unit 10: Function Operations
10.3 Composite Functions

Consider the functions $f(x) = 2x$ and $g(x) = x^2 + 2$. The output values of a function can become the input values for another function. Complete the mapping diagram below.

? $g(f(x)) = g(2x)$
$(g \circ f)(x) = (2x)^2 + 2$
$= 4x^2 + 2$



? $2x^2 + 4 = f(g(x))$
$(f \circ g)(x)$



* order matters!

Recall Function Notation:

If $f(x) = 2x$ then,

$f(-6) = 2(-6)$
$= -12$

$f(m) = 2m$

$f(☺) = 2☺$

$f(5x) = 2(5x)$
$= 10x$

$f(x^2 + 2) = 2(\quad)$

$2(x^2+2)$
$\underline{\quad\quad}_{g(x)} = 2x^2 + 4$] $f(g(x))$

Substituting one function into another is called **composition of functions** and results in a **composite function**.

* $(f \circ g)(x) \neq (fg)(x)$

Notation: $f(g(x))$ OR $(f \circ g)(x)$

"f of g of x" "f composed with g"

"f of g at x"

* generally
$f \circ g \neq g \circ f$

Ex. Use the functions: $f(x) = 4x$; $g(x) = x - 2$; and $h(x) = x^2$ to determine:

a) $f(g(x)) = f(x-2)$
$= 4(x-2)$
$= 4x - 8$

b) $g(f(x)) = g(4x)$
$= 4x - 2$

c) $(g \circ h)(x) = g(h(x))$
$= g(x^2)$
$= \boxed{x^2 - 2}$

d) $(g \circ h)(-2) = (-2)^2 - 2$
$= 2$

OR $g(h(-2))$
$= g((-2)^2)$
$= g(4)$
$= 4 - 2$
$= 2$

e) $h(h(x)) = h(x^2)$
$= (x^2)^2$
$= \boxed{x^4}$

f) $h(h(2)) = (2)^4$
$= 16$

Ex. Consider $f(x) = \sqrt{x-1}$ and $g(x) = x^2$.

a) Determine $(f \circ g)(x)$ and $(g \circ f)(x)$.

$f(g(x))$
$= f(x^2)$
$= \boxed{\sqrt{x^2-1}}$

$g(f(x))$
$= g(\sqrt{x-1})$
$= (\sqrt{x-1})^2 \leftarrow$
$= x-1$

b) Determine the domain of $f(x)$, $g(x)$, $(f \circ g)(x)$, and $(g \circ f)(x)$.

f $\boxed{x \geq 1}$ $\quad g$ $\boxed{\mathbb{R}}$

$f \circ g$ $\quad x^2-1 \geq 0$
$x^2 \geq 1$
$\underbrace{\boxed{x \geq 1 \text{ OR } x \leq -1}}_{|x| \geq 1}$

$g \circ f$ $\boxed{x \geq 1}$

Ex. If $h(x) = f(g(x))$, determine $f(x)$ and $g(x)$.

a) $h(x) = (x-2)^2 + (x-2) + 1$

$f(x) = x^2 + x + 1$ $\quad g(x) = x-2$

b) $h(x) = \sqrt{x^3+1}$

$f(x) = \sqrt{x}$ $\quad g(x) = x^3+1$ $\quad$ OR $\quad f(x) = \sqrt{x+1}$ $\quad g(x) = x^3$

Ex. Use the graphs of $f(x)$ and $g(x)$ to find:

a) $f(-1) = 5$

b) $g(0) = 2$

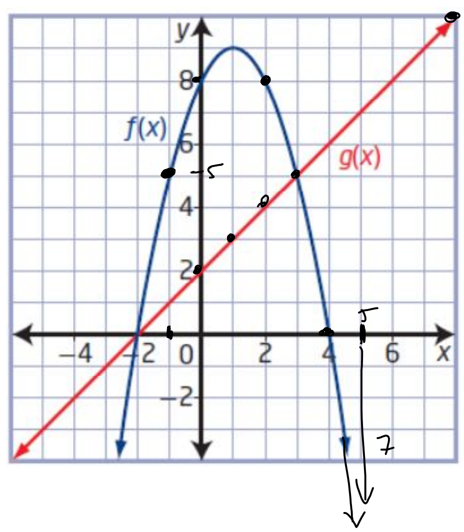
c) $f(g(0)) = f(2) = 8$

d) $g(f(0)) = g(8) = 10$

e) $(g \circ g)(1) = g(g(1)) = g(3) = 5$

f) $(f \circ f)(-1) = f(f(-1)) = f(5) = -7$

g) $(f \circ f \circ g \circ g)(0) = f(f(g(g(0))))$
$= f(f(g(2)))$
$= f(f(4))$
$= f(0) = 8$



Recall the inverse function $f^{-1}(x)$:

Ex. Use the function $f(x) = 4x + 3$ to find:

a) $f^{-1}(x)$

$y = 4x + 3$
$x = 4y + 3$
$y = \frac{x-3}{4}$
$\boxed{f^{-1}(x) = \frac{x-3}{4}}$

b) $f(f^{-1}(x))$

$= f\left(\frac{x-3}{4}\right)$
$= \cancel{4}\left(\frac{x-3}{\cancel{4}}\right) + 3$
$= x - 3 + 3$
$= x$

c) $f^{-1}(f(x))$

$= f^{-1}(4x+3)$
$= \frac{(4x+3)-3}{4}$
$= \frac{4x}{4}$
$= x$

* Two functions f + g are inverses of each other iff $f \circ g = x$ + $g \circ f = x$

Practice: pg. 507/ # 1 – 12, 14, 17, 19, 20, C2