Unit 1: Function Transformations

1.4 Inverse of a Relation

Every operation in math has an inverse operation:

$\times$ & $\div$; $+$ & $-$; $\square^2$ & $\sqrt{\ }$; sine & arcsine
$\sin^{-1}$

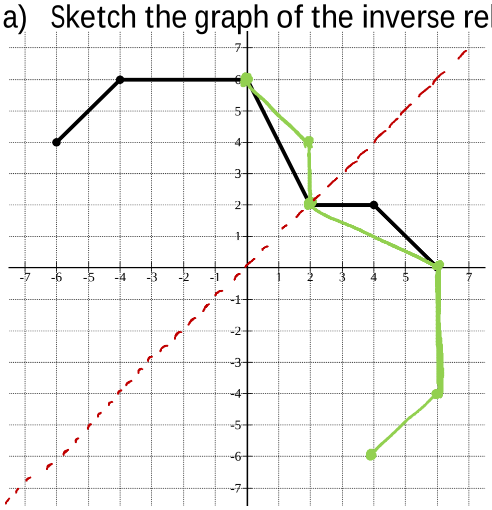
Inverse of a Function:

- if f is a function with domain A and range B, the inverse function, if it exists, is denoted by f^{-1} and has domain B and range A
- f^{-1} maps y to x if and only if f maps x to y
- Mapping notation: $(x, y) \Rightarrow (y, x)$
- Note: the -1 in f^{-1} does NOT represent an exponent; ie. $f^{-1}(x) \neq \frac{1}{f(x)}$
- f^{-1} is read as "f inverse"

Graphing an Inverse:

Ex. Consider the graph of the relation shown.

a) Sketch the graph of the inverse relation on the same grid.



x	y
-6	4
-4	6
0	6
2	2
4	2
6	0

$\Rightarrow$ inverse

y	x	
4	-6	$\rightarrow (4,-6)$
6	-4	$\rightarrow (6,-4)$
6	0	
2	2	
2	4	
0	6	

Note: the inverse graph is another type of reflection.

$y=x$ is the line of reflection

* under an inverse, invariant points are found on the line $y=x$

b) State the domain and range of the relation and its inverse.

	Domain	Range
Relation	$\{x \mid -6 \le x \le 6, x \in \mathbb{R}\}$ OR $[-6, 6]$	$\{y \mid 0 \le y \le 6, y \in \mathbb{R}\}$ OR $[0, 6]$
Inverse Relation	$\{x \mid 0 \le x \le 6, x \in \mathbb{R}\}$ $[0, 6]$	$\{y \mid -6 \le y \le 6, y \in \mathbb{R}\}$ $[-6, 6]$

c) Determine whether the relation and its inverse are functions.

The relation is a function, it passes the vertical line test.

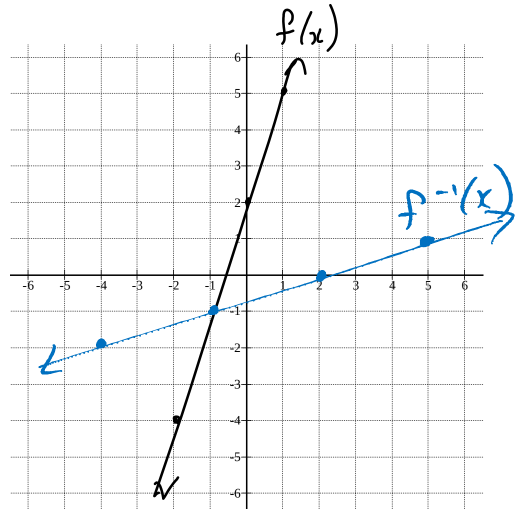
The inverse is ~~not~~ a function, it fails the vertical line test.

* Horizontal Line Test

The inverse of a given relation is a function only if a horizontal line intersects the relation no more than once.

The Equation of the Inverse

Ex. Consider the function $f(x) = 3x + 2$



a) Graph the function $f(x)$. Is the inverse of $f(x)$ a function?

Yes, $f(x)$ passes the horizontal line test

b) Sketch the graph of the inverse of $f(x)$ on the same grid.

c) Algebraically determine the equation of the inverse of $f(x)$.

$f(x) = 3x+2$

$y = 3x+2$

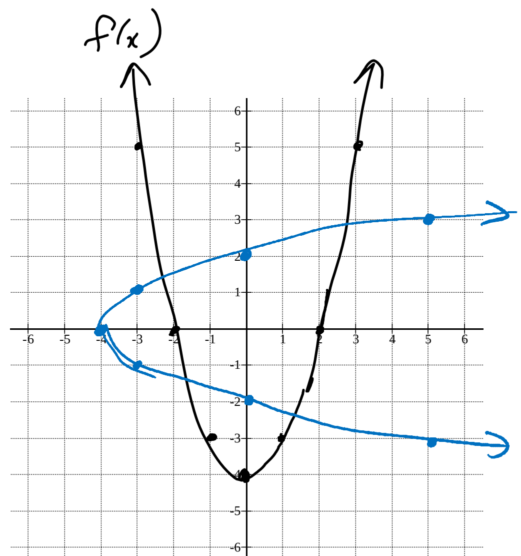
$x = 3y+2$

$y = \frac{x-2}{3}$ $\Rightarrow$ $\boxed{f^{-1}(x) = \frac{x-2}{3}}$

* switch x + y and solve for y

Restrict the Domain

Ex. Consider the function $f(x) = x^2 - 4$



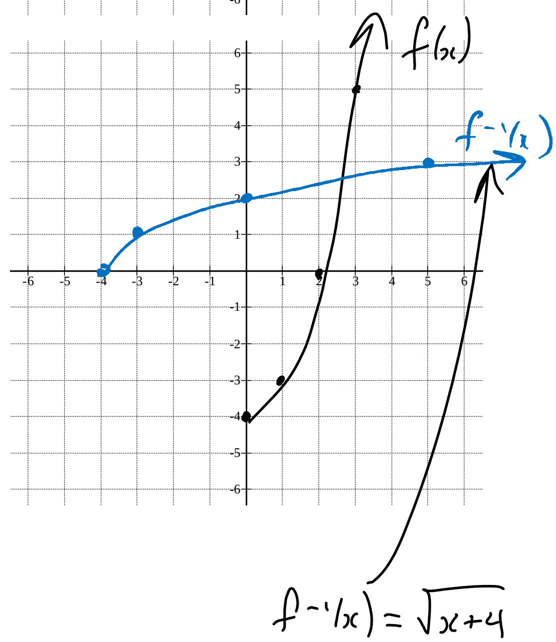
a) Graph the function $f(x)$. Is the inverse of $f(x)$ a function?

No, it fails the horizontal line test.

b) Sketch the graph of the inverse of $f(x)$ on the same grid.

c) Describe how the domain of $f(x)$ could be restricted so that the inverse of $f(x)$ is a function. [demonstrate on a grid] * eliminate ½ of $f(x)$

$\boxed{f(x) = x^2 - 4 \ ; \ \{x \mid x \geq 0, x \in \mathbb{R}\}}$



d) Algebraically determine the equation of the inverse of $f(x)$.

$f(x) = x^2 - 4$

$y = x^2 - 4$

$x = y^2 - 4$

$y^2 = x+4$

$\boxed{y = \pm\sqrt{x+4}}$

Practice: pg. 51/# 2 – 7, 9, 10, 12, 14 – 16, 19 – 21