Key Ideas

- Given the sequence $t_1, t_2, t_3, t_4, \ldots, t_n$ the associated series is $S_n = t_1 + t_2 + t_3 + t_4 + \cdots + t_n$.
- For the general arithmetic series,
  $t_1 + (t_1 + d) + (t_1 + 2d) + \cdots + (t_1 + [n - 1]d)$ or
  $t_1 + (t_1 + d) + (t_1 + 2d) + \cdots + (t_n - d) + t_n$,
  the sum of the first n terms is
  $S_n = \frac{n}{2}[2t_1 + (n - 1)d]$ or $S_n = \frac{n}{2}(t_1 + t_n)$,
  where t_1 is the first term
  n is number of terms
  d is the common difference
  t_n is the nth term
  S_n is the sum to n terms

Check Your Understanding

Practise

1. Determine the sum of each arithmetic series.

a) $5 + 8 + 11 + \cdots + 53$

b) $7 + 14 + 21 + \cdots + 98$

c) $8 + 3 + (-2) + \cdots + (-102)$

d) $\frac{2}{3} + \frac{5}{3} + \frac{8}{3} + \cdots + \frac{41}{3}$

2. For each of the following arithmetic series, determine the values of t_1 and d, and the value of S_n to the indicated sum.

a) $1 + 3 + 5 + \cdots (S_8)$

b) $40 + 35 + 30 + \cdots (S_{11})$

c) $\frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \cdots (S_7)$

d) $(-3.5) + (-1.25) + 1 + \cdots (S_6)$

3. Determine the sum, S_n, for each arithmetic sequence described.

a) $t_1 = 7$, $t_n = 79$, $n = 8$

b) $t_1 = 58$, $t_n = -7$, $n = 26$

c) $t_1 = -12$, $t_n = 51$, $n = 10$

d) $t_1 = 12$, $d = 8$, $n = 9$

e) $t_1 = 42$, $d = -5$, $n = 14$

4. Determine the value of the first term, t_1, for each arithmetic series described.

a) $d = 6$, $S_n = 574$, $n = 14$

b) $d = -6$, $S_n = 32$, $n = 13$

c) $d = 0.5$, $S_n = 218.5$, $n = 23$

d) $d = -3$, $S_n = 279$, $n = 18$

5. For the arithmetic series, determine the value of n.

a) $t_1 = 8$, $t_n = 68$, $S_n = 608$

b) $t_1 = -6$, $t_n = 21$, $S_n = 75$

6. For each series find t_{10} and S_{10}.

a) $5 + 10 + 15 + \cdots$

b) $10 + 7 + 4 + \cdots$

c) $(-10) + (-14) + (-18) + \cdots$

d) $2.5 + 3 + 3.5 + \cdots$

Apply

7. a) Determine the sum of all the multiples of 4 between 1 and 999.

b) What is the sum of the multiples of 6 between 6 and 999?

8. It's About Time, in Langley, British Columbia, is Canada's largest custom clock manufacturer. They have a grandfather clock that, on the hours, chimes the number of times that corresponds to the time of day. For example, at 4:00 p.m., it chimes 4 times. How many times does the clock chime in a 24-h period?

9. A training program requires a pilot to fly circuits of an airfield. Each day, the pilot flies three more circuits than the previous day. On the fifth day, the pilot flew 14 circuits. How many circuits did the pilot fly
   a) on the first day?
   b) in total by the end of the fifth day?
   c) in total by the end of the nth day?

10. The second and fifth terms of an arithmetic series are 40 and 121, respectively. Determine the sum of the first 25 terms of the series.

11. The sum of the first five terms of an arithmetic series is 85. The sum of the first six terms is 123. What are the first four terms of the series?

12. Galileo noticed a relationship between the distance travelled by a falling object and time. Suppose data show that when an object is dropped from a particular height it moves approximately 5 m during the first second of its fall, 15 m during the second second, 25 m during the third second, 35 during the fourth second, and so on. The formula describing the approximate distance, d, the object is from its starting position n seconds after it has been dropped is $d(n) = 5n^2$.
   a) Using the general formula for the sum of a series, derive the formula $d(n) = 5n^2$.
   b) Demonstrate algebraically, using $n = 100$, that the sum of the series $5 + 15 + 25 + \cdots$ is equivalent to $d(n) = 5n^2$.

13. At the sixth annual Vancouver Canstruction® Competition, architects and engineers competed to see whose team could build the most spectacular structure using little more than cans of food.

A Breach in Hunger

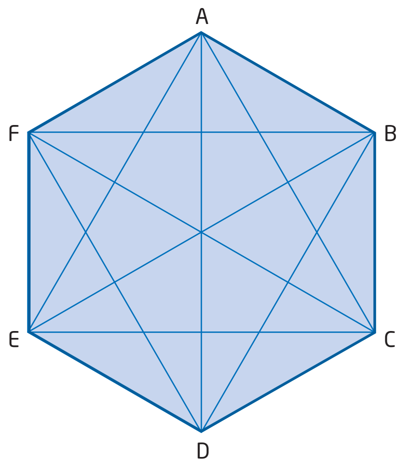
The UnBEARable Truth

Stores often stack cans for display purposes, although their designs are not usually as elaborate as the ones shown above. To calculate the number of cans in a display, an arithmetic series may be used. Suppose a store wishes to stack the cans in a pattern similar to the one shown. This display has one can at the top and each row thereafter adds one can. If there are 18 rows, how many cans in total are there in the display?

Did You Know?

The Vancouver Canstruction® Competition aids in the fight against hunger. At the end of the competition, all canned food is donated to food banks.

14. The number of handshakes between 6 people where everyone shakes hands with everyone else only once may be modelled using a hexagon. If you join each of the 6 vertices in the hexagon to every other point in the hexagon, there are 1 + 2 + 3 + 4 + 5 lines. Therefore, there are 15 lines.



a) What does the series 1 + 2 + 3 + 4 + 5 represent?

b) Write the series if there are 10 people in the room and everyone shakes hands with everyone else in the room once.

c) How many handshakes occur in a room of 30 people?

d) Describe a similar situation in which this method of determining the number of handshakes may apply.

15. The first three terms of an arithmetic sequence are given by x, $(2x - 5)$, 8.6.

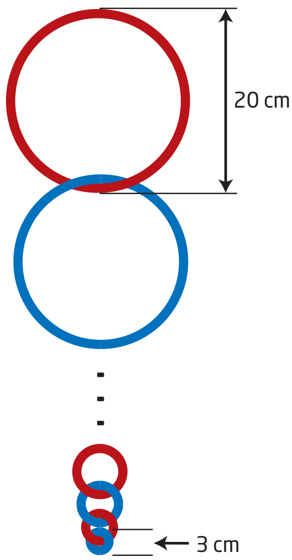
a) Determine the first term and the common difference for the sequence.

b) Determine the 20th term of the sequence.

c) Determine the sum of the first 20 terms of the series.

Extend

16. A number of interlocking rings each 1 cm thick are hanging from a peg. The top ring has an outside diameter of 20 cm. The outside diameter of each of the outer rings is 1 cm less than that of the ring above it. The bottom ring has an outside diameter of 3 cm. What is the distance from the top of the top ring to the bottom of the bottom ring?



17. Answer the following as either true or false. Justify your answers.

a) Doubling each term in an arithmetic series will double the sum of the series.

b) Keeping the first term constant and doubling the number of terms will double the sum of the series.

c) If each term of an arithmetic sequence is multiplied by a fixed number, the resulting sequence will always be an arithmetic sequence.

18. The sum of the first n terms of an arithmetic series is $S_n = 2n^2 + 5n$.

a) Determine the first three terms of this series.

b) Determine the sum of the first 10 terms of the series using the arithmetic sum formula.

c) Determine the sum of the first 10 terms of the series using the given formula.

d) Using the general formula for the sum of an arithmetic series, show how the formulas in parts b) and c) are equal.

19. Nathan Gerelus is a Manitoba farmer preparing to harvest his field of wheat. Nathan begins harvesting the crop at 11:00 a.m., after the morning dew has evaporated. By the end of the first hour he harvests 240 bushels of wheat. Nathan challenges himself to increase the number of bushels harvested by the end of each hour. Suppose that this increase produces an arithmetic series where Nathan harvests 250 bushels in the second hour, 260 bushels in the third hour, and so on.

a) Write the series that would illustrate the amount of wheat that Nathan has harvested by the end of the seventh hour.

b) Write the general sum formula that represents the number of bushels of wheat that Nathan took off the field by the end of the nth hour.

c) Determine the total number of bushels harvested by the end of the seventh hour.

d) State any assumptions that you made.

20. The 15th term in an arithmetic sequence is 43 and the sum of the first 15 terms of the series is 120. Determine the first three terms of the series.

Create Connections

21. An arithmetic series was defined where $t_1 = 12$, $n = 16$, $d = 6$, and $t_n = 102$. Two students were asked to determine the sum of the series. Their solutions are shown below.

Pierre's solution:

$$S_n = \frac{n}{2}(t_1 + t_n)$$
$$S_{16} = \frac{16}{2}(12 + 102)$$
$$S_{16} = 8(114)$$
$$S_{16} = 912$$

Jeanette's solution:

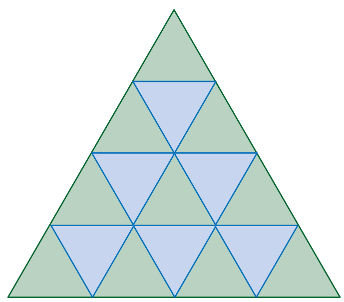
$$S_n = \frac{n}{2}[2t_1 + (n - 1)d]$$
$$S_{16} = \frac{16}{2}[2(12) + (16 - 1)6]$$
$$S_{16} = 8(24 + 90)$$
$$S_{16} = 912$$

Both students arrived at a correct answer. Explain how both formulas lead to the correct answer.

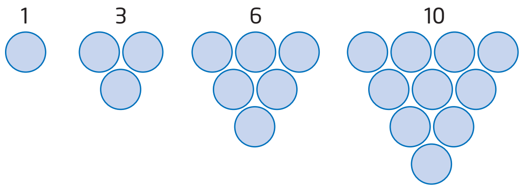
22. The triangular arrangement shown consists of a number of unit triangles. A unit triangle has side lengths equal to 1. The series for the total number of unit triangles in the diagram is $1 + 3 + 5 + 7$.

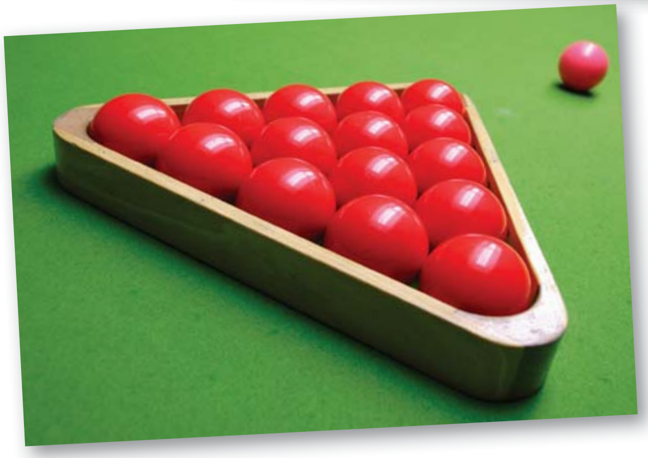
a) How many unit triangles are there if there are 10 rows in the triangular arrangement?

b) Using the sum of a series, show how the sum of the blue unit triangles plus the sum of the green unit triangles results in your answer from part a).



23. Bowling pins and snooker balls are often arranged in a triangular formation. A triangular number is a number that can be represented by a triangular array of dots. Each triangular number is an arithmetic series. The sequence 1, (1 + 2), (1 + 2 + 3), (1 + 2 + 3 + 4), … gives the first four triangular numbers as 1, 3, 6, and 10.





a) What is the tenth triangular number?

b) Use the general formula for the sum of an arithmetic series to show that the nth triangular number is $\frac{n}{2}(n + 1)$.

Project Corner — Diamond Mining

- In 1991, the first economic diamond deposit was discovered in the Lac de Gras area of the Northwest Territories. In October 1998, Ekati diamond mine opened about 300 km northeast of Yellowknife.
- By April 1999, the mine had produced one million carats. Ekati's average production over its projected 20-year life is expected to be 3 to 5 million carats per year.
- Diavik, Canada's second diamond mine, began production in January 2003. During its projected 20-year life, average diamond production from this mine is expected to be about 8 million carats per year, which represents about 6% of the world's total supply.

A polar bear diamond is a certified Canadian diamond mined, cut, and polished in Yellowknife.

Key Ideas

- A geometric sequence is a sequence in which each term, after the first term, is found by multiplying the previous term by a non-zero constant, r, called the common ratio.
- The common ratio of successive terms of a geometric sequence can be found by dividing any two consecutive terms, $r = \frac{t_n}{t_{n-1}}$.
- The general term of a geometric sequence is
  $t_n = t_1 r^{n-1}$
  where t_1 is the first term
  n is the number of terms
  r is the common ratio
  t_n is the general term or nth term

Check Your Understanding

Practise

1. Determine if the sequence is geometric. If it is, state the common ratio and the general term in the form $t_n = t_1 r^{n-1}$.

a) 1, 2, 4, 8, …

b) 2, 4, 6, 8, …

c) 3, −9, 27, −81, …

d) 1, 1, 2, 4, 8, …

e) 10, 15, 22.5, 33.75, …

f) −1, −5, −25, −125, …

2. Copy and complete the following table for the given geometric sequences.

	Geometric Sequence	Common Ratio	6th Term	10th Term
a)	6, 18, 54, ...			
b)	1.28, 0.64, 0.32, ...			
c)	$\frac{1}{5}, \frac{3}{5}, \frac{9}{5}, \ldots$			

3. Determine the first four terms of each geometric sequence.

a) $t_1 = 2, r = 3$

b) $t_1 = -3, r = -4$

c) $t_1 = 4, r = -3$

d) $t_1 = 2, r = 0.5$

4. Determine the missing terms, t_2, t_3, and t_4, in the geometric sequence in which $t_1 = 8.1$ and $t_5 = 240.1$.

5. Determine a formula for the nth term of each geometric sequence.

a) $r = 2, t_1 = 3$

b) 192, −48, 12, −3, …

c) $t_3 = 5, t_6 = 135$

d) $t_1 = 4, t_{13} = 16\ 384$

Apply

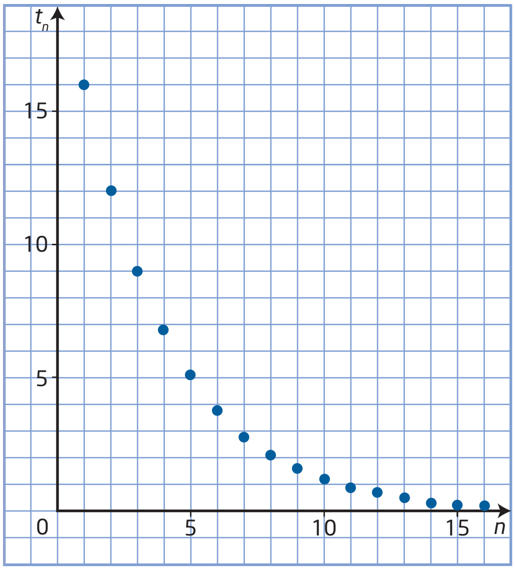
6. Given the following geometric sequences, determine the number of terms, n.

	Table A			
	First Term, t_1	Common Ratio, r	nth Term, t_n	Number of Terms, n
a)	5	3	135	
b)	−2	−3	−1458	
c)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{48}$	
d)	4	4	4096	
e)	$-\frac{1}{6}$	2	$-\frac{128}{3}$	
f)	$\frac{p^2}{2}$	$\frac{p}{2}$	$\frac{p^9}{256}$	

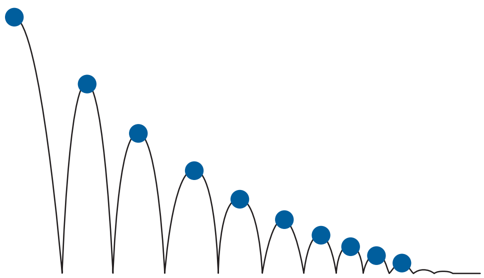
7. The following sequence is geometric. What is the value of y?

$3, 12, 48, 5y + 7, \ldots$

8. The following graph illustrates a geometric sequence. List the first three terms for the sequence and state the general term that describes the sequence.



9. A ball is dropped from a height of 3.0 m. After each bounce it rises to 75% of its previous height.



a) Write the first term and the common ratio of the geometric sequence.

b) Write the general term for the sequence in part a).

c) What height does the ball reach after the 6th bounce?

d) After how many bounces will the ball reach a height of approximately 40 cm?

10. The colour of some clothing fades over time when washed. Suppose a pair of jeans fades by 5% with each washing.

a) What percent of the colour remains after one washing?

b) If $t_1 = 100$, what are the first four terms of the sequence?

c) What is the value of r for your geometric sequence?

d) What percent of the colour remains after 10 washings?

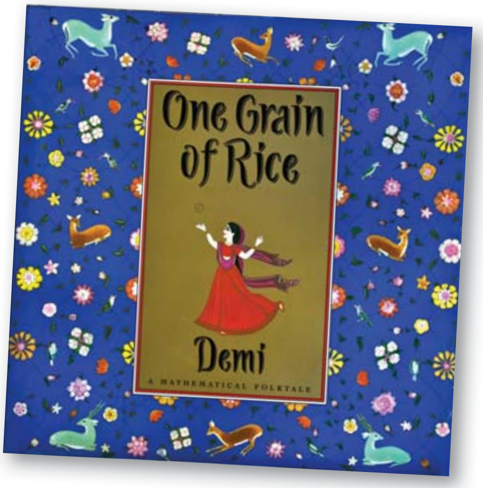
e) How many washings would it take so that only 25% of the original colour remains in the jeans? What assumptions did you make?

11. Pincher Creek, in the foothills of the Rocky Mountains in southern Alberta, is an ideal location to harness the wind power of the chinook winds that blow through the mountain passes. Kinetic energy from the moving air is converted to electricity by wind turbines. In 2004, the turbines generated 326 MW of wind energy, and it is projected that the amount will be 10 000 MW per year by 2010. If this growth were modelled by a geometric sequence, determine the value of the annual growth rate from 2004 to 2010.

Did You Know?

In an average year, a single 660-kW wind turbine produces 2000 MW of electricity, enough power for over 250 Canadian homes. Using wind to produce electricity rather than burning coal will leave 900 000 kg of coal in the ground and emit 2000 tonnes fewer greenhouse gases annually. This has the same positive impact as taking 417 cars off the road or planting 10 000 trees.

12. The following excerpt is taken from the book *One Grain of Rice* by Demi.



Long ago in India, there lived a raja who believed that he was wise and fair. But every year he kept nearly all of the people's rice for himself. Then when famine came, the raja refused to share the rice, and the people went hungry. Then a village girl named Rani devises a clever plan. She does a good deed for the raja, and in return, the raja lets her choose her reward. Rani asks for just one grain of rice, doubled every day for thirty days.

a) Write the sequence of terms for the first five days that Rani would receive the rice.

b) Write the general term that relates the number of grains of rice to the number of days.

c) Use the general term to determine the number of grains of rice that Rani would receive on the 30th day.

13. The Franco-Manitoban community of St-Pierre-Jolys celebrates Les Folies Grenouilles annually in August. Some of the featured activities include a slow pitch tournament, a parade, fireworks, and the Canadian National Frog Jumping Championships. During the competition, competitor's frogs have five chances to reach their maximum jump. One year, a frog by the name of Georges, achieved the winning jump in his 5th try. Georges' first jump was 191.41 cm, his second jump was 197.34 cm, and his third was 203.46 cm. The pattern of Georges' jumps approximated a geometric sequence.

a) By what ratio did Georges improve his performance with each jump? Express your answer to three decimal places.

b) How far was Georges' winning jump? Express your answer to the nearest tenth of a centimetre.

c) The world record frog jump is held by a frog named Santjie of South Africa. Santjie jumped approximately 10.2 m. If Georges, from St-Pierre-Jolys, had continued to increase his jumps following this same geometric sequence, how many jumps would Georges have needed to complete to beat Santjie's world record jump?

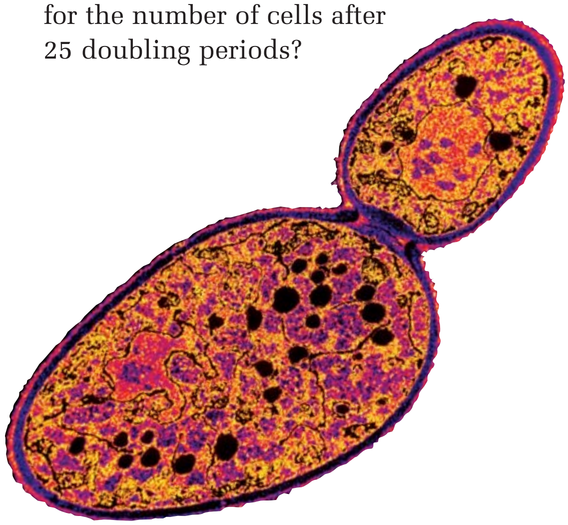
14. Bread and bread products have been part of our diet for centuries. To help bread rise, yeast is added to the dough. Yeast is a living unicellular micro-organism about one hundredth of a millimetre in size. Yeast multiplies by a biochemical process called budding. After mitosis and cell division, one cell results in two cells with exactly the same characteristics.

a) Write a sequence for the first six terms that describes the cell growth of yeast, beginning with a single cell.

b) Write the general term for the growth of yeast.

c) How many cells would there be after 25 doublings?

d) What assumptions would you make for the number of cells after 25 doubling periods?



15. The Arctic Winter Games is a high profile sports competition for northern and arctic athletes. The premier sports are the Dene and Inuit games, which include the arm pull, the one foot high kick, the two foot high kick, and the Dene hand games. The games are held every two years. The first Arctic Winter Games, held in 1970, drew 700 competitors. In 2008, the games were held in Yellowknife and drew 2000 competitors. If the number of competitors grew geometrically from 1970 to 2008, determine the annual rate of growth in the number of competitors from one Arctic Winter Games to the next. Express your answer to the nearest tenth of a percent.

16. Jason Annahatak entered the Russian sledge jump competition at the Arctic Winter Games, held in Yellowknife. Suppose that to prepare for this event, Jason started training by jumping 2 sledges each day for the first week, 4 sledges each day for the second week, 8 sledges each day for the third week, and so on. During the competition, Jason jumped 142 sledges. Assuming he continued his training pattern, how many weeks did it take him to reach his competition number of 142 sledges?

Did You Know?

Sledge jump starts from a standing position. The athlete jumps consecutively over 10 sledges placed in a row, turns around using one jumping movement, and then jumps back over the 10 sledges. This process is repeated until the athlete misses a jump or touches a sledge.

17. At Galaxyland in the West Edmonton Mall, a boat swing ride has been modelled after a basic pendulum design. When the boat first reaches the top of the swing, this is considered to be the beginning of the first swing. A swing is completed when the boat changes direction. On each successive completed swing, the boat travels 96% as far as on the previous swing. The ride finishes when the arc length through which the boat travels is 30 m. If it takes 20 swings for the boat to reach this arc length, determine the arc length through which the boat travels on the first swing. Express your answer to the nearest tenth of a metre.

18. The Russian nesting doll or Matryoshka had its beginnings in 1890. The dolls are made so that the smallest doll fits inside a larger one, which fits inside a larger one, and so on, until all the dolls are hidden inside the largest doll. In a set of 50 dolls, the tallest doll is 60 cm and the smallest is 1 cm. If the decrease in doll size approximates a geometric sequence, determine the common ratio. Express your answer to three decimal places.

19. The primary function for our kidneys is to filter our blood to remove any impurities. Doctors take this into account when prescribing the dosage and frequency of medicine. A person's kidneys filter out 18% of a particular medicine every two hours.

a) How much of the medicine remains after 12 h if the initial dosage was 250 mL? Express your answer to the nearest tenth of a millilitre.

b) When there is less than 20 mL left in the body, the medicine becomes ineffective and another dosage is needed. After how many hours would this happen?

Did You Know?

Every day, a person's kidneys process about 190 L of blood to remove about 1.9 L of waste products and extra water.

20. The charge in a car battery, when the car is left to sit, decreases by about 2% per day and can be modelled by the formula $C = 100(0.98)^d$, where d is the time, in days, and C is the approximate level of charge, as a percent.

a) Copy and complete the chart to show the percent of charge remaining in relation to the time passed.

Time, d (days)	Charge Level, C (%)
0	100
1	
2	
3	

b) Write the general term of this geometric sequence.

c) Explain how this formula is different from the formula $C = 100(0.98)^d$.

d) How much charge is left after 10 days?

21. A coiled basket is made using dried pine needles and sinew. The basket is started from the centre using a small twist and spirals outward and upward to shape the basket. The circular coiling of the basket approximates a geometric sequence, where the radius of the first coil is 6 mm.

a) If the ratio of consecutive coils is 1.22, calculate the radius for the 8th coil.

b) If there are 18 coils, what is the circumference of the top coil of the basket?

Extend

22. Demonstrate that 6^a, 6^b, 6^c, … forms a geometric sequence when a, b, c, … forms an arithmetic sequence.

23. If $x + 2$, $2x + 1$, and $4x - 3$ are three consecutive terms of a geometric sequence, determine the value of the common ratio and the three given terms.

24. On a six-string guitar, the distance from the nut to the bridge is 38 cm. The distance from the first fret to the bridge is 35.87 cm, and the distance from the second fret to the bridge is 33.86 cm. This pattern approximates a geometric sequence.

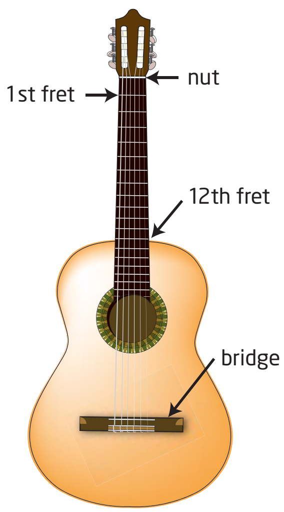
a) What is the distance from the 8th fret to the bridge?

b) What is the distance from the 12th fret to the bridge?

c) Determine the distance from the nut to the first fret.

d) Determine the distance from the first fret to the second fret.

e) Write the sequence for the first three terms of the distances between the frets. Is this sequence geometric or arithmetic? What is the common ratio or common difference?



Create Connections

25. Alex, Mala, and Paul were given the following problem to solve in class.

An aquarium that originally contains 40 L of water loses 8% of its water to evaporation every day. Determine how much water will be in the aquarium at the beginning of the 7th day.

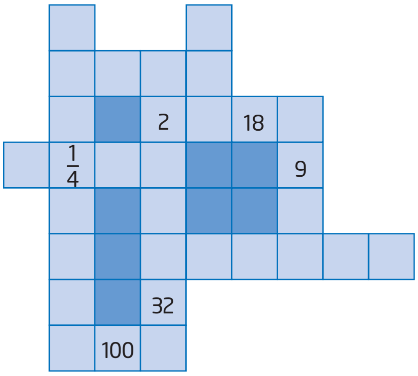
The three students' solutions are shown below. Which approach to the solution is correct? Justify your reasoning.

Alex's solution:
Alex believed that the sequence was geometric, where $t_1 = 40$, $r = 0.08$, and $n = 7$. He used the general formula $t_n = t_1r^{n-1}$.
$t_n = t_1r^{n-1}$
$t_n = 40(0.08)^{n-1}$
$t_7 = 40(0.08)^{7-1}$
$t_7 = 40(0.08)^6$
$t_7 = 0.000\ 01$
There will be 0.000 01 L of water in the tank at the beginning of the 7th day.

Mala's solution:
Mala believed that the sequence was geometric, where $t_1 = 40$, $r = 0.92$, and $n = 7$. She used the general formula $t_n = t_1r^{n-1}$.
$t_n = t_1r^{n-1}$
$t_n = 40(0.92)^{n-1}$
$t_7 = 40(0.92)^{7-1}$
$t_7 = 40(0.92)^6$
$t_7 = 24.25$
There will be 24.25 L of water in the tank at the beginning of the 7th day.

Paul's solution:
Paul believed that the sequence was arithmetic, where $t_1 = 40$ and $n = 7$. To calculate the value of d, Paul took 8% of 40 = 3.2. He reasoned that this would be a negative constant since the water was gradually disappearing. He used the general formula $t_n = t_1 + (n - 1)d$.
$t_n = t_1 + (n - 1)d$
$t_n = 40 + (n - 1)(-3.2)$
$t_7 = 40 + (7 - 1)(-3.2)$
$t_7 = 40 + (6)(-3.2)$
$t_7 = 20.8$
There will be 20.8 L of water in the tank at the beginning of the 7th day.

26. Copy the puzzle. Fill in the empty boxes with positive numbers so that each row and column forms a geometric sequence.



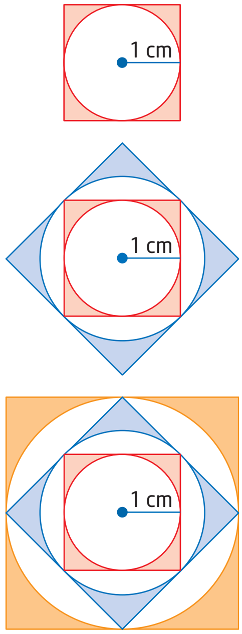
27. A square has an inscribed circle of radius 1 cm.

a) What is the area of the red portion of the square, to the nearest hundredth of a square centimetre?

b) If another square with an inscribed circle is drawn around the original, what is the area of the blue region, to the nearest hundredth of a square centimetre?

c) If another square with an inscribed circle is drawn around the squares, what is the area of the orange region, to the nearest hundredth of a square centimetre?

d) If this pattern were to continue, what would be the area of the newly coloured region for the 8th square, to the nearest hundredth of a square centimetre?



Project Corner — Forestry

- Canada has 402.1 million hectares (ha) of forest and other wooded lands. This value represents 41.1% of Canada's total surface area of 979.1 million hectares.
- Annually, Canada harvests 0.3% of its commercial forest area. In 2007, 0.9 million hectares were harvested.
- In 2008, British Columbia planted its 6 billionth tree seedling since the 1930s, as part of its reforestation programs.

Key Ideas

- A geometric series is the expression for the sum of the terms of a geometric sequence.
  For example, $5 + 10 + 20 + 40 + \cdots$ is a geometric series.
- The general formula for the sum of the first n terms of a geometric series with the first term, t_1, and the common ratio, r, is
  $S_n = \dfrac{t_1(r^n - 1)}{r - 1}$, $r \neq 1$
- A variation of this formula may be used when the first term, t_1, the common ratio, r, and the nth term, t_n, are known, but the number of terms, n, is not known.
  $S_n = \dfrac{rt_n - t_1}{r - 1}$, $r \neq 1$

Check Your Understanding

Practise

1. Determine whether each series is geometric. Justify your answer.

a) $4 + 24 + 144 + 864 + \cdots$

b) $-40 + 20 - 10 + 5 - \cdots$

c) $3 + 9 + 18 + 54 + \cdots$

d) $10 + 11 + 12.1 + 13.31 + \cdots$

2. For each geometric series, state the values of t_1 and r. Then determine each indicated sum. Express your answers as exact values in fraction form and to the nearest hundredth.

a) $6 + 9 + 13.5 + \cdots$ (S_{10})

b) $18 - 9 + 4.5 + \cdots$ (S_{12})

c) $2.1 + 4.2 + 8.4 + \cdots$ (S_9)

d) $0.3 + 0.003 + 0.000\ 03 + \cdots$ (S_{12})

3. What is S_n for each geometric series described? Express your answers as exact values in fraction form.

a) $t_1 = 12$, $r = 2$, $n = 10$

b) $t_1 = 27$, $r = \frac{1}{3}$, $n = 8$

c) $t_1 = \frac{1}{256}$, $r = -4$, $n = 10$

d) $t_1 = 72$, $r = \frac{1}{2}$, $n = 12$

4. Determine S_n for each geometric series. Express your answers to the nearest hundredth, if necessary.

a) $27 + 9 + 3 + \cdots + \frac{1}{243}$

b) $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \cdots + \frac{128}{6561}$

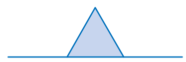
c) $t_1 = 5$, $t_n = 81\ 920$, $r = 4$

d) $t_1 = 3$, $t_n = 46\ 875$, $r = -5$

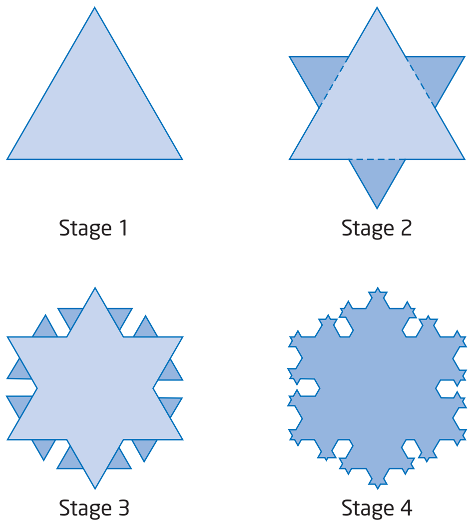
5. What is the value of the first term for each geometric series described? Express your answers to the nearest tenth, if necessary.
   - **a)** $S_n = 33$, $t_n = 48$, $r = -2$
   - **b)** $S_n = 443$, $n = 6$, $r = \frac{1}{3}$
6. The sum of $4 + 12 + 36 + 108 + \cdots + t_n$ is 4372. How many terms are in the series?
7. The common ratio of a geometric series is $\frac{1}{3}$ and the sum of the first 5 terms is 121.
   - **a)** What is the value of the first term?
   - **b)** Write the first 5 terms of the series.
8. What is the second term of a geometric series in which the third term is $\frac{9}{4}$ and the sixth term is $-\frac{16}{81}$? Determine the sum of the first 6 terms. Express your answer to the nearest tenth.

Apply

9. A fan-out system is used to contact a large group of people. The person in charge of the contact committee relays the information to four people. Each of these four people notifies four more people, who in turn each notify four more people, and so on.
   - **a)** Write the corresponding series for the number of people contacted.
   - **b)** How many people are notified after 10 levels of this system?
10. A tennis ball dropped from a height of 20 m bounces to 40% of its previous height on each bounce. The total vertical distance travelled is made up of upward bounces and downward drops. Draw a diagram to represent this situation. What is the total vertical distance the ball has travelled when it hits the floor for the sixth time? Express your answer to the nearest tenth of a metre.
11. Celia is training to run a marathon. In the first week she runs 25 km and increases this distance by 10% each week. This situation may be modelled by the series $25 + 25(1.1) + 25(1.1)^2 + \cdots$. She wishes to continue this pattern for 15 weeks. How far will she have run in total when she completes the 15th week? Express your answer to the nearest tenth of a kilometre.
12. **MINI LAB** Building the Koch snowflake is a step-by-step process.
   - Start with an equilateral triangle. (Stage 1)
   - In the middle of each line segment forming the sides of the triangle, construct an equilateral triangle with side length equal to $\frac{1}{3}$ of the length of the line segment.



   - Delete the base of this new triangle. (Stage 2)
   - For each line segment in Stage 2, construct an equilateral triangle, deleting its base. (Stage 3)
   - Repeat this process for each line segment, as you move from one stage to the next.



a) Work with a partner. Use dot paper to draw three stages of the Koch snowflake.

b) Copy and complete the following table.

Stage Number	Length of Each Line Segment	Number of Line Segments	Perimeter of Snowflake
1	1	3	3
2	$\frac{1}{3}$	12	4
3	$\frac{1}{9}$		
4			
5			

c) Determine the general term for the length of each line segment, the number of line segments, and the perimeter of the snowflake.

d) What is the total perimeter of the snowflake up to Stage 6?

13. An advertising company designs a campaign to introduce a new product to a metropolitan area. The company determines that 1000 people are aware of the product at the beginning of the campaign. The number of new people aware increases by 40% every 10 days during the advertising campaign. Determine the total number of people who will be aware of the product after 100 days.

14. Bead working has a long history among Canada's Indigenous peoples. Floral designs are the predominate patterns found among people of the boreal forests and northern plains. Geometric patterns are found predominately in the Great Plains. As bead work continues to be popular, traditional patterns are being exchanged among people in all regions. Suppose a set of 10 beads were laid in a line where each successive bead had a diameter that was $\frac{3}{4}$ of the diameter of the previous bead. If the first bead had a diameter of 24 mm, determine the total length of the line of beads. Express your answer to the nearest millimetre.

Did You Know?

Wampum belts consist of rows of beads woven together. Weaving traditionally involves stringing the beads onto twisted plant fibres, and then securing them to animal sinew.

Wanuskewin Native Heritage Park, Cree Nation, Saskatchewan

15. When doctors prescribe medicine at equally spaced time intervals, they are aware that the body metabolizes the drug gradually. After some period of time, only a certain percent of the original amount remains. After each dose, the amount of the drug in the body is equal to the amount of the given dose plus the amount remaining from the previous doses. The amount of the drug present in the body after the nth dose is modelled by a geometric series where t_1 is the prescribed dosage and r is the previous dose remaining in the body.

Suppose a person with an ear infection takes a 200-mg ampicillin tablet every 4 h. About 12% of the drug in the body at the start of a four-hour period is still present at the end of that period. What amount of ampicillin is in the body, to the nearest tenth of a milligram,

a) after taking the third tablet?

b) after taking the sixth tablet?

Extend

16. Determine the number of terms, n, if $3 + 3^2 + 3^3 + \cdots + 3^n = 9840$.

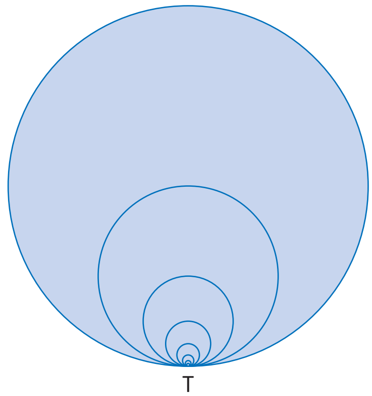
17. The third term of a geometric series is 24 and the fourth term is 36. Determine the sum of the first 10 terms. Express your answer as an exact fraction.

18. Three numbers, a, b, and c, form a geometric series so that $a + b + c = 35$ and $abc = 1000$. What are the values of a, b, and c?

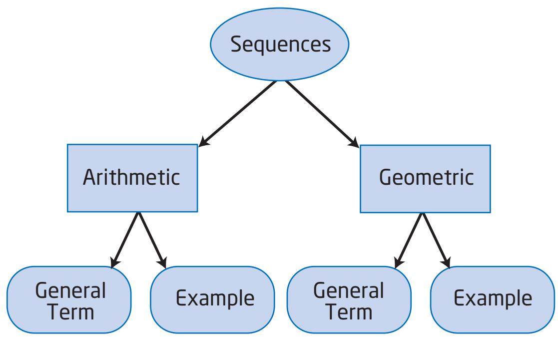
19. The sum of the first 7 terms of a geometric series is 89, and the sum of the first 8 terms is 104. What is the value of the eighth term?

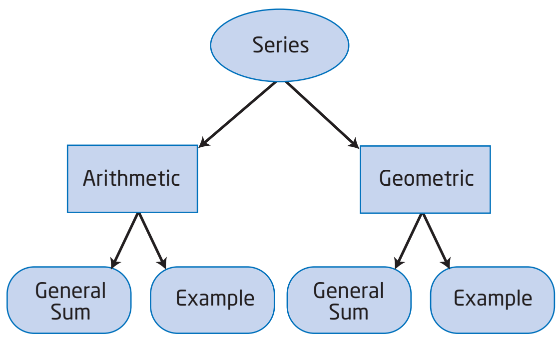
Create Connections

20. A fractal is created as follows: A circle is drawn with radius 8 cm. Another circle is drawn with half the radius of the previous circle. The new circle is tangent to the previous circle at point T as shown. Suppose this pattern continues through five steps. What is the sum of the areas of the circles? Express your answer as an exact fraction.



21. Copy the following flowcharts. In the appropriate segment of each chart, give a definition, a general term or sum, or an example, as required.





22. Tom learned that the monarch butterfly lays an average of 400 eggs. He decided to calculate the growth of the butterfly population from a single butterfly by using the logic that the first butterfly produced 400 butterflies. Each of those butterflies would produce 400 butterflies, and this pattern would continue. Tom wanted to estimate how many butterflies there would be in total in the fifth generation following this pattern. His calculation is shown below.

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_5 = \frac{1(400^5 - 1)}{400 - 1}$$

$$S_5 \approx 2.566 \times 10^{10}$$

Tom calculated that there would be approximately 2.566×10^{10} monarch butterflies in the fifth generation.

a) What assumptions did Tom make in his calculations?

b) Do you agree with the method Tom used to arrive at the number of butterflies? Explain.

c) Would this be a reasonable estimate of the total number of butterflies in the fifth generation? Explain.

d) Explain a method you would use to calculate the number of butterflies in the fifth generation.

Did You Know?

According to the American Indian Butterfly Legend:

If anyone desires a wish to come true they must first capture a butterfly and whisper that wish to it.

Since a butterfly can make no sound, the butterfly cannot reveal the wish to anyone but the Great Spirit who hears and sees all.

In gratitude for giving the beautiful butterfly its freedom, the Great Spirit always grants the wish.

So, according to legend, by making a wish and giving the butterfly its freedom, the wish will be taken to the heavens and be granted.

Project Corner — Oil Discovery

- The first oil well in Canada was discovered by James Miller Williams in 1858 near Oil Springs, Ontario. The oil was taken to Hamilton, Ontario, where it was refined into lamp oil. This well produced 37 barrels a day. By 1861 there were 400 wells in the area.
- In 1941, Alberta's population was approximately 800 000. By 1961, it was about 1.3 million.
- In February 1947, oil was struck in Leduc, Alberta. Leduc was the largest discovery in Canada in 33 years. By the end of 1947, 147 more wells were drilled in the Leduc-Woodbend oilfield.
- With these oil discoveries came accelerated population growth. In 1941, Leduc was inhabited by 871 people. By 1951, its population had grown to 1842.
- Leduc #1 was capped in 1974, after producing 300 000 barrels of oil and 9 million cubic metres of natural gas.

Key Ideas

- An infinite geometric series is a geometric series that has an infinite number of terms; that is, the series has no last term.
- An infinite series is said to be convergent if its sequence of partial sums approaches a finite number. This number is the sum of the infinite series. An infinite series that is not convergent is said to be divergent.
- An infinite geometric series has a sum when $-1 < r < 1$ and the sum is given by $S_\infty = \dfrac{t_1}{1 - r}$.

Check Your Understanding

Practise

1. State whether each infinite geometric series is convergent or divergent.

a) $t_1 = -3, r = 4$

b) $t_1 = 4, r = -\dfrac{1}{4}$

c) $125 + 25 + 5 + \cdots$

d) $(-2) + (-4) + (-8) + \cdots$

e) $\dfrac{243}{3125} - \dfrac{81}{625} + \dfrac{27}{25} - \dfrac{9}{5} + \cdots$

2. Determine the sum of each infinite geometric series, if it exists.

a) $t_1 = 8, r = -\dfrac{1}{4}$

b) $t_1 = 3, r = \dfrac{4}{3}$

c) $t_1 = 5, r = 1$

d) $1 + 0.5 + 0.25 + \cdots$

e) $4 - \dfrac{12}{5} + \dfrac{36}{25} - \dfrac{108}{125} + \cdots$

3. Express each of the following as an infinite geometric series. Determine the sum of the series.

a) $0.\overline{87}$

b) $0.4\overline{37}$

4. Does $0.999... = 1$? Support your answer.

5. What is the sum of each infinite geometric series?

a) $5 + 5\left(\dfrac{2}{3}\right) + 5\left(\dfrac{2}{3}\right)^2 + 5\left(\dfrac{2}{3}\right)^3 + \cdots$

b) $1 + \left(-\dfrac{1}{4}\right) + \left(-\dfrac{1}{4}\right)^2 + \left(-\dfrac{1}{4}\right)^3 + \cdots$

c) $7 + 7\left(\dfrac{1}{2}\right) + 7\left(\dfrac{1}{2}\right)^2 + 7\left(\dfrac{1}{2}\right)^3 + \cdots$

Apply

6. The sum of an infinite geometric series is 81, and its common ratio is $\dfrac{2}{3}$. What is the value of the first term? Write the first three terms of the series.

7. The first term of an infinite geometric series is -8, and its sum is $-\dfrac{40}{3}$. What is the common ratio? Write the first four terms of the series.

8. In its first month, an oil well near Virden, Manitoba produced 24 000 barrels of crude. Every month after that, it produced 94% of the previous month's production.

a) If this trend continued, what would be the lifetime production of this well?

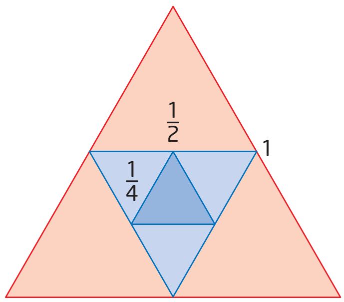
b) What assumption are you making? Is your assumption reasonable?

9. The infinite series given by $1 + 3x + 9x^2 + 27x^3 + \cdots$ has a sum of 4. What is the value of x? List the first four terms of the series.

10. The sum of an infinite series is twice its first term. Determine the value of the common ratio.

11. Each of the following represents an infinite geometric series. For what values of x will each series be convergent?
    a) $5 + 5x + 5x^2 + 5x^3 + \cdots$
    b) $1 + \frac{x}{3} + \frac{x^2}{9} + \frac{x^3}{27} + \cdots$
    c) $2 + 4x + 8x^2 + 16x^3 + \cdots$

12. Each side of an equilateral triangle has length of 1 cm. The midpoints of the sides are joined to form an inscribed equilateral triangle. Then, the midpoints of the sides of that triangle are joined to form another triangle. If this process continues forever, what is the sum of the perimeters of the triangles?



13. The length of the initial swing of a pendulum is 50 cm. Each successive swing is 0.8 times the length of the previous swing. If this process continues forever, how far will the pendulum swing?

14. Andrew uses the formula for the sum of an infinite geometric series to evaluate $1 + 1.1 + 1.21 + 1.331 + \cdots$. He calculates the sum of the series to be 10. Is Andrew's answer reasonable? Explain.

15. A ball is dropped from a height of 16 m. The ball rebounds to one half of its previous height each time it bounces. If the ball keeps bouncing, what is the total vertical distance the ball travels?

16. A pile driver pounds a metal post into the ground. With the first impact, the post moves 30 cm; with the second impact it moves 27 cm. Predict the total distance that the post will be driven into the ground if
    a) the distances form a geometric sequence and the post is pounded 8 times
    b) the distances form a geometric sequence and the post is pounded indefinitely

17. Dominique and Rita are discussing the series $-\frac{1}{3} + \frac{4}{9} - \frac{16}{27} + \cdots$. Dominique says that the sum of the series is $-\frac{1}{7}$. Rita says that the series is divergent and has no sum.
    a) Who is correct?
    b) Explain your reasoning.

18. A hot air balloon rises 25 m in its first minute of flight. Suppose that in each succeeding minute the balloon rises only 80% as high as in the previous minute. What would be the balloon's maximum altitude?

Hot air balloon rising over Calgary.

Extend

19. A square piece of paper with a side length of 24 cm is cut into four small squares, each with side lengths of 12 cm. Three of these squares are placed side by side. The remaining square is cut into four smaller squares, each with side lengths of 6 cm. Three of these squares are placed side by side with the bigger squares. The fourth square is cut into four smaller squares and three of these squares are placed side by side with the bigger squares. Suppose this process continues indefinitely. What is the length of the arrangement of squares?

20. The sum of the series
$0.98 + 0.98^2 + 0.98^3 + \cdots + 0.98^n = 49$.
The sum of the series
$0.02 + 0.0004 + 0.000\,008 + \cdots = \frac{1}{49}$.
The common ratio in the first series is 0.98 and the common ratio in the second series is 0.02. The sum of these ratios is equal to 1. Suppose that $\frac{1}{z} = x + x^2 + x^3 + \cdots$, where z is an integer and $x = \frac{1}{z + 1}$.

a) Create another pair of series that would follow this pattern, where the sum of the common ratios of the two series is 1.

b) Determine the sum of each series using the formula for the sum of an infinite series.

Create Connections

21. Under what circumstances will an infinite geometric series converge?

22. The first two terms of a series are 1 and $\frac{1}{4}$. Determine a formula for the sum of n terms if the series is

a) an arithmetic series

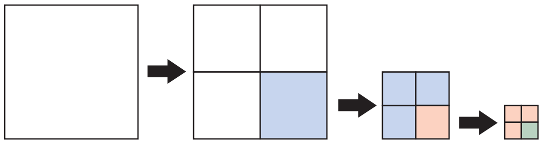
b) a geometric series

c) an infinite geometric series

23. **MINI LAB** Work in a group of three.

Step 1 Begin with a large sheet of grid paper and draw a square. Assume that the area of this square is 1.

Step 2 Cut the square into 4 equal parts. Distribute one part to each member of your group. Cut the remaining part into 4 equal parts. Again distribute one part to each group member. Subdivide the remaining part into 4 equal parts. Suppose you could continue this pattern indefinitely.



Step 3 Write a sequence for the fraction of the original square that each student received at each stage.

n	1	2	3	4
Fraction of Paper				

Step 4 Write the total area of paper each student has as a series of partial sums. What do you expect the sum to be?

Project Corner — Petroleum

- The Athabasca Oil Sands have estimated oil reserves in excess of that of the rest of the world. These reserves are estimated to be 1.6 trillion barrels.
- Canada is the seventh largest oil producing country in the world. In 2008, Canada produced an average of 438 000 m^3 per day of crude oil, crude bitumen, and natural gas.
- As Alberta's reserves of light crude oil began to deplete, so did production. By 1997, Alberta's light crude oil production totalled 37.3 million cubic metres. This production has continued to decline each year since, falling to just over half of its 1990 total at 21.7 million cubic metres in 2005.

Chapter 1 Review

1.1 Arithmetic Sequences, pages 6–21

1. Determine whether each of the following sequences is arithmetic. If it is arithmetic, state the common difference.

a) 36, 40, 44, 48, …

b) $-35, -40, -45, -50, \ldots$

c) 1, 2, 4, 8, …

d) $8.3, 4.3, 0.3, -3, -3.7, \ldots$

2. Match the equation for the nth term of an arithmetic sequence to the correct sequence.

a) 18, 30, 42, 54, 66, … **A** $t_n = 3n + 1$

b) 7, 12, 17, 22, … **B** $t_n = -4(n + 1)$

c) 2, 4, 6, 8, … **C** $t_n = 12n + 6$

d) $-8, -12, -16, -20, \ldots$ **D** $t_n = 5n + 2$

e) 4, 7, 10, 13, … **E** $t_n = 2n$

3. Consider the sequence 7, 14, 21, 28, …. Determine whether each of the following numbers is a term of this sequence. Justify your answer. If the number is a term of the sequence, determine the value of n for that term.

a) 98 **b)** 110

c) 378 **d)** 575

4. Two sequences are given:
Sequence 1 is 2, 9, 16, 23, …
Sequence 2 is 4, 10, 16, 22, …

a) Which of the following statements is correct?

A t_{17} is greater in sequence 1.

B t_{17} is greater in sequence 2.

C t_{17} is equal in both sequences.

b) On a grid, sketch a graph of each sequence. Does the graph support your answer in part a)? Explain.

5. Determine the tenth term of the arithmetic sequence in which the first term is 5 and the fourth term is 17.

6. The Gardiner Dam, located 100 km south of Saskatoon, Saskatchewan, is the largest earth-filled dam in the world. Upon its opening in 1967, engineers discovered that the pressure from Lake Diefenbaker had moved the clay-based structure 200 cm downstream. Since then, the dam has been moving at a rate of 2 cm per year. Determine the distance the dam will have moved downstream by the year 2020.

1.2 Arithmetic Series, pages 22–31

7. Determine the indicated sum for each of the following arithmetic series

a) $6 + 9 + 12 + \cdots (S_{10})$

b) $4.5 + 8 + 11.5 + \cdots (S_{12})$

c) $6 + 3 + 0 + \cdots (S_{10})$

d) $60 + 70 + 80 + \cdots (S_{20})$

8. The sum of the first 12 terms of an arithmetic series is 186, and the 20th term is 83. What is the sum of the first 40 terms?

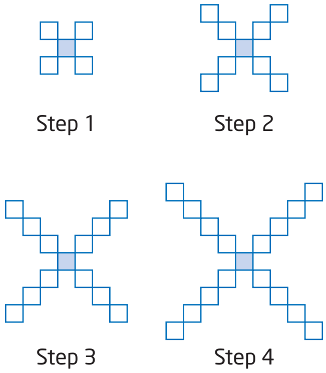
9. You have taken a job that requires being in contact with all the people in your neighbourhood. On the first day, you are able to contact only one person. On the second day, you contact two more people than you did on the first day. On day three, you contact two more people than you did on the previous day. Assume that the pattern continues.

a) How many people would you contact on the 15th day?

b) Determine the total number of people you would have been in contact with by the end of the 15th day.

c) How many days would you need to contact the 625 people in your neighbourhood?

10. A new set of designs is created by the addition of squares to the previous pattern.



a) Determine the total number of squares in the 15th step of this design.

b) Determine the total number of squares required to build all 15 steps.

11. A concert hall has 10 seats in the first row. The second row has 12 seats. If each row has 2 seats more than the row before it and there are 30 rows of seats, how many seats are in the entire concert hall?

1.3 Geometric Sequences, pages 32–45

12. Determine whether each of the following sequences is geometric. If it is geometric, determine the common ratio, r, the first term, t_1, and the general term of the sequence.

a) $3, 6, 10, 15, \dots$

b) $1, -2, 4, -8, \dots$

c) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

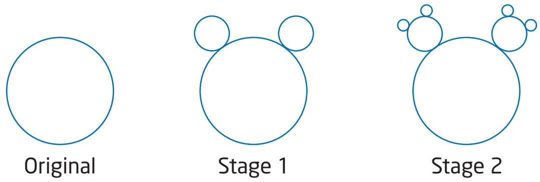
d) $\frac{16}{9}, -\frac{3}{4}, 1, \dots$

13. A culture initially has 5000 bacteria, and the number increases by 8% every hour.

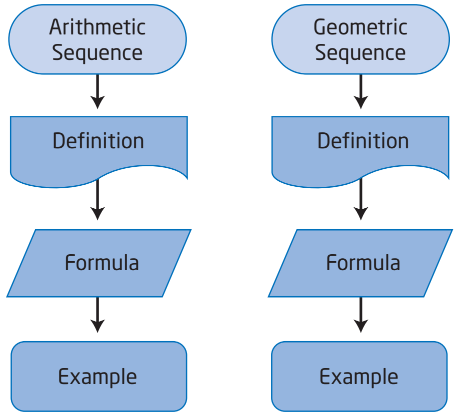
a) How many bacteria are present at the end of 5 h?

b) Determine a formula for the number of bacteria present after n hours.

14. In the Mickey Mouse fractal shown below, the original diagram has a radius of 81 cm. Each successive circle has a radius $\frac{1}{3}$ of the previous radius. What is the circumference of the smallest circle in the 4th stage?



15. Use the following flowcharts to describe what you know about arithmetic and geometric sequences.



1.4 Geometric Series, pages 46–57

16. Decide whether each of the following statements relates to an arithmetic series or a geometric series.

a) A sum of terms in which the difference between consecutive terms is constant.

b) A sum of terms in which the ratio of consecutive terms is constant.

c) $S_n = \frac{t_1(r^n - 1)}{r - 1}, r \neq 1$

d) $S_n = \frac{n[2t_1 + (n - 1)d]}{2}$

e) $\frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 + \cdots$

f) $\frac{1}{4} + \frac{1}{6} + \frac{1}{9} + \frac{2}{27} + \cdots$

17. Determine the sum indicated for each of the following geometric series.

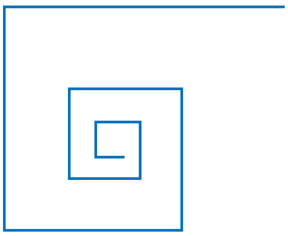
a) $6 + 9 + 13.5 + \cdots (S_{10})$

b) $18 + 9 + 4.5 + \cdots (S_{12})$

c) $6000 + 600 + 60 + \cdots (S_{20})$

d) $80 + 20 + 5 + \cdots (S_9)$

18. A student programs a computer to draw a series of straight lines with each line beginning at the end of the previous line and at right angles to it. The first line is 4 mm long. Each subsequent line is 25% longer than the previous one, so that a spiral shape is formed as shown.



a) What is the length, in millimetres, of the eighth straight line drawn by the program? Express your answer to the nearest tenth of a millimetre.

b) Determine the total length of the spiral, in metres, when 20 straight lines have been drawn. Express your answer to the nearest hundredth of a metre.

1.5 Infinite Geometric Series, pages 58–65

19. Determine the sum of each of the following infinite geometric series.

a) $5 + 5\left(\frac{2}{3}\right) + 5\left(\frac{2}{3}\right)^2 + 5\left(\frac{2}{3}\right)^3 + \cdots$

b) $1 + \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^3 + \cdots$

20. For each of the following series, state whether it is convergent or divergent. For those that are convergent, determine the sum.

a) $8 + 4 + 2 + 1 + \cdots$

b) $8 + 12 + 27 + 40.5 + \cdots$

c) $-42 + 21 - 10.5 + 5.25 - \cdots$

d) $\frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{32} + \cdots$

21. Given the infinite geometric series: $7 - 2.8 + 1.12 - 0.448 + \cdots$

a) What is the common ratio, r?

b) Determine S_1, S_2, S_3, S_4, and S_5.

c) What is the particular value that the sums are approaching?

d) What is the sum of the series?

22. Draw four squares adjacent to each other. The first square has a side length of 1 unit, the second has a side length of $\frac{1}{2}$ unit, the third has a side length of $\frac{1}{4}$ unit, and the fourth has a side length of $\frac{1}{8}$ unit.

a) Calculate the area of each square. Do the areas form a geometric sequence? Justify your answer.

b) What is the total area of the four squares?

c) If the process of adding squares with half the side length of the previous square continued indefinitely, what would the total area of all the squares be?

23. a) Copy and complete each of the following statements.

- A series is geometric if there is a common ratio r such that ______.
- An infinite geometric series converges if ______.
- An infinite geometric series diverges if ______.

b) Give two examples of convergent infinite geometric series one with positive common ratio and one with negative common ratio. Determine the sum of each of your series.

Chapter 1 Practice Test

Multiple Choice

For # 1 to #5, choose the best answer.

1. What are the missing terms of the arithmetic sequence ■, 3, 9, ■, ■?

A 1, 27, 81 **B** 9, 3, 9

C −6, 12, 17 **D** −3, 15, 21

2. Marc has set up in his father's grocery store a display of cans as shown in the diagram. The top row (Row 1) has 1 can and each successive row has 3 more cans than the previous row. Which expression would represent the number of cans in row n?

A $S_n = 3n + 1$ **B** $t_n = 3n - 2$

C $t_n = 3n + 2$ **D** $S_n = 3n - 3$

3. What is the sum of the first five terms of the geometric series $16\,807 - 2401 + 343 - \cdots$?

A 19 607 **B** 14 707

C 16 807.29 **D** 14 706.25

4. The numbers represented by a, b, and c are the first three terms of an arithmetic sequence. The number c, when expressed in terms of a and b, would be represented by

A $a + b$ **B** $2b - a$

C $a + (n - 1)b$ **D** $2a + b$

5. The 20th term of a geometric sequence is 524 288 and the 14th term is 8192. The value of the third term could be

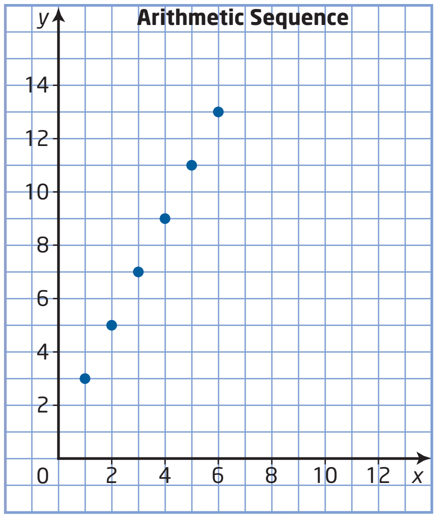
A 4 only **B** 8 only

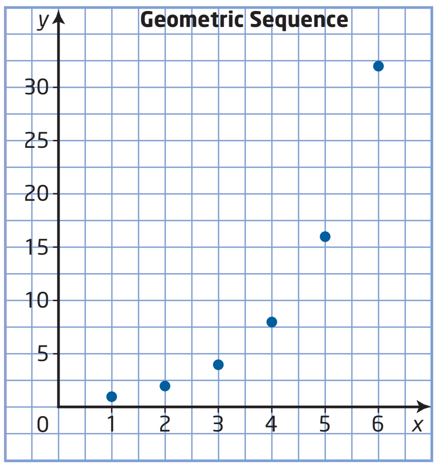
C +4 or −4 **D** +8 or −8

Short Answer

6. A set of hemispherical bowls are made so they can be nested for easy storage. The largest bowl has a radius of 30 cm and each successive bowl has a radius 90% of the preceding one. What is the radius of the tenth bowl?

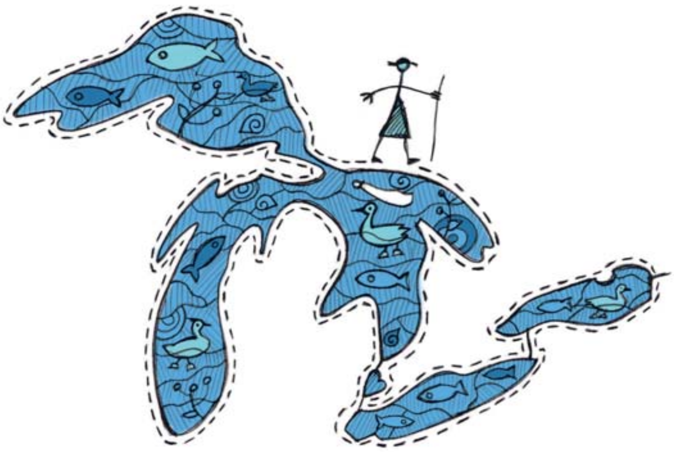
7. Use the following graphs to compare and contrast an arithmetic and a geometric sequence.





8. If 3, A, 27 is an arithmetic sequence and 3, B, 27 is a geometric sequence where $B > 0$, then what are the values of A and B?

9. Josephine Mandamin, an Anishinabe elder from Thunder Bay, Ontario, set out to walk around the Great Lakes to raise awareness about the quality of water in the lakes. In six years, she walked 17 000 km. If Josephine increased the number of kilometres walked per week by 2% every week, how many kilometres did she walk in the first week?



10. Consider the sequence 5, ■, ■, ■, ■, 160.
   a) Assume the sequence is arithmetic. Determine the unknown terms of the sequence.
   b) What is the general term of the arithmetic sequence?
   c) Assume the sequence is geometric. Determine the unknown terms of the sequence.
   d) What is the general term of the geometric sequence?

Extended Response

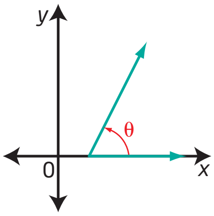
11. Scientists have been measuring the continental drift between Europe and North America for about 25 years. The data collected show that the continents are moving apart at a steady rate of about 17 mm per year.
   a) According to the Pangaea theory, Europe and North America were connected at one time. Assuming this theory is correct, write an arithmetic sequence that describes how far apart the continents were at the end of each of the first five years after separation.
   b) Determine the general term that describes the arithmetic sequence.
   c) Approximately how many years did it take to separate to the current distance of 6000 km? Express your answer to the nearest million years.
   d) What assumptions did you make in part c)?

12. Photodynamic therapy is used in patients with certain types of disease. A doctor injects a patient with a drug that is attracted to the diseased cells. The diseased cells are then exposed to red light from a laser. This procedure targets and destroys diseased cells while limiting damage to surrounding healthy tissue. The drug remains in the normal cells of the body and must be bleached out by exposure to the sun. A patient must be exposed to the sun for 30 s on the first day, and then increase the exposure by 30 s every day until a total of 30 min is reached.
   a) Write the first five terms of the sequence of sun exposure times.
   b) Is the sequence arithmetic or geometric?
   c) How many days are required to reach the goal of 30 min of exposure to the sun?
   d) What is the total number of minutes of sun exposure when a patient reaches the 30 min goal?

Check Your Understanding

Practise

1. Is each angle, θ, in standard position? Explain.

a)


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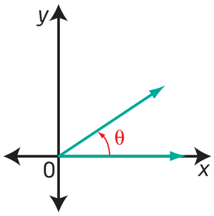

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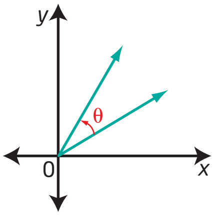
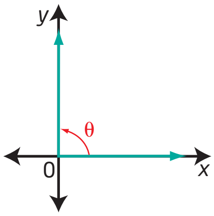
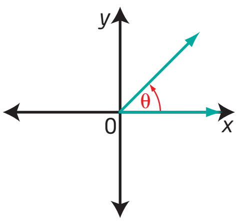
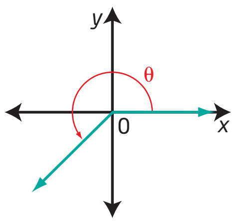


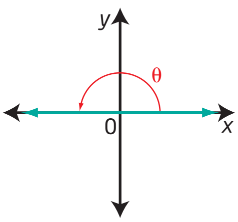
d)


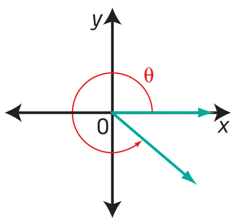
2. Without measuring, match each angle with a diagram of the angle in standard position.

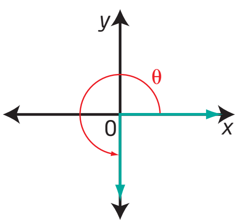
a) 150° **b)** 180°
c) 45° **d)** 320°
e) 215° **f)** 270°

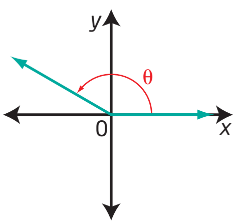
A


B


C


D


E


F


3. In which quadrant does the terminal arm of each angle in standard position lie?

a) 48° **b)** 300°
c) 185° **d)** 75°
e) 220° **f)** 160°

4. Sketch an angle in standard position with each given measure.

a) 70° **b)** 310°
c) 225° **d)** 165°

5. What is the reference angle for each angle in standard position?

a) 170° **b)** 345°
c) 72° **d)** 215°

6. Determine the measure of the three other angles in standard position, $0° < \theta < 360°$, that have a reference angle of

a) 45° **b)** 60°
c) 30° **d)** 75°

7. Copy and complete the table. Determine the measure of each angle in standard position given its reference angle and the quadrant in which the terminal arm lies.

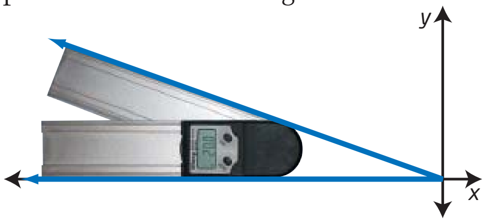
	Reference Angle	Quadrant	Angle in Standard Position
a)	72°	IV	
b)	56°	II	
c)	18°	III	
d)	35°	IV	

8. Copy and complete the table without using a calculator. Express each ratio using exact values.

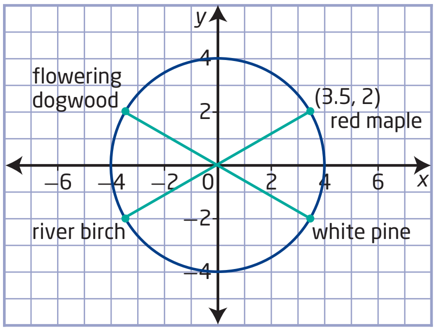
θ	sin θ	cos θ	tan θ
30°			
45°			
60°			

Apply

9. A digital protractor is used in woodworking. State the measure of the angle in standard position when the protractor has a reading of 20.4°.



10. Paul and Gail decide to use a Cartesian plane to design a landscape plan for their yard. Each grid mark represents a distance of 10 m. Their home is centred at the origin. There is a red maple tree at the point (3.5, 2). They will plant a flowering dogwood at a point that is a reflection in the y-axis of the position of the red maple. A white pine will be planted so that it is a reflection in the x-axis of the position of the red maple. A river birch will be planted so that it is a reflection in both the x-axis and the y-axis of the position of the red maple.



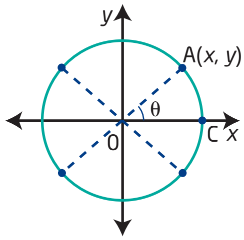
a) Determine the coordinates of the trees that Paul and Gail wish to plant.

b) Determine the angles in standard position if the lines drawn from the house to each of the trees are terminal arms. Express your answers to the nearest degree.

c) What is the actual distance between the red maple and the white pine?

11. A windshield wiper has a length of 50 cm. The wiper rotates from its resting position at 30°, in standard position, to 150°. Determine the exact horizontal distance that the tip of the wiper travels in one swipe.

12. Suppose A(x, y) is a point on the terminal arm of ∠AOC in standard position.

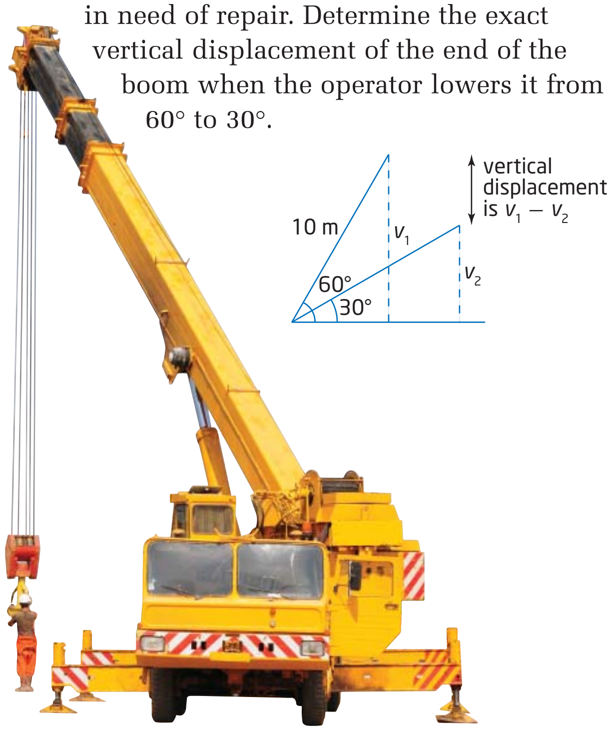


a) Determine the coordinates of points A′, A″, and A‴, where

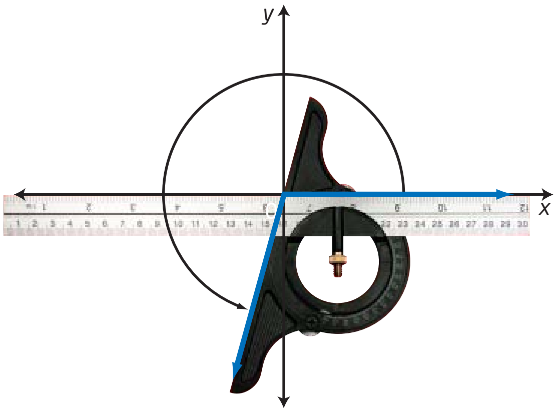
- A′ is the image of A reflected in the x-axis
- A″ is the image of A reflected in the y-axis
- A‴ is the image of A reflected in both the x-axis and the y-axis

b) Assume that each angle is in standard position and ∠AOC = θ. What are the measures, in terms of θ, of the angles that have A′, A″, and A‴ on their terminal arms?

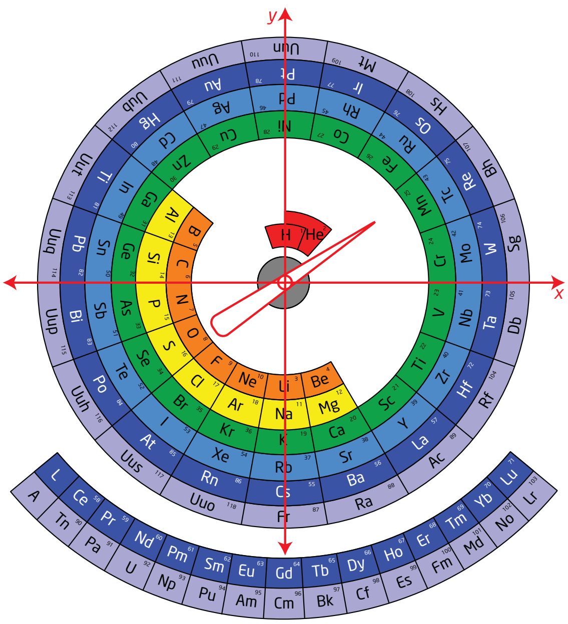
13. A 10-m boom lifts material onto a roof in need of repair. Determine the exact vertical displacement of the end of the boom when the operator lowers it from 60° to 30°.



14. Engineers use a bevel protractor to measure the angle and the depth of holes, slots, and other internal features. A bevel protractor is set to measure an angle of 72°. What is the measure of the angle in standard position of the lower half of the ruler, used for measuring the depth of an object?

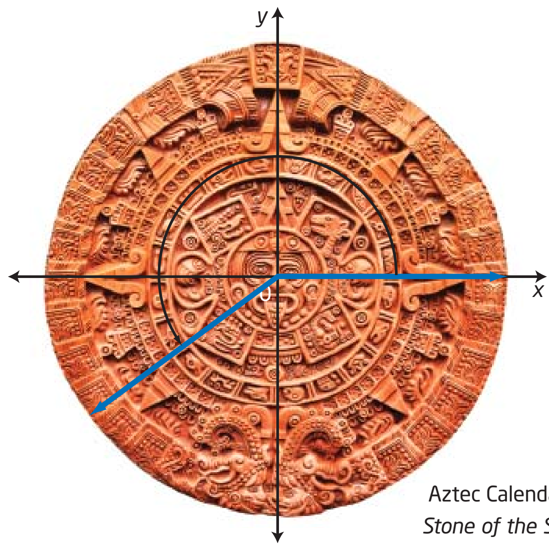


15. Researcher Mohd Abubakr developed a circular periodic table. He claims that his model gives a better idea of the size of the elements. Joshua and Andrea decided to make a spinner for the circular periodic table to help them study the elements for a quiz. They will spin the arm and then name the elements that the spinner lands on. Suppose the spinner lands so that it forms an angle in standard position of 110°. Name one of the elements it may have landed on.



16. The Aztec people of pre-Columbian Mexico used the Aztec Calendar. It consisted of a 365-day calendar cycle and a 260-day ritual cycle. In the stone carving of the calendar, the second ring from the centre showed the days of the month, numbered from one to 20.

Suppose the Aztec Calendar was placed on a Cartesian plane, as shown.



Aztec Calendar–
Stone of the Sun

a) The blue angle marks the passing of 12 days. Determine the measure of the angle.

b) How many days would have passed if the angle had been drawn in quadrant II, using the same reference angle as in part a)?

c) Keeping the same reference angle, how many days would have passed if the angle had been drawn in quadrant IV?

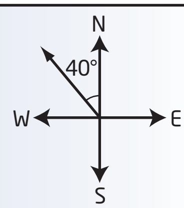
17. Express each direction as an angle in standard position. Sketch each angle.

a) N20°E　　**b)** S50°W

c) N80°W　　**d)** S15°E

Did You Know?

Directions are defined as a measure either east or west from north and south, measured in degrees. N40°W means to start from north and measure 40° toward the west.



Extend

18. You can use trigonometric ratios to design robotic arms. A robotic arm is motorized so that the angle, θ, increases at a constant rate of 10° per second from an initial angle of 0°. The arm is kept at a constant length of 45 cm to the tip of the fingers.

a) Let h represent the height of the robotic arm, measured at its fingertips. When $\theta = 0°$, h is 12 cm. Construct a table, using increments of 15°, that lists the angle, θ, and the height, h, for $0° \leq \theta \leq 90°$.

b) Does a constant increase in the angle produce a constant increase in the height? Justify your answer.

c) What conjecture would you make if θ were extended beyond 90°?

Did You Know?

A conjecture is a general conclusion based on a number of individual facts or results. In 1997, the American Mathematical Society published Beal's Conjecture. It states: If $A^x + B^y = C^z$, where A, B, C, x, y, and z are positive integers and x, y, and z are greater than 2, then A, B, and C must have a common prime factor. Andy Beal has offered a prize for a proof or counterexample of his conjecture.

Web Link

To learn about Beal's Conjecture and prize, go to www.mhrprecalc11.ca and follow the links.

19. Suppose two angles in standard position are supplementary and have terminal arms that are perpendicular. What are the measures of the angles?

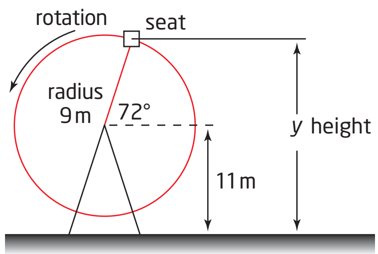
20. Carl and a friend are on the Antique Ferris Wheel Ride at Calaway Park in Calgary. The ride stops to unload the riders. Carl's seat forms an angle of 72° with a horizontal axis running through the centre of the Ferris wheel.

a) If the radius of the Ferris wheel is 9 m and the centre of the wheel is 11 m above the ground, determine the height of Carl's seat above the ground.

b) Suppose the Ferris wheel travels at four revolutions per minute and the operator stops the ride in 5 s.

i) Determine the angle in standard position of the seat that Carl is on at this second stop. Consider the horizontal central axis to be the x-axis.

ii) Determine the height of Carl's seat at the second stop.



Did You Know?

The first Ferris wheel was built for the 1853 World's Fair in Chicago. The wheel was designed by George Washington Gale Ferris. It had 36 gondola seats and reached a height of 80 m.

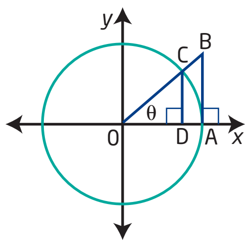
21. An angle in standard position is shown. Suppose the radius of the circle is 1 unit.

a) Which distance represents $\sin \theta$?

A OD **B** CD **C** OC **D** BA

b) Which distance represents $\tan \theta$?

A OD **B** CD **C** OC **D** BA



Create Connections

22. A point P(x, y) lies on the terminal arm of an angle θ. The distance from P to the origin is r units. Create a formula that links x, y, and r.

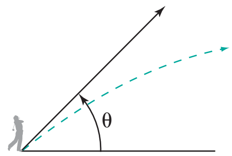
23. a) Copy and complete the table. Use a calculator and round ratios to four decimal places.

θ	20°	40°	60°	80°
sin θ				
sin (180° − θ)				
sin (180° + θ)				
sin (360° − θ)				

b) Make a conjecture about the relationships between sin θ, sin (180° − θ), sin (180° + θ), and sin (360° − θ).

c) Would your conjecture hold true for values of cosine and tangent? Explain your reasoning.

24. Daria purchased a new golf club. She wants to know the distance that she will be able to hit the ball with this club. She recalls from her physics class that the distance, d, a ball travels can be modelled by the formula $d = \frac{V^2 \cos \theta \sin \theta}{16}$, where V is the initial velocity, in feet per second, and θ is the angle of elevation.



a) The radar unit at the practice range indicates that the initial velocity is 110 ft/s and that the ball is hit at an angle of 30° to the ground. Determine the exact distance that Daria hit the ball with this driver.

b) To get a longer hit than that in part a), should Daria increase or decrease the angle of the hit? Explain.

c) What angle of elevation do you think would produce a hit that travels the greatest distance? Explain your reasoning.

Project Corner — Prospecting

- Prospecting is exploring an area for natural resources, such as oil, gas, minerals, precious metals, and mineral specimens. Prospectors travel through the countryside, often through creek beds and along ridgelines and hilltops, in search of natural resources.

Web Link

To search for locations of various minerals in Canada, go to www.mhrprecalc11.ca and follow the links.

Check Your Understanding

Practise

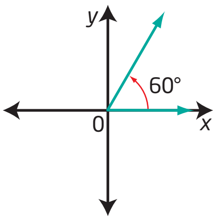
1. Sketch an angle in standard position so that the terminal arm passes through each point.

a) $(2, 6)$ **b)** $(-4, 2)$

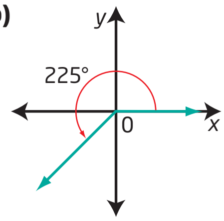
c) $(-5, -2)$ **d)** $(-1, 0)$

2. Determine the exact values of the sine, cosine, and tangent ratios for each angle.

a)



b)



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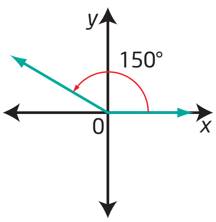

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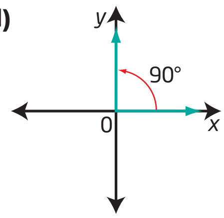
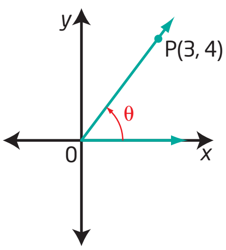


3. The coordinates of a point P on the terminal arm of each angle are shown. Write the exact trigonometric ratios $\sin \theta$, $\cos \theta$, and $\tan \theta$ for each.

a)



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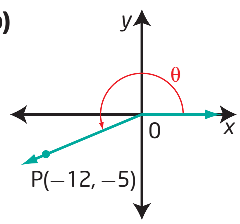

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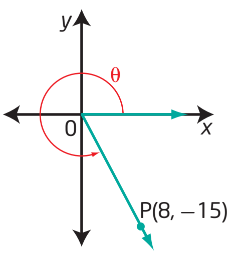

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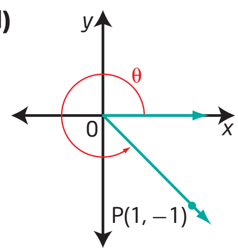


4. For each description, in which quadrant does the terminal arm of angle θ lie?

a) $\cos \theta < 0$ and $\sin \theta > 0$

b) $\cos \theta > 0$ and $\tan \theta > 0$

c) $\sin \theta < 0$ and $\cos \theta < 0$

d) $\tan \theta < 0$ and $\cos \theta > 0$

5. Determine the exact values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ if the terminal arm of an angle in standard position passes through the given point.

a) $P(-5, 12)$

b) $P(5, -3)$

c) $P(6, 3)$

d) $P(-24, -10)$

6. Without using a calculator, state whether each ratio is positive or negative.

a) $\sin 155°$

b) $\cos 320°$

c) $\tan 120°$

d) $\cos 220°$

7. An angle is in standard position such that $\sin \theta = \frac{5}{13}$.

a) Sketch a diagram to show the two possible positions of the angle.

b) Determine the possible values of θ, to the nearest degree, if $0° \leq \theta < 360°$.

8. An angle in standard position has its terminal arm in the stated quadrant. Determine the exact values for the other two primary trigonometric ratios for each.

	Ratio Value	Quadrant
a)	$\cos \theta = -\frac{2}{3}$	II
b)	$\sin \theta = \frac{3}{5}$	I
c)	$\tan \theta = -\frac{4}{5}$	IV
d)	$\sin \theta = -\frac{1}{3}$	III
e)	$\tan \theta = 1$	III

9. Solve each equation, for $0° \leq \theta < 360°$, using a diagram involving a special right triangle.

a) $\cos \theta = \frac{1}{2}$ **b)** $\cos \theta = -\frac{1}{\sqrt{2}}$

c) $\tan \theta = -\frac{1}{\sqrt{3}}$ **d)** $\sin \theta = -\frac{\sqrt{3}}{2}$

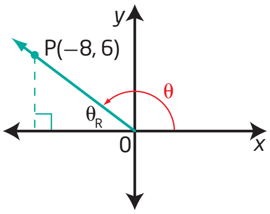
e) $\tan \theta = \sqrt{3}$ **f)** $\tan \theta = -1$

10. Copy and complete the table using the coordinates of a point on the terminal arm.

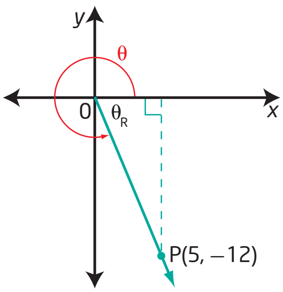
θ	sin θ	cos θ	tan θ
0°			
90°			
180°			
270°			
360°			

11. Determine the values of x, y, r, $\sin \theta$, $\cos \theta$, and $\tan \theta$ in each.

a)



b)



Apply

12. Point P(−9, 4) is on the terminal arm of an angle θ.

a) Sketch the angle in standard position.

b) What is the measure of the reference angle, to the nearest degree?

c) What is the measure of θ, to the nearest degree?

13. Point P(7, −24) is on the terminal arm of an angle, θ.

a) Sketch the angle in standard position.

b) What is the measure of the reference angle, to the nearest degree?

c) What is the measure of θ, to the nearest degree?

14. a) Determine sin θ when the terminal arm of an angle in standard position passes through the point P(2, 4).

b) Extend the terminal arm to include the point Q(4, 8). Determine sin θ for the angle in standard position whose terminal arm passes through point Q.

c) Extend the terminal arm to include the point R(8, 16). Determine sin θ for the angle in standard position whose terminal arm passes through point R.

d) Explain your results from parts a), b), and c). What do you notice? Why does this happen?

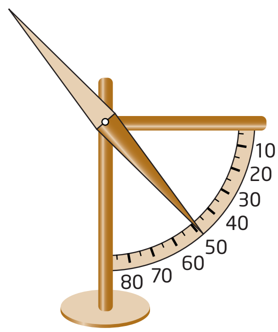
15. The point P(k, 24) is 25 units from the origin. If P lies on the terminal arm of an angle, θ, in standard position, $0° \leq \theta < 360°$, determine

a) the measure(s) of θ

b) the sine, cosine, and tangent ratios for θ

16. If $\cos \theta = \frac{1}{5}$ and $\tan \theta = 2\sqrt{6}$, determine the exact value of sin θ.

17. The angle between the horizontal and Earth's magnetic field is called the angle of dip. Some migratory birds may be capable of detecting changes in the angle of dip, which helps them navigate. The angle of dip at the magnetic equator is 0°, while the angle at the North and South Poles is 90°. Determine the exact values of sin θ, cos θ, and tan θ for the angles of dip at the magnetic equator and the North and South Poles.



18. Without using technology, determine whether each statement is true or false. Justify your answer.

a) $\sin 151° = \sin 29°$

b) $\cos 135° = \sin 225°$

c) $\tan 135° = \tan 225°$

d) $\sin 60° = \cos 330°$

e) $\sin 270° = \cos 180°$

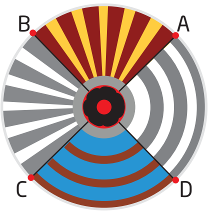
19. Copy and complete the table. Use exact values. Extend the table to include the primary trigonometric ratios for all angles in standard position, $90° \leq \theta \leq 360°$, that have the same reference angle as those listed for quadrant I.

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°			
30°			
45°			
60°			
90°			

20. Alberta Aboriginal Tourism designed a circular icon that represents both the Métis and First Nations communities of Alberta. The centre of the icon represents the collection of all peoples' perspectives and points of view relating to Aboriginal history, touching every quadrant and direction.

a) Suppose the icon is placed on a coordinate plane with a reference angle of 45° for points A, B, C, and D. Determine the measure of the angles in standard position for points A, B, C, and D.

b) If the radius of the circle is 1 unit, determine the coordinates of points A, B, C, and D.



21. Explore patterns in the sine, cosine, and tangent ratios.

a) Copy and complete the table started below. List the sine, cosine, and tangent ratios for θ in increments of 15° for $0° \leq \theta \leq 180°$. Where necessary, round values to four decimal places.

Angle	Sine	Cosine	Tangent
0°			
15°			
30°			
45°			
60°			

b) What do you observe about the sine, cosine, and tangent ratios as θ increases?

c) What comparisons can you make between the sine and cosine ratios?

d) Determine the signs of the ratios as you move from quadrant I to quadrant II.

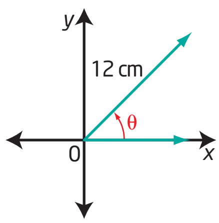
e) Describe what you expect will happen if you expand the table to include quadrant III and quadrant IV.

Extend

22. a) The line $y = 6x$, for $x \geq 0$, creates an acute angle, θ, with the x-axis. Determine the sine, cosine, and tangent ratios for θ.

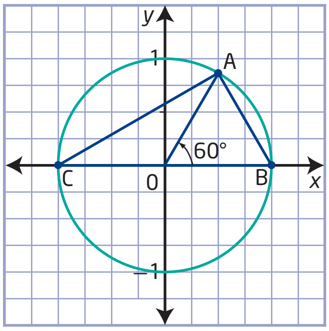
b) If the terminal arm of an angle, θ, lies on the line $4y + 3x = 0$, for $x \geq 0$, determine the exact value of $\tan \theta + \cos \theta$.

23. Consider an angle in standard position with $r = 12$ cm. Describe how the measures of x, y, $\sin \theta$, $\cos \theta$, and $\tan \theta$ change as θ increases continuously from 0° to 90°.



24. Suppose θ is a positive acute angle and $\cos \theta = a$. Write an expression for $\tan \theta$ in terms of a.

25. Consider an angle of 60° in standard position in a circle of radius 1 unit. Points A, B, and C lie on the circumference, as shown. Show that the lengths of the sides of △ABC satisfy the Pythagorean Theorem and that ∠CAB = 90°.



Create Connections

26. Explain how you can use reference angles to determine the trigonometric ratios of any angle, θ.

27. Point P(−5, −9) is on the terminal arm of an angle, θ, in standard position. Explain the role of the reference triangle and the reference angle in determining the value of θ.

28. Explain why there are exactly two non-quadrantal angles between 0° and 360° that have the same sine ratio.

29. Suppose that θ is an angle in standard position with $\cos \theta = -\frac{1}{2}$ and $\sin \theta = -\frac{\sqrt{3}}{2}$, $0° \leq \theta < 360°$. Determine the measure of θ. Explain your reasoning, including diagrams.

30. **MINI LAB** Use dynamic geometry software to explore the trigonometric ratios.

Step 1 **a)** Draw a circle with a radius of 5 units and centre at the origin.

b) Plot a point A on the circle in quadrant I. Join point A and the origin by constructing a line segment. Label this distance r.

Step 2 **a)** Record the x-coordinate and the y-coordinate for point A.

b) Construct a formula to calculate the sine ratio of the angle in standard position whose terminal arm passes through point A. Use the measure and calculate features of your software to determine the sine ratio of this angle.

c) Repeat step b) to determine the cosine ratio and tangent ratio of the angle in standard position whose terminal arm passes through point A.

Step 3 Animate point A. Use the motion controller to slow the animation. Pause the animation to observe the ratios at points along the circle.

Step 4 **a)** What observations can you make about the sine, cosine, and tangent ratios as point A moves around the circle?

b) Record where the sine and cosine ratios are equal. What is the measure of the angle at these points?

c) What do you notice about the signs of the ratios as point A moves around the circle? Explain.

d) For several choices for point A, divide the sine ratio by the cosine ratio. What do you notice about this calculation? Is it true for all angles as A moves around the circle?

Check Your Understanding

Practise

Where necessary, round lengths to the nearest tenth of a unit and angle measures to the nearest degree.

1. Solve for the unknown side or angle in each.

a) $\dfrac{a}{\sin 35°} = \dfrac{10}{\sin 40°}$

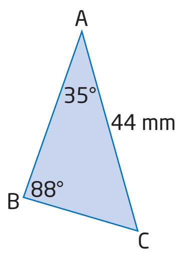
b) $\dfrac{b}{\sin 48°} = \dfrac{65}{\sin 75°}$

c) $\dfrac{\sin \theta}{12} = \dfrac{\sin 50°}{65}$

d) $\dfrac{\sin A}{25} = \dfrac{\sin 62°}{32}$

2. Determine the length of AB in each.

a)



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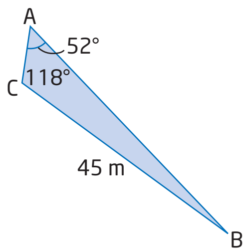
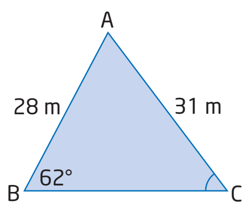


3. Determine the value of the marked unknown angle in each.

a)



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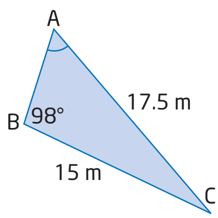
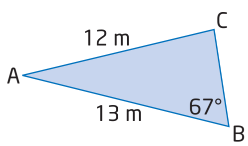


4. Determining the lengths of all three sides and the measures of all three angles is called solving a triangle. Solve each triangle.

a)



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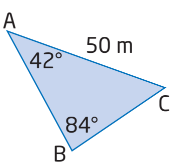

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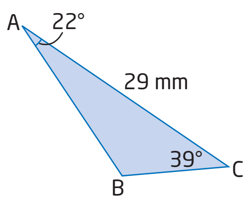

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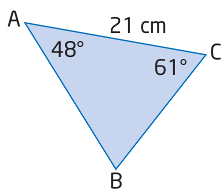


5. Sketch each triangle. Determine the measure of the indicated side.

a) In △ABC, ∠A = 57°, ∠B = 73°, and AB = 24 cm. Find the length of AC.

b) In △ABC, ∠B = 38°, ∠C = 56°, and BC = 63 cm. Find the length of AB.

c) In △ABC, ∠A = 50°, ∠B = 50°, and AC = 27 m. Find the length of AB.

d) In △ABC, ∠A = 23°, ∠C = 78°, and AB = 15 cm. Find the length of BC.

6. For each triangle, determine whether there is no solution, one solution, or two solutions.

a) In △ABC, ∠A = 39°, a = 10 cm, and b = 14 cm.

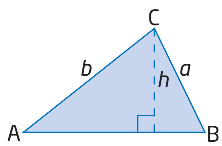
b) In △ABC, ∠A = 123°, a = 23 cm, and b = 12 cm.

c) In △ABC, ∠A = 145°, a = 18 cm, and b = 10 cm.

d) In △ABC, ∠A = 124°, a = 1 cm, and b = 2 cm.

7. In each diagram, h is an altitude. Describe how ∠A, sides a and b, and h are related in each diagram.

a)



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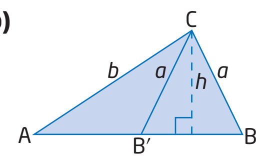

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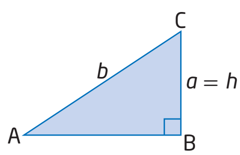

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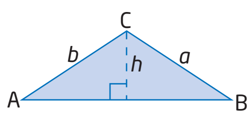
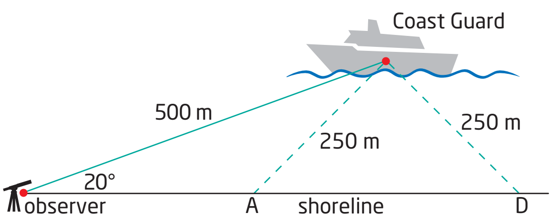


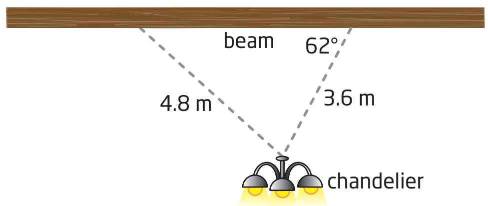
8. Determine the unknown side and angles in each triangle. If two solutions are possible, give both.
   a) In $\triangle ABC$, $\angle C = 31°$, $a = 5.6$ cm, and $c = 3.9$ cm.
   b) In $\triangle PQR$, $\angle Q = 43°$, $p = 20$ cm, and $q = 15$ cm.
   c) In $\triangle XYZ$, $\angle X = 53°$, $x = 8.5$ cm, and $z = 12.3$ cm.
9. In $\triangle ABC$, $\angle A = 26°$ and $b = 120$ cm. Determine the range of values of a for which there is
   a) one oblique triangle
   b) one right triangle
   c) two oblique triangles
   d) no triangle

Apply

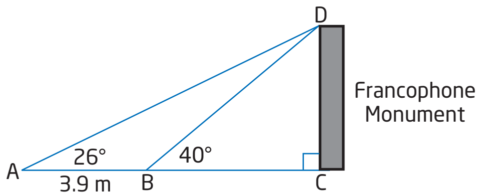
10. A hot-air balloon is flying above BC Place Stadium. Maria is standing due north of the stadium and can see the balloon at an angle of inclination of 64°. Roy is due south of the stadium and can see the balloon at an angle of inclination of 49°. The horizontal distance between Maria and Roy is 500 m.
    a) Sketch a diagram to represent the given information.
    b) Determine the distance that the hot air balloon is from Maria.
11. The Canadian Coast Guard Pacific Region is responsible for more than 27 000 km of coastline. The rotating spotlight from the Coast Guard ship can illuminate up to a distance of 250 m. An observer on the shore is 500 m from the ship. His line of sight to the ship makes an angle of 20° with the shoreline. What length of shoreline is illuminated by the spotlight?



12. A chandelier is suspended from a horizontal beam by two support chains. One of the chains is 3.6 m long and forms an angle of 62° with the beam. The second chain is 4.8 m long. What angle does the second chain make with the beam?



13. Nicolina wants to approximate the height of the Francophone Monument in Edmonton. From the low wall surrounding the statue, she measures the angle of elevation to the top of the monument to be 40°. She measures a distance 3.9 m farther away from the monument and measures the angle of elevation to be 26°. Determine the height of the Francophone Monument.



Did You Know?

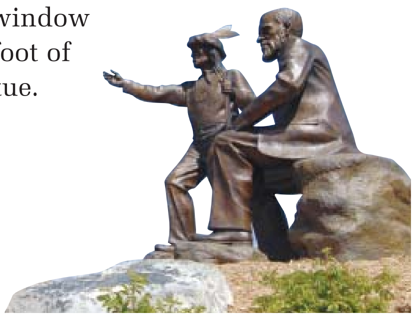
The Francophone Monument located at the Legislature Grounds in Edmonton represents the union of the fleur de lis and the wild rose. This monument celebrates the contribution of francophones to Alberta's heritage.

14. From the window of his hotel in Saskatoon, Max can see statues of Chief Whitecap of the Whitecap First Nation and John Lake, leader of the Temperance Colonists, who founded Saskatoon. The angle formed by Max's lines of sight to the top and to the foot of the statue of Chief Whitecap is 3°. The angle of depression of Max's line of sight to the top of the statue is 21°. The horizontal distance between Max and the front of the statue is 66 m.

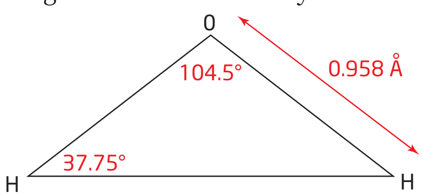
a) Sketch a diagram to represent this problem.

b) Determine the height of the statue of Chief Whitecap.

c) Determine the line-of-sight distance from where Max is standing at the window to the foot of the statue.

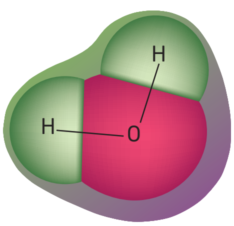


15. The chemical formula for water, H_2O, tells you that one molecule of water is made up of two atoms of hydrogen and one atom of oxygen bonded together. The nuclei of the atoms are separated by the distance shown, in angstroms. An angstrom is a unit of length used in chemistry.



a) Determine the distance, in angstroms (Å), between the two hydrogen atoms.

b) Given that 1 Å = 0.01 mm, what is the distance between the two hydrogen atoms, in millimetres?



16. A hang-glider is a triangular parachute made of nylon or Dacron fabric. The pilot of a hang-glider flies through the air by finding updrafts and wind currents. The nose angle for a hang-glider may vary. The nose angle for the T2 high-performance glider ranges from 127° to 132°. If the length of the wing is 5.1 m, determine the greatest and least wingspans of the T2 glider.

Did You Know?

Cochrane, Alberta, located 22 km west of Calgary, is considered to be the perfect location for hang-gliding. The prevailing winds are heated as they cross the prairie. Then they are forced upward by hills, creating powerful thermals that produce the long flights so desired by flyers.

The Canadian record for the longest hang-gliding flight is a distance of 332.8 km, flown from east of Beiseker, Alberta, to northwest of Westlock, in May 1989, by Willi Muller.

17. On his trip to Somerset Island, Nunavut, Armand joined an informative tour near Fort Ross. During the group's first stop, Armand spotted a cairn at the top of a hill, a distance of 500 m away. The group made a second stop at the bottom of the hill. From this point, the cairn is 360 m away. The angle between the cairn, Armand's first stop, and his second stop is 35°.

a) Explain why there are two possible locations for Armand's second stop.

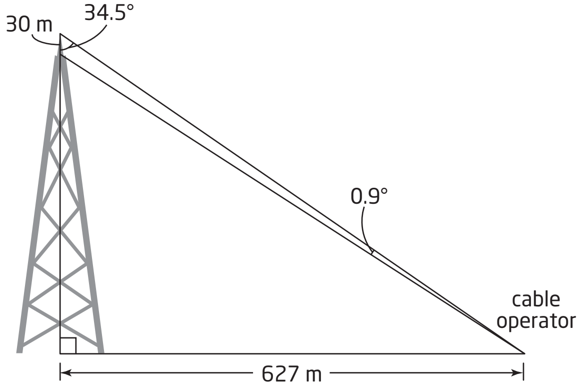
b) Sketch a diagram to illustrate each possible location.

c) Determine the possible distances between Armand's first and second stops.

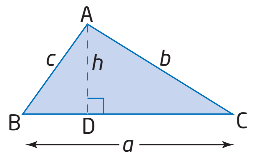
Did You Know?

A cairn is a pile of stones placed as a memorial, tomb, or landmark.

18. Radio towers, designed to support broadcasting and telecommunications, are among the tallest non-natural structures. Construction and maintenance of radio towers is rated as one of the most dangerous jobs in the world. To change the antenna of one of these towers, a crew sets up a system of pulleys. The diagram models the machinery and cable set-up. Suppose the height of the antenna is 30 m. Determine the total height of the structure.



19. Given an obtuse △ABC, copy and complete the table. Indicate the reasons for each step for the proof of the sine law.

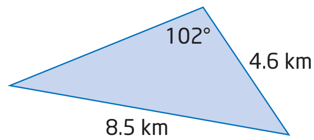


Statements	Reasons
$\sin C = \frac{h}{b}$ $\quad$ $\sin B = \frac{h}{c}$	
$h = b \sin C$ $\quad$ $h = c \sin B$	
$b \sin C = c \sin B$	
$\frac{\sin C}{c} = \frac{\sin B}{b}$	

Extend

20. Use the sine law to prove that if the measures of two angles of a triangle are equal, then the lengths of the sides opposite those angles are equal. Use a sketch in your explanation.

21. There are about 12 reported oil spills of 4000 L or more each day in Canada. Oil spills, such as occurred after the train derailment near Wabamun Lake, Alberta, can cause long-term ecological damage. To contain the spilled oil, floating booms are placed in the water. Suppose for the cleanup of the 734 000 L of oil at Wabamun, the floating booms used approximated an oblique triangle with the measurements shown. Determine the area of the oil spill at Wabamun.



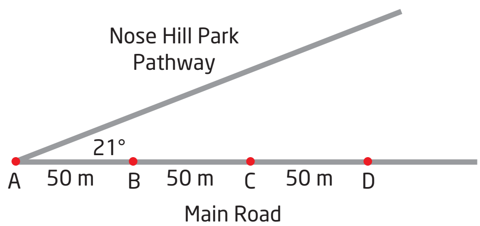
22. For each of the following, include a diagram in your solution.

a) Determine the range of values side a can have so that △ABC has two solutions if ∠A = 40° and b = 50.0 cm.

b) Determine the range of values side a can have so that △ABC has no solutions if ∠A = 56° and b = 125.7 cm.

c) △ABC has exactly one solution. If ∠A = 57° and b = 73.7 cm, what are the values of side a for which this is possible?

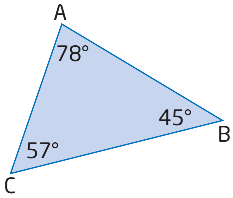
23. Shawna takes a pathway through Nose Hill Park in her Calgary neighbourhood. Street lights are placed 50 m apart on the main road, as shown. The light from each streetlight illuminates a distance along the ground of up to 60 m. Determine the distance from A to the farthest point on the pathway that is lit.



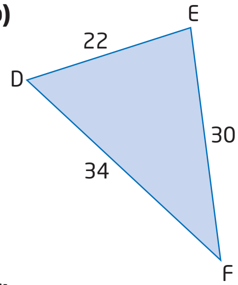
Create Connections

24. Explain why the sine law cannot be used to solve each triangle.

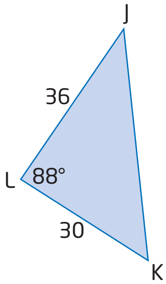
a)



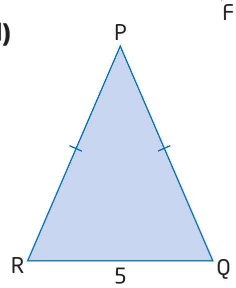
b)



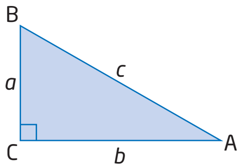
c)



d)

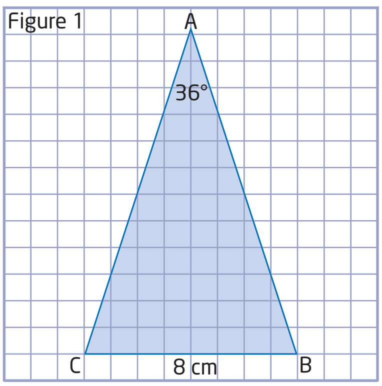


25. Explain how you could use a right △ABC to partially develop the sine law.



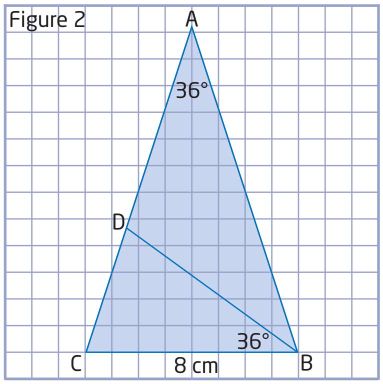
26. A golden triangle is an isosceles triangle in which the ratio of the length of the longer side to the length of the shorter side is the golden ratio, $\frac{\sqrt{5}+1}{2}:1$. The golden ratio is found in art, in math, and in architecture. In golden triangles, the vertex angle measures 36° and the two base angles measure 72°.

a) The triangle in Figure 1 is a golden triangle. The base measures 8 cm. Use the sine law to determine the length of the two equal sides of the triangle.

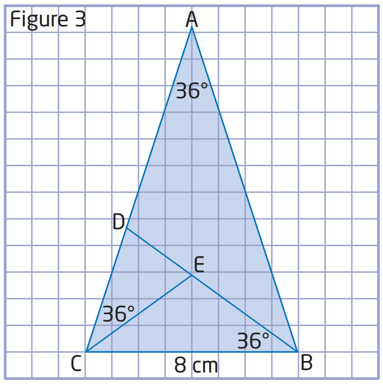


b) Use the golden ratio to determine the exact lengths of the two equal sides.

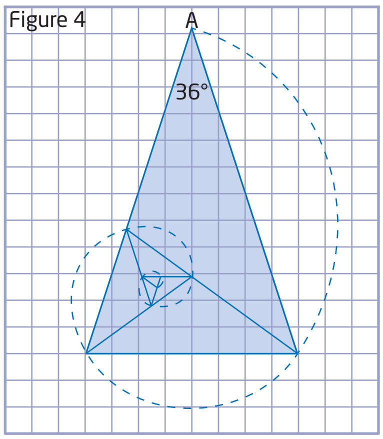
c) If you bisect one base angle of a golden triangle, you create another golden triangle similar to the first, as in Figure 2. Determine the length of side CD.



d) This pattern may be repeated, as in Figure 3. Determine the length of DE.

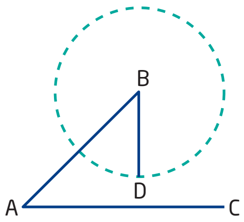


e) Describe how the spiral in Figure 4 is created.



27. Complete a concept map to show the conditions necessary to be able to use the sine law to solve triangles.

28. MINI LAB Work with a partner to explore conditions for the ambiguous case of the sine law.



Materials

- ruler
- compass
- string
- scissors

Step 1 Draw a line segment AC. Draw a second line segment AB that forms an acute angle at A. Draw a circle, using point B as the centre, such that the circle does not touch the line segment AC. Draw a radius and label the point intersecting the circle D.

Step 2 Cut a piece of string that is the length of the radius BD. Hold one end at the centre of the circle and turn the other end through the arc of the circle.

a) Can a triangle be formed under these conditions?

b) Make a conjecture about the number of triangles formed and the conditions necessary for this situation.

Step 3 Extend the circle so that it just touches the line segment AC at point D. Cut a piece of string that is the length of the radius. Hold one end at the centre of the circle and turn the other end through the arc of the circle.

a) Can a triangle be formed under these conditions?

b) Make a conjecture about the number of triangles formed and the conditions necessary for this situation.

Step 4 Extend the circle so that it intersects line segment AC at two distinct points. Cut a piece of string that is the length of the radius. Hold one end at the centre of the circle and turn the other end through the arc of the circle.

a) Can a triangle be formed under these conditions?

b) Make a conjecture about the number of triangles formed and the conditions necessary for this situation.

Step 5 Cut a piece of string that is longer than the segment AB. Hold one end at B and turn the other end through the arc of a circle.

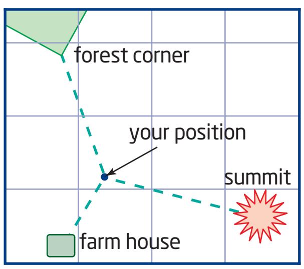
a) Can a triangle be formed under these conditions?

b) Make a conjecture about the number of triangles formed and the conditions necessary for this situation.

Step 6 Explain how varying the measure of ∠A would affect your conjectures.

Project Corner — Triangulation

- Triangulation is a method of determining your exact location based on your position relative to three other established locations using angle measures or bearings.
- Bearings are angles measured in degrees from the north line in a clockwise direction. A bearing usually has three digits. For instance, north is 000°, east is 090°, south is 180°, and southwest is 225°.
- A bearing of 045° is the same as N45°E.
- How could you use triangulation to help you determine the location of your resource?



The six parts of the triangle are as follows:

$\angle A = 144.8°$ $a = 11$

$\angle B = 15.2°$ $b = 5$

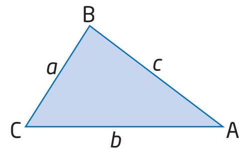
$\angle C = 20°$ $c = 6.5$

Your Turn

In $\triangle ABC$, $a = 9$, $b = 7$, and $\angle C = 33.6°$. Sketch a diagram and determine the length of the unknown side and the measures of the unknown angles, to the nearest tenth.

Key Ideas

- Use the cosine law to find the length of an unknown side of any triangle when you know the lengths of two sides and the measure of the angle between them.
- The cosine law states that for any $\triangle ABC$, the following relationships exist:
  $a^2 = b^2 + c^2 - 2bc \cos A$
  $b^2 = a^2 + c^2 - 2ac \cos B$
  $c^2 = a^2 + b^2 - 2ab \cos C$



- Use the cosine law to find the measure of an unknown angle of any triangle when the lengths of three sides are known.
- Rearrange the cosine law to solve for a particular angle.
  For example, $\cos A = \dfrac{a^2 - b^2 - c^2}{-2bc}$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.
- Use the cosine law in conjunction with the sine law to solve a triangle.

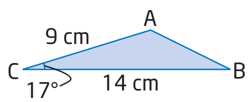
Check Your Understanding

Practise

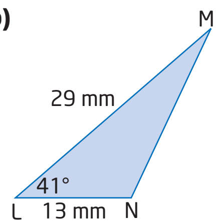
Where necessary, round lengths to the nearest tenth of a unit and angles to the nearest degree, unless otherwise stated.

1. Determine the length of the third side of each triangle.

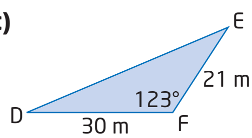
a)



b)

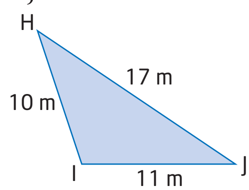


c)

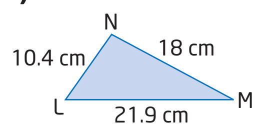


2. Determine the measure of the indicated angle.

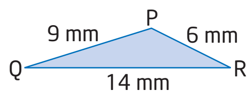
a) $\angle J$



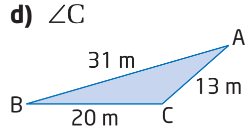
b) $\angle L$



c) $\angle P$

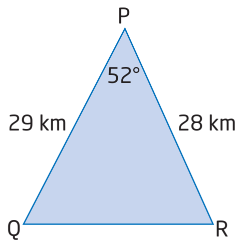


d) $\angle C$

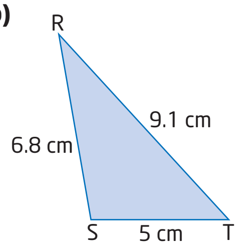


3. Determine the lengths of the unknown sides and the measures of the unknown angles.

a)



b)



4. Make a sketch to show the given information for each △ABC. Then, determine the indicated value.

a) AB = 24 cm, AC = 34 cm, and ∠A = 67°. Determine the length of BC.

b) AB = 15 m, BC = 8 m, and ∠B = 24°. Determine the length of AC.

c) AC = 10 cm, BC = 9 cm, and ∠C = 48°. Determine the length of AB.

d) AB = 9 m, AC = 12 m, and BC = 15 m. Determine the measure of ∠B.

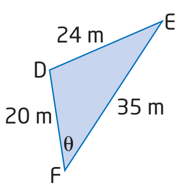
e) AB = 18.4 m, BC = 9.6 m, and AC = 10.8 m. Determine the measure of ∠A.

f) AB = 4.6 m, BC = 3.2 m, and AC = 2.5 m. Determine the measure of ∠C.

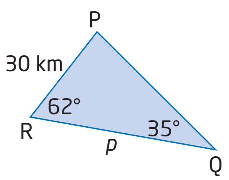
Apply

5. Would you use the sine law or the cosine law to determine each indicated side length or angle measure? Give reasons for your choice.

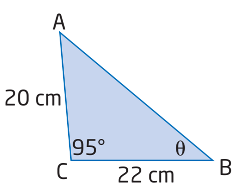
a) ∠F



b) p

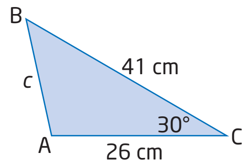


c) ∠B

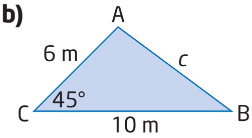


6. Determine the length of side c in each △ABC, to the nearest tenth.

a)



b)



7. In a parallelogram, the measure of the obtuse angle is 116°. The adjacent sides, containing the angle, measure 40 cm and 22 cm, respectively. Determine the length of the longest diagonal.

8. The longest tunnel in North America could be built through the mountains of the Kicking Horse Canyon, near Golden, British Columbia. The tunnel would be on the Trans-Canada highway connecting the Prairies with the west coast. Suppose the surveying team selected a point A, 3000 m away from the proposed tunnel entrance and 2000 m from the tunnel exit. If ∠A is measured as 67.7°, determine the length of the tunnel, to the nearest metre.

Web Link

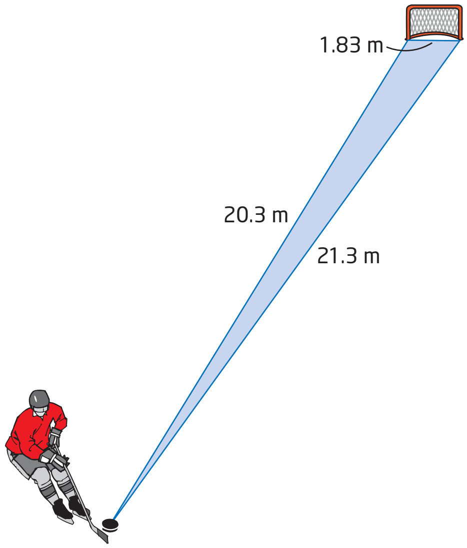
To learn more about the history of the Kicking Horse Pass and proposed plans for a new tunnel, go to www.mhrprecalc11.ca and follow the links.

9. Thousands of Canadians are active in sailing clubs. In the Paralympic Games, there are competitions in the single-handed, double-handed, and three-person categories. A sailing race course often follows a triangular route around three buoys. Suppose the distances between the buoys of a triangular course are 8.56 km, 5.93 km, and 10.24 km. Determine the measure of the angle at each of the buoys.

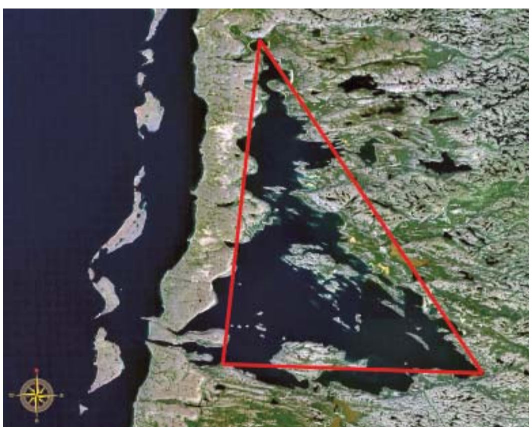
Did You Know?

Single-handed sailing means that one person sails the boat. Double-handed refers to two people. Buoys are floating markers, anchored to keep them in place. The oldest record of buoys being used to warn of rock hazards is in the thirteenth century on the Guadalquivir River near Seville in Spain.

10. The Canadian women's national ice hockey team has won numerous international competitions, including gold medals at the 2002, 2006, and 2010 Winter Olympics. A player on the blue line shoots a puck toward the 1.83-m-wide net from a point 20.3 m from one goal post and 21.3 m from the other. Within what angle must she shoot to hit the net? Answer to the nearest tenth of a degree.

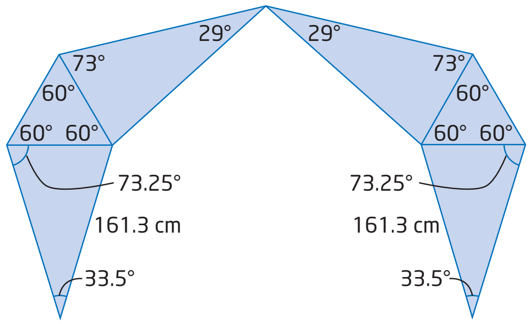


11. One of the best populated sea-run brook trout areas in Canada is Lac Guillaume-Delisle in northern Québec. Also known as Richmond Gulf, it is a large triangular-shaped lake. Suppose the sides forming the northern tip of the lake are 65 km and 85 km in length, and the angle at the northern tip is 7.8°. Determine the width of the lake at its base.



12. An aircraft-tracking station determines the distances from a helicopter to two aircraft as 50 km and 72 km. The angle between these two distances is 49°. Determine the distance between the two aircraft.

13. Tony Smith was born in 1912 in South Orange, New Jersey. As a child, Tony suffered from tuberculosis. He spent his time playing and creating with medicine boxes. His sculpture, Moondog, consists of several equilateral and isosceles triangles that combine art and math. Use the information provided on the diagram to determine the maximum width of Moondog.



Moondog by Tony Smith

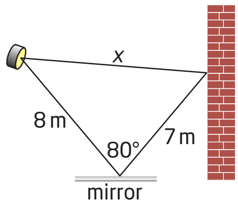
14. Julia and Isaac are backpacking in Banff National Park. They walk 8 km from their base camp heading N42°E. After lunch, they change direction to a bearing of 137° and walk another 5 km.

a) Sketch a diagram of their route.

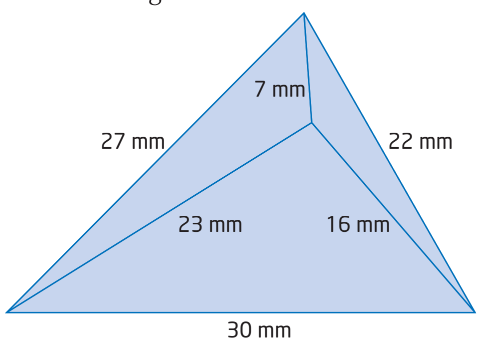
b) How far are Julia and Isaac from their base camp?

c) At what bearing must they travel to return to their base camp?

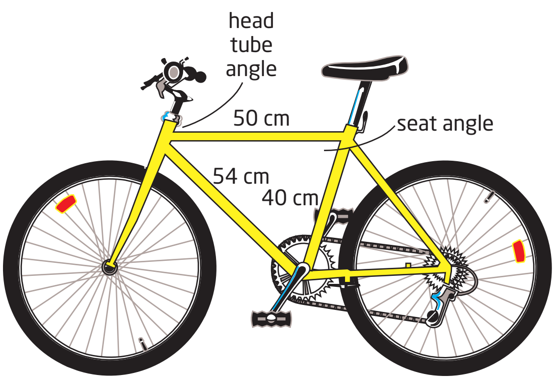
15. A spotlight is 8 m away from a mirror on the floor. A beam of light shines into the mirror, forming an angle of 80° with the reflected light. The light is reflected a distance of 7 m to shine onto a wall. Determine the distance from the spotlight to the point where the light is reflected onto the wall.



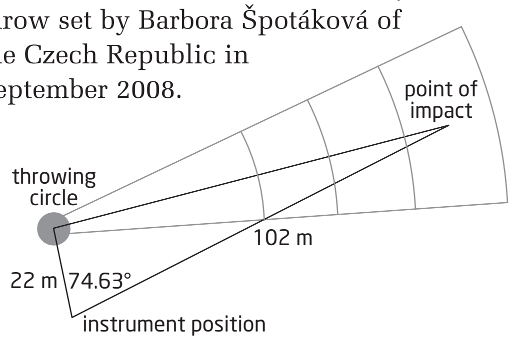
16. Erica created this design for part of a company logo. She needs to determine the accuracy of the side lengths. Explain how you could use the cosine law to verify that the side lengths shown are correct.



17. The sport of mountain biking became popular in the 1970s. The mountain bike was designed for off-road cycling. The geometry of the mountain bike contains two triangles designed for the safety of the rider. The seat angle and the head tube angle are critical angles that affect the position of the rider and the performance of the bike. Calculate the interior angles of the frame of the mountain bike shown.

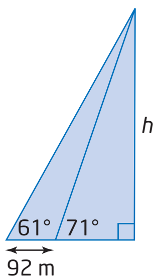


18. Distances in the throwing events at the Olympic games, traditionally measured with a tape measure, are now found with a piece of equipment called the Total Station. This instrument measures the angles and distances for events such as the shot put, the discus, and the javelin. Use the measurements shown to determine the distance, to the nearest hundredth of a metre, of the world record for the javelin throw set by Barbora Špotáková of the Czech Republic in September 2008.

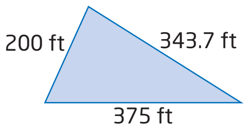


19. The Bermuda Triangle is an unmarked area in the Atlantic Ocean where there have been reports of unexplained disappearances of boats and planes and problems with radio communications. The triangle is an isosceles triangle with vertices at Miami, Florida, San Juan, Puerto Rico, and at the island of Bermuda. Miami is approximately 1660 km from both Bermuda and San Juan. If the angle formed by the sides of the triangle connecting Miami to Bermuda and Miami to San Juan is 55.5°, determine the distance from Bermuda to San Juan. Express your answer to the nearest kilometre.

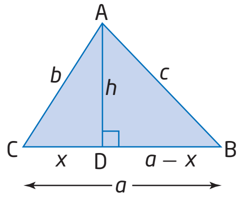
20. Della Falls in Strathcona Provincial Park on Vancouver Island is the highest waterfall in Canada. A surveyor measures the angle to the top of the falls to be 61°. He then moves in a direct line toward the falls a distance of 92 m. From this closer point, the angle to the top of the falls is 71°. Determine the height of Della Falls.



21. The floor of the Winnipeg Art Gallery is in the shape of a triangle with sides measuring 343.7 ft, 375 ft, and 200 ft. Determine the measures of the interior angles of the building.



22. Justify each step of the proof of the cosine law.



Statement	Reason
$c^2 = (a - x)^2 + h^2$	
$c^2 = a^2 - 2ax + x^2 + h^2$	
$b^2 = x^2 + h^2$	
$c^2 = a^2 - 2ax + b^2$	
$\cos C = \frac{x}{b}$	
$x = b \cos C$	
$c^2 = a^2 - 2a(b \cos C) + b^2$	
$c^2 = a^2 + b^2 - 2ab \cos C$	

Extend

23. Two ships leave port at 4 p.m. One is headed N38°E and is travelling at 11.5 km/h. The other is travelling at 13 km/h, with heading S47°E. How far apart are the two ships at 6 p.m.?

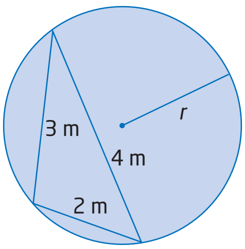
24. Is it possible to draw a triangle with side lengths 7 cm, 8 cm, and 16 cm? Explain why or why not. What happens when you use the cosine law with these numbers?

25. The hour and minute hands of a clock have lengths of 7.5 cm and 15.2 cm, respectively. Determine the straight-line distance between the tips of the hands at 1:30 p.m., if the hour hand travels 0.5° per minute and the minute hand travels 6° per minute.

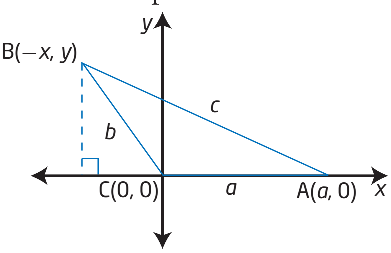
26. Graph A(−5, −4), B(8, 2), and C(2, 7) on a coordinate grid. Extend BC to intersect the y-axis at D. Find the measure of the interior angle ∠ABC and the measure of the exterior angle ∠ACD.

27. Researchers at Queen's University use a combination of genetics, bear tracks, and feces to estimate the numbers of polar bears in an area and gather information about their health, gender, size, and age. Researchers plan to set up hair traps around King William Island, Nunavut. The hair traps, which look like fences, will collect polar bear hair samples for analysis. Suppose the hair traps are set up in the form of $\triangle ABC$, where $\angle B = 40°$, $c = 40.4$ km, and $a = 45.9$ km. Determine the area of the region.

28. If the sides of a triangle have lengths 2 m, 3 m, and 4 m, what is the radius of the circle circumscribing the triangle?

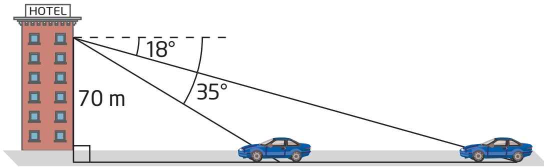


29. $\triangle ABC$ is placed on a Cartesian grid with $\angle BCA$ in standard position. Point B has coordinates $(-x, y)$. Use primary trigonometric ratios and the Pythagorean Theorem to prove the cosine law.



Create Connections

30. The Delta Regina Hotel is the tallest building in Saskatchewan. From the window of her room on the top floor, at a height of 70 m above the ground, Rian observes a car moving toward the hotel. If the angle of depression of the car changes from 18° to 35° during the time Rian is observing it, determine the distance the car has travelled.

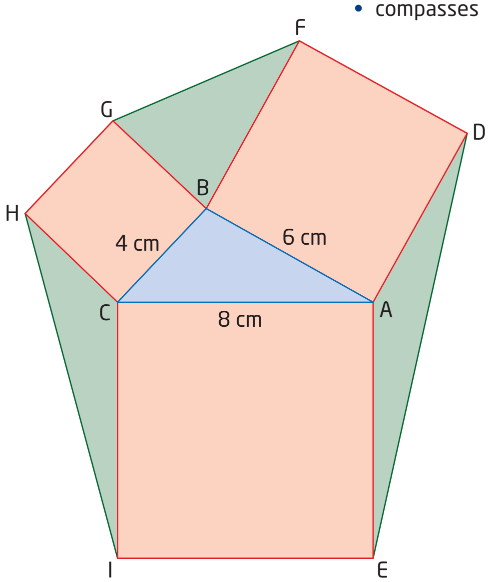


31. Given $\triangle ABC$ where $\angle C = 90°$, $a = 12.2$ cm, $b = 8.9$ cm, complete the following.
   a) Use the cosine law to find c^2.
   b) Use the Pythagorean Theorem to find c^2.
   c) Compare and contrast the cosine law with the Pythagorean Theorem.
   d) Explain why the two formulas are the same in a right triangle.

32. When solving triangles, the first step is choosing which method is best to begin with. Copy and complete the following table. Place the letter of the method beside the information given. There may be more than one answer.
   A primary trigonometric ratios
   B sine law
   C cosine law
   D none of the above

Concept Summary for Solving a Triangle	
Given	**Begin by Using the Method of**
Right triangle	
Two angles and any side	
Three sides	
Three angles	
Two sides and the included angle	
Two sides and the angle opposite one of them	

33. MINI LAB



Materials

- ruler
- protractor
- compasses

Step 1 **a)** Construct $\triangle ABC$ with side lengths $a = 4$ cm, $b = 8$ cm, and $c = 6$ cm.

b) On each side of the triangle, construct a square.

c) Join the outside corners of the squares to form three new triangles.

Step 2 **a)** In $\triangle ABC$, determine the measures of $\angle A$, $\angle B$, and $\angle C$.

b) Explain why pairs of angles, such as $\angle ABC$ and $\angle GBF$, have a sum of 180°. Determine the measures of $\angle GBF$, $\angle HCI$, and $\angle DAE$.

c) Determine the lengths of the third sides, GF, DE, and HI, of $\triangle BGF$, $\triangle ADE$, and $\triangle CHI$.

Step 3 For each of the four triangles, draw an altitude.

a) Use the sine ratio to determine the measure of each altitude.

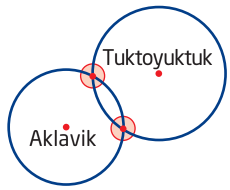
b) Determine the area of each triangle.

Step 4 What do you notice about the areas of the triangles? Explain why you think this happens. Will the result be true for any $\triangle ABC$?

Project Corner — Trilateration

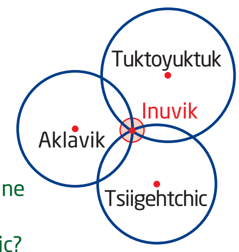
- GPS receivers work on the principle of trilateration. The satellites circling Earth use 3-D trilateration to pinpoint locations. You can use 2-D trilateration to see how the principle works.

- Suppose you are 55 km from Aklavik, NT. Knowing this tells you that you are on a circle with radius 55 km centred at Aklavik.
- If you also know that you are 127 km from Tuktoyuktuk, you have two circles that intersect and you must be at one of these intersection points.



- If you are told that you are 132 km from Tsiigehtchic, a third circle will intersect with one of the other two points of intersection, telling you that your location is at Inuvik.
- How can you use the method of trilateration to pinpoint the location of your resource?

If you know the angle at Inuvik, how can you determine the distance between Tuktoyuktuk and Tsiigehtchic?



Chapter 2 Review

Where necessary, express lengths to the nearest tenth and angles to the nearest degree.

2.1 Angles in Standard Position, pages 74–87

1. Match each term with its definition from the choices below.

a) angle in standard position

b) reference angle

c) exact value

d) sine law

e) cosine law

f) terminal arm

g) ambiguous case

A a formula that relates the lengths of the sides of a triangle to the sine values of its angles

B a value that is not an approximation and may involve a radical

C the final position of the rotating arm of an angle in standard position

D the acute angle formed by the terminal arm and the *x*-axis

E an angle whose vertex is at the origin and whose arms are the *x*-axis and the terminal arm

F a formula that relates the lengths of the sides of a triangle to the cosine value of one of its angles

G a situation that is open to two or more interpretations

2. Sketch each angle in standard position. State which quadrant the angle terminates in and the measure of the reference angle.

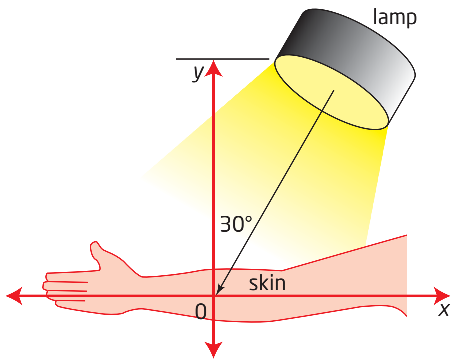
a) 200°

b) 130°

c) 20°

d) 330°

3. A heat lamp is placed above a patient's arm to relieve muscle pain. According to the diagram, would you consider the reference angle of the lamp to be 30°? Explain your answer.



4. Explain how to determine the measure of all angles in standard position, $0° \leq \theta < 360°$, that have 35° for their reference angle.

5. Determine the exact values of the sine, cosine, and tangent ratios for each angle.

a) 225°

b) 120°

c) 330°

d) 135°

2.2 Trigonometric Ratios of Any Angle, pages 88–99

6. The point Q(−3, 6) is on the terminal arm of an angle, θ.

a) Draw this angle in standard position.

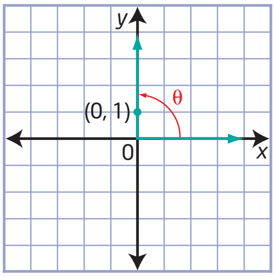
b) Determine the exact distance from the origin to point Q.

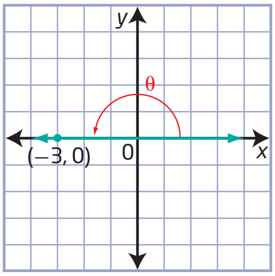
c) Determine the exact values for sin θ, cos θ, and tan θ.

d) Determine the value of θ.

7. A reference angle has a terminal arm that passes through the point P(2, −5). Identify the coordinates of a corresponding point on the terminal arm of three angles in standard position that have the same reference angle.

8. Determine the values of the primary trigonometric ratios of θ in each case.

a) 

b) 

9. Determine the exact value of the other two primary trigonometric ratios given each of the following.

a) $\sin \theta = -\frac{3}{5}$, $\cos \theta < 0$

b) $\cos \theta = \frac{1}{3}$, $\tan \theta < 0$

c) $\tan \theta = \frac{12}{5}$, $\sin \theta > 0$

10. Solve for all values of θ, $0° \leq \theta < 360°$, given each trigonometric ratio value.

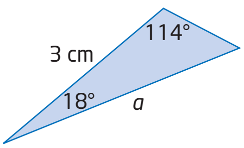
a) $\tan \theta = -1.1918$

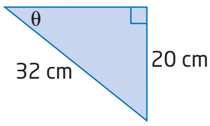
b) $\sin \theta = -0.3420$

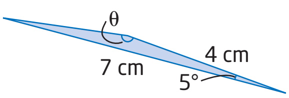
c) $\cos \theta = 0.3420$

2.3 The Sine Law, pages 100–113

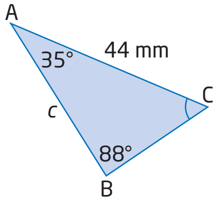
11. Does each triangle contain sufficient information for you to determine the unknown variable using the sine law? Explain why or why not.

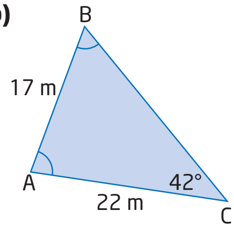
a) 

b) 

c) 

12. Determine the length(s) of the indicated side(s) and the measure(s) of the indicated angle(s) in each triangle.

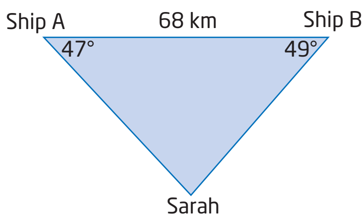
a) 

b) 

13. In △PQR, ∠P = 63.5°, ∠Q = 51.2°, and $r = 6.3$ cm. Sketch a diagram and find the measures of the unknown sides and angle.

14. In travelling to Jasper from Edmonton, you notice a mountain directly in front of you. The angle of elevation to the peak is 4.1°. When you are 21 km closer to the mountain, the angle of elevation is 8.7°. Determine the approximate height of the mountain.

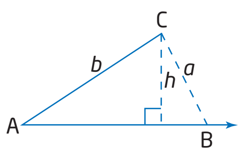
15. Sarah runs a deep-sea-fishing charter. On one of her expeditions, she has travelled 40 km from port when engine trouble occurs. There are two Search and Rescue (SAR) ships, as shown below.



a) Which ship is closer to Sarah? Use the sine law to determine her distance from that ship.

b) Verify your answer in part a) by using primary trigonometric ratios.

16. Given the measure of ∠A and the length of side b in △ABC, explain the restrictions on the lengths of sides a and b for the problem to have no solution, one solution, and two solutions.



17. A passenger jet is at cruising altitude heading east at 720 km/h. The pilot, wishing to avoid a thunderstorm, changes course by heading N70°E. The plane travels in this direction for 1 h, before turning to head toward the original path. After 30 min, the jet makes another turn onto its original path.

    a) Sketch a diagram to represent the distances travelled by the jet to avoid the thunderstorm.

    b) What heading, east of south, did the plane take, after avoiding the storm, to head back toward the original flight path?

    c) At what distance, east of the point where it changed course, did the jet resume its original path?

2.4 The Cosine Law, pages 114–125

18. Explain why each set of information does not describe a triangle that can be solved.

    a) $a = 7$, $b = 2$, $c = 4$

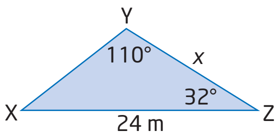
    b) $\angle A = 85°$, $b = 10$, $\angle C = 98°$

    c) $a = 12$, $b = 20$, $c = 8$

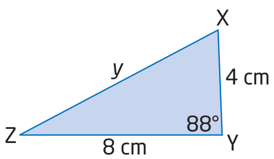
    d) $\angle A = 65°$, $\angle B = 82°$, $\angle C = 35°$

19. Would you use the sine law or the cosine law to find each indicated side length? Explain your reasoning.

    a)

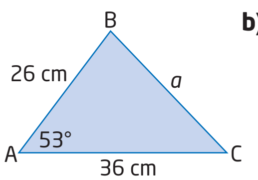


    b)

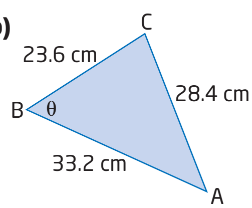


20. Determine the value of the indicated variable.

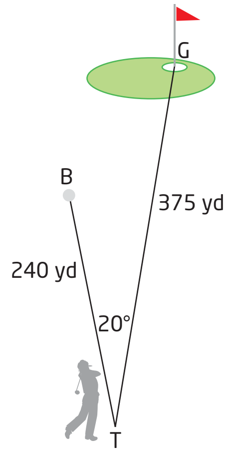
    a)



    b)



21. The 12th hole at a golf course is a 375-yd straightaway par 4. When Darla tees off, the ball travels 20° to the left of the line from the tee to the hole. The ball stops 240 yd from the tee (point B). Determine how far the ball is from the centre of the hole.



22. Sketch a diagram of each triangle and solve for the indicated value(s).

    a) In △ABC, AB = 18.4 m, BC = 9.6 m, and AC = 10.8 m. Determine the measure of ∠A.

    b) In △ABC, AC = 10 cm, BC = 9 cm, and ∠C = 48°. Determine the length of AB.

    c) Solve △ABC, given that AB = 15 m, BC = 8 m, and ∠B = 24°.

23. Two boats leave a dock at the same time. Each travels in a straight line but in different directions. The angle between their courses measures 54°. One boat travels at 48 km/h and the other travels at 53.6 km/h.

    a) Sketch a diagram to represent the situation.

    b) How far apart are the two boats after 4 h?

24. The sides of a parallelogram are 4 cm and 6 cm in length. One angle of the parallelogram is 58° and a second angle is 122°.

    a) Sketch a diagram to show the given information.

    b) Determine the lengths of the two diagonals of the parallelogram.

Chapter 2 Practice Test

Multiple Choice

For #1 to #5, choose the best answer.

1. Which angle in standard position has a different reference angle than all the others?

A 125° **B** 155°
C 205° **D** 335°

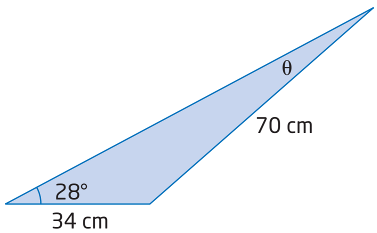
2. Which angle in standard position does not have a reference angle of 55°?

A 35° **B** 125°
C 235° **D** 305°

3. Which is the exact value of cos 150°?

A $\frac{1}{2}$ **B** $\frac{\sqrt{3}}{2}$
C $-\frac{\sqrt{3}}{2}$ **D** $-\frac{1}{2}$

4. The equation that could be used to determine the measure of angle θ is



A $\frac{\sin \theta}{70} = \frac{\sin 28°}{34}$

B $\frac{\sin \theta}{34} = \frac{\sin 28°}{70}$

C $\cos \theta = \frac{70^2 + 34^2 - 28^2}{2(70)(34)}$

D $\theta^2 = 34^2 + 70^2 - 2(34)(70)\cos 28°$

5. For which of these triangles must you consider the ambiguous case?

A In △ABC, $a = 16$ cm, $b = 12$ cm, and $c = 5$ cm.

B In △DEF, ∠D = 112°, $e = 110$ km, and $f = 65$ km.

C In △ABC, ∠B = 35°, $a = 27$ m, and $b = 21$ m.

D In △DEF, ∠D = 108°, ∠E = 52°, and $f = 15$ cm.

Short Answer

6. The point P(2, b) is on the terminal arm of an angle, θ, in standard position. If $\cos \theta = \frac{1}{\sqrt{10}}$ and tan θ is negative, what is the value of b?

7. Oak Bay in Victoria, is in the direction of N57°E from Ross Bay. A sailboat leaves Ross Bay in the direction of N79°E. After sailing for 1.9 km, the sailboat turns and travels 1.1 km to reach Oak Bay.

a) Sketch a diagram to represent the situation.

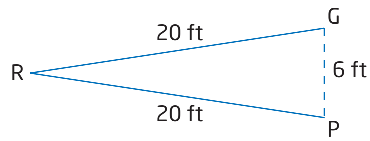
b) What is the distance between Ross Bay and Oak Bay?

8. In △ABC, $a = 10$, $b = 16$, and ∠A = 30°.

a) How many distinct triangles can be drawn given these measurements?

b) Determine the unknown measures in △ABC.

9. Rudy is 20 ft from each goal post when he shoots the puck along the ice toward the goal. The goal is 6 ft wide. Within what angle must he fire the puck to have a hope of scoring a goal?

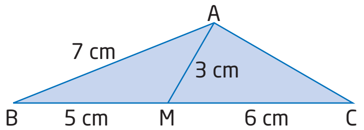


10. In △PQR, ∠P = 56°, $p = 10$ cm, and $q = 12$ cm.

a) Sketch a diagram of the triangle.

b) Determine the length of the unknown side and the measures of the unknown angles.

11. In $\triangle$ABC, M is a point on BC such that BM = 5 cm and MC = 6 cm. If AM = 3 cm and AB = 7 cm, determine the length of AC.

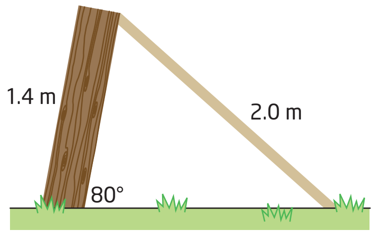


12. A fence that is 1.4 m tall has started to lean and now makes an angle of 80° with the ground. A 2.0-m board is jammed between the top of the fence and the ground to prop the fence up.

a) What angle, to the nearest degree, does the board make with the ground?

b) What angle, to the nearest degree, does the board make with the top of the fence?

c) How far, to the nearest tenth of a metre, is the bottom of the board from the base of the fence?



Extended Response

13. Explain, using examples, how to determine all angles $0° \leq \theta < 360°$ with a given reference angle θ_R.

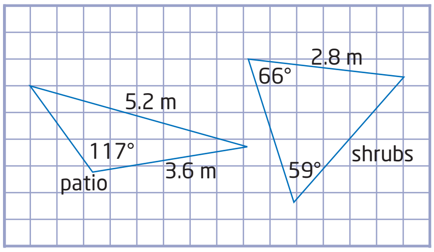
14. A softball diamond is a square measuring 70 ft on each side, with home plate and the three bases as vertices. The centre of the pitcher's mound is located 50 ft from home plate on a direct line from home to second base.

a) Sketch a diagram to represent the given information.

b) Show that first base and second base are not the same distance from the pitcher's mound.

15. Describe when to use the sine law and when to use the cosine law to solve a given problem.

16. As part of her landscaping course at the Northern Alberta Institute of Technology, Justine spends a summer redesigning her aunt's backyard. She chooses triangular shapes for her theme. Justine knows some of the measurements she needs in her design. Determine the unknown side lengths in each triangle.



17. The North Shore Rescue (NSR) is a mountain search and rescue team based in Vancouver. On one of their training exercises, the team splits up into two groups. From their starting point, the groups head off at an angle of 129° to each other. The Alpha group walks east for 3.8 km before losing radio contact with the Beta group. If their two-way radios have a range of 6.2 km, how far could the Beta group walk before losing radio contact with the Alpha group?

Cumulative Review, Chapters 1–2

Chapter 1 Sequences and Series

1. Match each term to the correct expression.

a) arithmetic sequence

b) geometric sequence

c) arithmetic series

d) geometric series

e) convergent series

A $3, 7, 11, 15, 19, \ldots$

B $5 + 1 + \frac{1}{5} + \frac{1}{25} + \cdots$

C $1 + 2 + 4 + 8 + 16 + \cdots$

D $1, 3, 9, 27, 81, \ldots$

E $2 + 5 + 8 + 11 + 14 + \cdots$

2. Classify each sequence as arithmetic or geometric. State the value of the common difference or common ratio. Then, write the next three terms in each sequence.

a) $27, 18, 12, 8, \ldots$

b) $17, 14, 11, 8, \ldots$

c) $-21, -16, -11, -6, \ldots$

d) $3, -6, 12, -24, \ldots$

3. For each arithmetic sequence, determine the general term. Express your answer in simplified form.

a) $18, 15, 12, 9, \ldots$

b) $1, \frac{5}{2}, 4, \frac{11}{2}, \ldots$

4. Use the general term to determine t_{20} in the geometric sequence $2, -4, 8, -16, \ldots$.

5. a) What is S_{12} for the arithmetic series with a common difference of 3 and $t_{12} = 31$?

b) What is S_5 for a geometric series where $t_1 = 4$ and $t_{10} = 78\ 732$?

6. Phytoplankton, or algae, is a nutritional supplement used in natural health programs. Canadian Pacific Phytoplankton Ltd. is located in Nanaimo, British Columbia. The company can grow 10 t of marine phytoplankton on a regular 11-day cycle. Assume this cycle continues.

a) Create a graph showing the amount of phytoplankton produced for the first five cycles of production.

b) Write the general term for the sequence produced.

c) How does the general term relate to the characteristics of the linear function described by the graph?

7. The Living Shangri-La is the tallest building in Metro Vancouver. The ground floor of the building is 5.8 m high, and each floor above the ground floor is 3.2 m high. There are 62 floors altogether, including the ground floor. How tall is the building?

8. Tristan and Julie are preparing a math display for the school open house. Both students create posters to debate the following question:

Does $0.999\ldots = 1$?

Julie's Poster

$0.999\ldots \neq 1$
$0.999\ldots = 0.999\ 999\ 999\ 999\ 9\ldots$

The decimal will continue to infinity and will never reach exactly one.

Tristan's Poster

$0.999\ldots = 1$
Rewrite 0.999... in expanded form.

$$\frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \cdots$$

This can be written as a geometric series where $t_1 = \frac{9}{10}$ and $r =$

a) Finish Tristan's poster by determining the value of the common ratio and then finding the sum of the infinite geometric series.

b) Which student do you think correctly answered the question?

Chapter 2 Trigonometry

9. Determine the exact distance, in simplified form, from the origin to a point P(−2, 4) on the terminal arm of an angle.

10. Point P(15, 8) is on the terminal arm of angle θ. Determine the exact values for sin θ, cos θ, and tan θ.

11. Sketch each angle in standard position and determine the measure of the reference angle.

a) 40°

b) 120°

c) 225°

d) 300°

12. The clock tower on the post office in Battleford, Saskatchewan, demonstrates the distinctive Romanesque Revival style of public buildings designed in the early decades of the twentieth century. The Battleford Post Office is similar in design to the post offices in Humboldt and Melfort. These three buildings are the only surviving post offices of this type in the Prairie Provinces.

a) What is the measure of the angle, θ, created between the hands of the clock when it reads 3 o'clock?

b) If the length of the minute hand is 2 ft, sketch a diagram to represent the clock face on a coordinate grid with the centre at the origin. Label the coordinates represented by the tip of the minute hand.

c) What are the exact values for sin θ, cos θ, and tan θ at 3 o'clock?

Battleford post office

13. Determine the exact value of each trigonometric ratio.

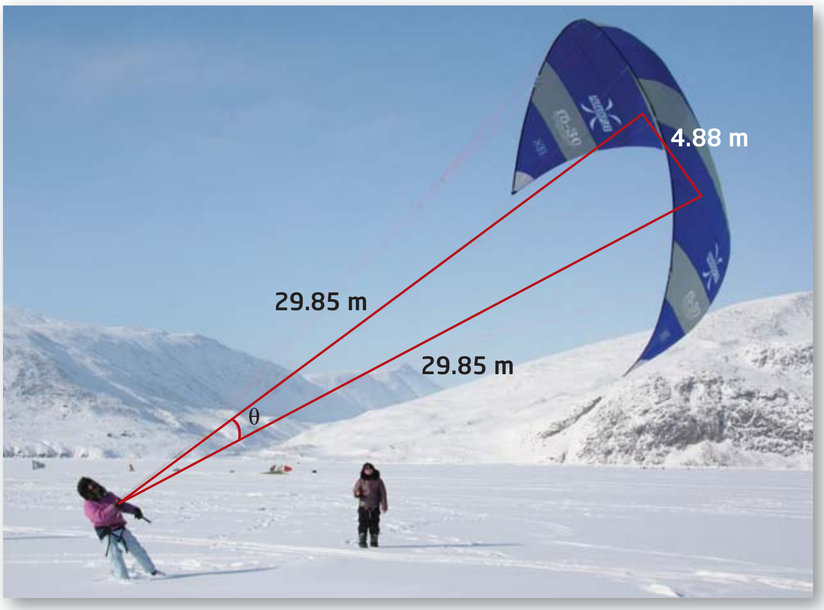
a) sin 405° **b)** cos 330°

c) tan 225° **d)** cos 180°

e) tan 150° **f)** sin 270°

14. Radio collars are used to track polar bears by sending signals via GPS to receiving stations. Two receiving stations are 9 km apart along a straight road. At station A, the signal from one of the collars comes from a direction of 49° from the road. At station B, the signal from the same collar comes from a direction of 65° from the road. Determine the distance the polar bear is from each of the stations.

15. The Arctic Wind Riders is a program developed to introduce youth in the communities of Northern Canada to the unique sport of Paraski. This sport allows participants to sail over frozen bays, rivers, and snowy tundra using wind power. The program has been offered to close to 700 students and young adults. A typical Paraski is shown below. Determine the measure of angle θ, to the nearest tenth of a degree.



Clara Akulukjuk, Pangnirtung, Nunavut, learning to Paraski.

16. Waterton Lakes National Park in Alberta is a popular site for birdwatching, with over 250 species of birds recorded. Chelsea spots a rare pileated woodpecker in a tree at an angle of elevation of 52°. After walking 16 m closer to the tree, she determines the new angle of elevation to be 70°.

a) Sketch and label a diagram to represent the situation.

b) What is the closest distance that Chelsea is from the bird, to the nearest tenth of a metre?

17. In △RST, RT = 2 m, ST = 1.4 m, and ∠R = 30°. Determine the measure of obtuse ∠S to the nearest tenth of a degree.

Unit 1 Test

Multiple Choice

For #1 to #5, choose the best answer.

1. Which of the following expressions could represent the general term of the sequence 8, 4, 0, …?

A $t_n = 8 + (n - 1)4$

B $t_n = 8 - (n - 1)4$

C $t_n = 4n + 4$

D $t_n = 8(-2)^{n-1}$

2. The expression for the 14th term of the geometric sequence x, x^3, x^5, … is

A x^{13}

B x^{14}

C x^{27}

D x^{29}

3. The sum of the series $6 + 18 + 54 + \cdots$ to n terms is 2184. How many terms are in the series?

A 5

B 7

C 8

D 6

4. Which angle has a reference angle of 55°?

A 35°

B 135°

C 235°

D 255°

5. Given the point $P(x, \sqrt{5})$ on the terminal arm of angle θ, where $\sin\theta = \frac{\sqrt{5}}{5}$ and $90° \le \theta \le 180°$, what is the exact value of cos θ?

A $\frac{3}{5}$

B $-\frac{3}{\sqrt{5}}$

C $\frac{2}{\sqrt{5}}$

D $-\frac{2\sqrt{5}}{5}$

Numerical Response

Complete the statements in #6 to #8.

6. A coffee shop is holding its annual fundraiser to help send a local child to summer camp. The coffee shop plans to donate a portion of the profit for every cup of coffee served. At the beginning of the day, the owner buys the first cup of coffee and donates $20 to the fundraiser. If the coffee shop regularly serves another 2200 cups of coffee in one day, they must collect $■ per cup to raise $350.

7. An angle of 315° drawn in standard position has a reference angle of ■°.

8. The terminal arm of an angle, θ, in standard position lies in quadrant IV, and it is known that $\sin\theta = -\frac{\sqrt{3}}{2}$. The measure of θ is ■.

Written Response

9. Jacques Chenier is one of Manitoba's premier children's entertainers. Jacques was a Juno Award Nominee for his album *Walking in the Sun*. He has performed in over 600 school fairs and festivals across the country. Suppose there were 150 people in the audience for his first performance. If this number increased by 5 for each of the next 14 performances, what total number of people attended the first 15 of Jacques Chenier's performances?

10. The third term in an arithmetic sequence is 4 and the seventh term in the sequence is 24.

a) Determine the value of the common difference.

b) What is the value of t_1?

c) Write the general term of the sequence.

d) What is the sum of the first 10 terms of the corresponding series?

11. A new car that is worth $35 000 depreciates 20% in the first year and 10% every year after that. About how much will the car be worth 7 years after it is purchased?

12. The Multiple Sclerosis Walk is a significant contributor to the Multiple Sclerosis Society's fundraising efforts to support research. One walker was sponsored $100 plus $5 for the first kilometre, $10 for the second kilometre, $15 for the third kilometre, and so on. How far would this walker need to walk to earn $150?

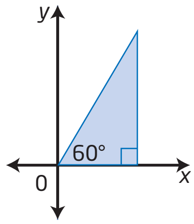
13. Les Jeux de la Francophonie Canadienne are held each summer to celebrate sport, leadership, and French culture. Badminton is one of the popular events at the games. There are 64 entrants in the boys' singles tournament. There are two players per game, and only the winner advances to the next round. The number of players in each round models a geometric sequence.

 a) Write the first four terms of the geometric sequence.

 b) Write the general term that could be used to determine the number of players in any round of the tournament.

 c) How many games must be played altogether to determine the winner of the boys' singles tournament?

14. A right triangle with a reference angle of 60° is drawn in standard position on a coordinate grid.

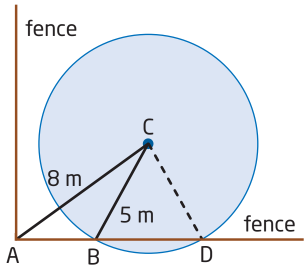


 a) Apply consecutive rotations of 60° counterclockwise to complete one 360° revolution about the origin.

 b) Write the sequence that represents the measures of the angles in standard position formed by the rotations.

 c) Write the general term for the sequence.

15. A circular water sprinkler in a backyard sprays a radius of 5 m. The sprinkler is placed 8 m from the corner of the lot.



 a) If the measure of ∠CAB is 32°, what is the measure of ∠CDA?

 b) What length of fence, to the nearest tenth of a metre, would get wet from the sprinkler?

16. A triangle has sides that measure 61 cm, 38 cm, and 43 cm. Determine the measure of the smallest angle in the triangle.

Check Your Understanding

Practise

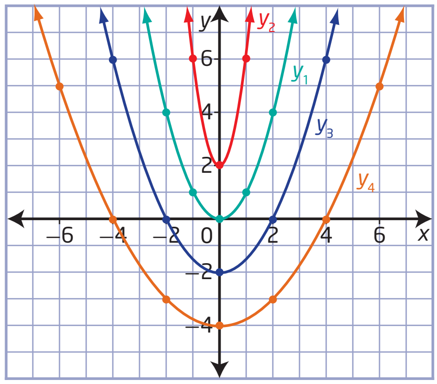
1. Describe how you can obtain the graph of each function from the graph of $f(x) = x^2$. State the direction of opening, whether it has a maximum or a minimum value, and the range for each.
   a) $f(x) = 7x^2$
   b) $f(x) = \frac{1}{6}x^2$
   c) $f(x) = -4x^2$
   d) $f(x) = -0.2x^2$

2. Describe how the graphs of the functions in each pair are related. Then, sketch the graph of the second function in each pair, and determine the vertex, the equation of the axis of symmetry, the domain and range, and any intercepts.
   a) $y = x^2$ and $y = x^2 + 1$
   b) $y = x^2$ and $y = (x - 2)^2$
   c) $y = x^2$ and $y = x^2 - 4$
   d) $y = x^2$ and $y = (x + 3)^2$

3. Describe how to sketch the graph of each function using transformations.
   a) $f(x) = (x + 5)^2 + 11$
   b) $f(x) = -3x^2 - 10$
   c) $f(x) = 5(x + 20)^2 - 21$
   d) $f(x) = -\frac{1}{8}(x - 5.6)^2 + 13.8$

4. Sketch the graph of each function. Identify the vertex, the axis of symmetry, the direction of opening, the maximum or minimum value, the domain and range, and any intercepts.
   a) $y = -(x - 3)^2 + 9$
   b) $y = 0.25(x + 4)^2 + 1$
   c) $y = -3(x - 1)^2 + 12$
   d) $y = \frac{1}{2}(x - 2)^2 - 2$

5. **a)** Write a quadratic function in vertex form for each parabola in the graph.



   b) Suppose four new parabolas open downward instead of upward but have the same shape and vertex as each parabola in the graph. Write a quadratic function in vertex form for each new parabola.
   c) Write the quadratic functions in vertex form of four parabolas that are identical to the four in the graph but translated 4 units to the left.
   d) Suppose the four parabolas in the graph are translated 2 units down. Write a quadratic function in vertex form for each new parabola.

6. For the function $f(x) = 5(x - 15)^2 - 100$, explain how you can identify each of the following without graphing.
   a) the coordinates of the vertex
   b) the equation of the axis of symmetry
   c) the direction of opening
   d) whether the function has a maximum or minimum value, and what that value is
   e) the domain and range
   f) the number of x-intercepts

7. Without graphing, identify the location of the vertex and the axis of symmetry, the direction of opening and the maximum or minimum value, the domain and range, and the number of x-intercepts for each function.

a) $y = -4x^2 + 14$

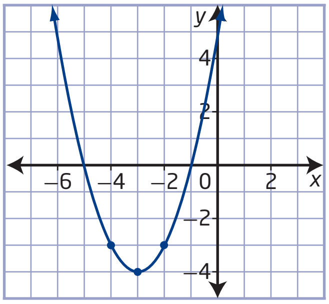
b) $y = (x + 18)^2 - 8$

c) $y = 6(x - 7)^2$

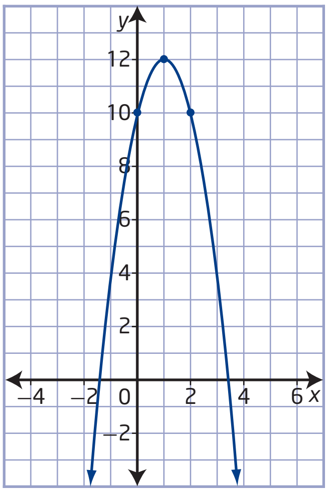
d) $y = -\frac{1}{9}(x + 4)^2 - 36$

8. Determine the quadratic function in vertex form for each parabola.

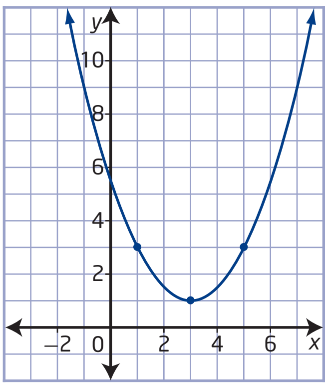
a)



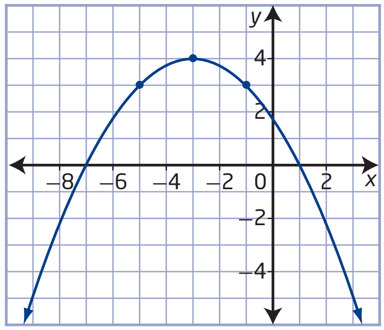
b)



c)



d)



9. Determine a quadratic function in vertex form that has the given characteristics.

a) vertex at (0, 0), passing through the point (6, −9)

b) vertex at (0, −6), passing through the point (3, 21)

c) vertex at (2, 5), passing through the point (4, −11)

d) vertex at (−3, −10), passing through the point (2, −5)

Apply

10. The point (4, 16) is on the graph of $f(x) = x^2$. Describe what happens to the point when each of the following sets of transformations is performed in the order listed. Identify the corresponding point on the transformed graph.

a) a horizontal translation of 5 units to the left and then a vertical translation of 8 units up

b) a multiplication of the y-values by a factor of $\frac{1}{4}$ and then a reflection in the x-axis

c) a reflection in the x-axis and then a horizontal translation of 10 units to the right

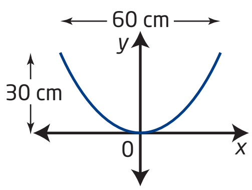
d) a multiplication of the y-values by a factor of 3 and then a vertical translation of 8 units down

11. Describe how to obtain the graph of $y = 20 - 5x^2$ using transformations on the graph of $y = x^2$.

12. Quadratic functions do not all have the same number of x-intercepts. Is the same true about y-intercepts? Explain.

13. A parabolic mirror was used to ignite the Olympic torch for the 2010 Winter Olympics in Vancouver and Whistler, British Columbia. Suppose its diameter is 60 cm and its depth is 30 cm.

a) Determine the quadratic function that represents its cross-sectional shape if the lowest point in the centre of the mirror is considered to be the origin, as shown.



b) How would the quadratic function be different if the outer edge of the mirror were considered the origin? Explain why there is a difference.

Did You Know?

Before the 2010 Winter Olympics began in Vancouver and Whistler, the Olympic torch was carried over 45 000 km for 106 days through every province and territory in Canada. The torch was initially lit in Olympia, Greece, the site of the ancient Olympic Games, before beginning its journey in Canada. The flame was lit using a special bowl-shaped reflector called a parabolic mirror that focuses the Sun's rays to a single point, concentrating enough heat to ignite the torch.

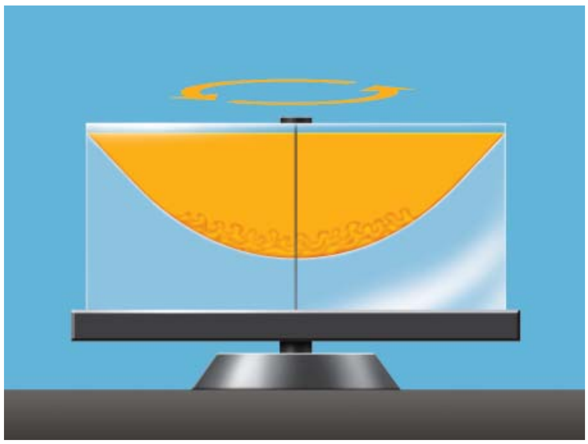
14. The finance team at an advertising company is using the quadratic function $N(x) = -2.5(x - 36)^2 + 20\,000$ to predict the effectiveness of a TV commercial for a certain product, where N is the predicted number of people who buy the product if the commercial is aired x times per week.

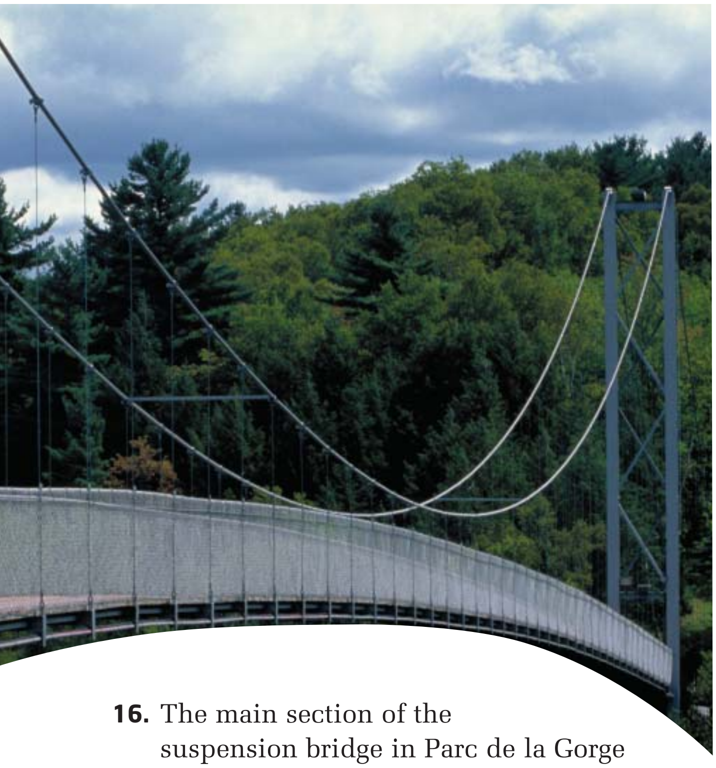
a) Explain how you could sketch the graph of the function, and identify its characteristics.

b) According to this model, what is the optimum number of times the commercial should be aired?

c) What is the maximum number of people that this model predicts will buy the product?

15. When two liquids that do not mix are put together in a container and rotated around a central axis, the surface created between them takes on a parabolic shape as they rotate. Suppose the diameter at the top of such a surface is 40 cm, and the maximum depth of the surface is 12 cm. Choose a location for the origin and write the function that models the cross-sectional shape of the surface.





16. The main section of the suspension bridge in Parc de la Gorge de Coaticook, Québec, has cables in the shape of a parabola. Suppose that the points on the tops of the towers where the cables are attached are 168 m apart and 24 m vertically above the minimum height of the cables.

a) Determine the quadratic function in vertex form that represents the shape of the cables. Identify the origin you used.

b) Choose two other locations for the origin. Write the corresponding quadratic function for the shape of the cables for each.

c) Use each quadratic function to determine the vertical height of the cables above the minimum at a point that is 35 m horizontally from one of the towers. Are your answers the same using each of your functions? Explain.

Did You Know?

The suspension bridge in Parc de la Gorge de Coaticook in Québec claims to be the longest pedestrian suspension bridge in the world.

17. During a game of tennis, Natalie hits the tennis ball into the air along a parabolic trajectory. Her initial point of contact with the tennis ball is 1 m above the ground. The ball reaches a maximum height of 10 m before falling toward the ground. The ball is again 1 m above the ground when it is 22 m away from where she hit it. Write a quadratic function to represent the trajectory of the tennis ball if the origin is on the ground directly below the spot from which the ball was hit.

Did You Know?

Tennis originated from a twelth-century French game called *jeu de paume*, meaning game of palm (of the hand). It was a court game where players hit the ball with their hands. Over time, gloves covered bare hands and, finally, racquets became the standard equipment. In 1873, Major Walter Wingfield invented a game called *sphairistike* (Greek for *playing ball*), from which modern outdoor tennis evolved.

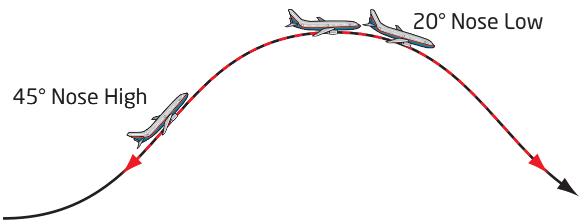
18. Water is spraying from a nozzle in a fountain, forming a parabolic path as it travels through the air. The nozzle is 10 cm above the surface of the water. The water achieves a maximum height of 100 cm above the water's surface and lands in the pool. The water spray is again 10 cm above the surface of the water when it is 120 cm horizontally from the nozzle. Write the quadratic function in vertex form to represent the path of the water if the origin is at the surface of the water directly below the nozzle.

19. The function $y = x^2 + 4$ represents a translation of 4 units up, which is in the positive direction. The function $y = (x + 4)^2$ represents a translation of 4 units to the left, which is in the negative direction. How can you explain this difference?

20. In the movie, *Apollo 13*, starring Tom Hanks, scenes were filmed involving weightlessness. Weightlessness can be simulated using a plane to fly a special manoeuvre. The plane follows a specific inverted parabolic arc followed by an upward-facing recovery arc. Suppose the parabolic arc starts when the plane is at 7200 m and takes it up to 10 000 m and then back down to 7200 m again. It covers approximately 16 000 m of horizontal distance in total.

a) Determine the quadratic function that represents the shape of the parabolic path followed by the plane if the origin is at ground level directly below where the plane starts the parabolic arc.

b) Identify the domain and range in this situation.



Did You Know?

Passengers can experience the feeling of *zero-g*, or weightlessness, for approximately 30 s during each inverted parabolic manoeuvre made. During the recovery arc, passengers feel almost *two-g*, or almost twice the sensation of gravity. In addition to achieving weightlessness, planes such as these are also able to fly parabolic arcs designed to simulate the gravity on the moon (one sixth of Earth's) or on Mars (one third of Earth's).

21. Determine a quadratic function in vertex form given each set of characteristics. Explain your reasoning.

a) vertex (6, 30) and a y-intercept of -24

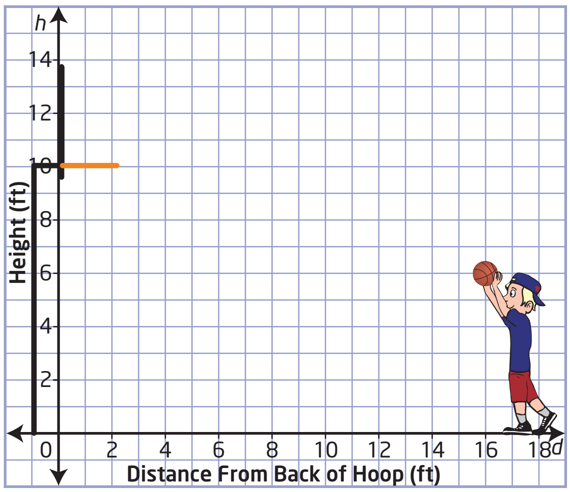
b) minimum value of -24 and x-intercepts at -21 and -5

Extend

22. a) Write quadratic functions in vertex form that represent three different trajectories the basketball shown can follow and pass directly through the hoop without hitting the backboard.

b) Which of your three quadratic functions do you think represents the most realistic trajectory for an actual shot? Explain your thoughts.

c) What do you think are a reasonable domain and range in this situation?



23. If the point (m, n) is on the graph of $f(x) = x^2$, determine expressions for the coordinates of the corresponding point on the graph of $f(x) = a(x - p)^2 + q$.

Create Connections

24. a) Write a quadratic function that is related to $f(x) = x^2$ by a change in width, a reflection, a horizontal translation, and a vertical translation.

b) Explain your personal strategy for accurately sketching the function.

25. Create your own specific examples of functions to explain how to determine the number of x-intercepts for quadratic functions of the form $f(x) = a(x - p)^2 + q$ without graphing.

26. **MINI LAB** Graphing a function like $y = -x^2 + 9$ will produce a curve that extends indefinitely. If only a portion of the curve is desired, you can state the function with a restriction on the domain. For example, to draw only the portion of the graph of $y = -x^2 + 9$ between the points where $x = -2$ and $x = 3$, write $y = -x^2 + 9$, $\{x \mid -2 \leq x \leq 3, x \in \mathrm{R}\}$.

Materials

- 0.5-cm grid paper

Create a line-art illustration of an object or design using quadratic and/or linear functions with restricted domains.

Step 1 Use a piece of 0.5-cm grid paper. Draw axes vertically and horizontally through the centre of the grid. Label the axes with a scale.

Step 2 Plan out a line-art drawing that you can draw using portions of the graphs of quadratic and linear functions. As you create your illustration, keep a record of the functions you use. Add appropriate restrictions to the domain to indicate the portion of the graph you want.

Step 3 Use your records to make a detailed and accurate list of instructions/functions (including restrictions) that someone else could use to recreate your illustration.

Step 4 Trade your functions/instructions list with a partner. See if you can recreate each other's illustration using only the list as a guide.

Project Corner — Parabolic Shape

- Many suspension bridge cables, the arches of bridges, satellite dishes, reflectors in headlights and spotlights, and other physical objects often appear to have parabolic shape.
- You can try to model a possible quadratic relationship by drawing a set of axes on an image of a physical object that appears to be quadratic in nature, and using one or more points on the curve.
- What images or objects can you find that might be quadratic?

Check Your Understanding

Practise

1. Which functions are quadratic? Explain.

a) $f(x) = 2x^2 + 3x$

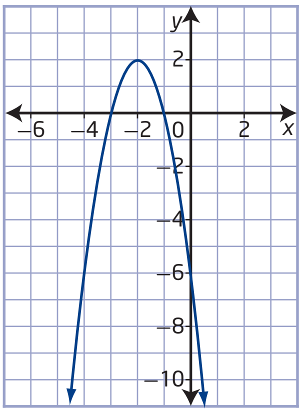
b) $f(x) = 5 - 3x$

c) $f(x) = x(x + 2)(4x - 1)$

d) $f(x) = (2x - 5)(3x - 2)$

2. For each graph, identify the following:
- the coordinates of the vertex
- the equation of the axis of symmetry
- the x-intercepts and y-intercept
- the maximum or minimum value and how it is related to the direction of opening
- the domain and range

a)



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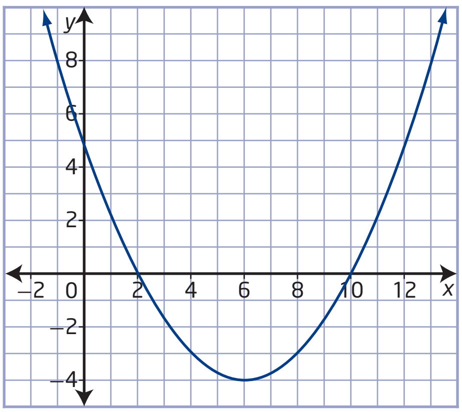
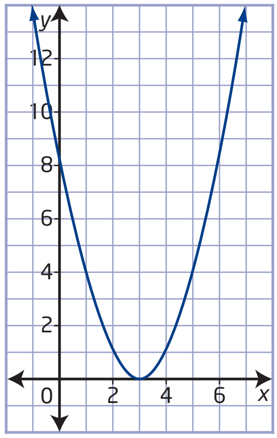


c)



3. Show that each function fits the definition of a quadratic function by writing it in standard form.

a) $f(x) = 5x(10 - 2x)$

b) $f(x) = (10 - 3x)(4 - 5x)$

4. Create a table of values and then sketch the graph of each function. Determine the vertex, the axis of symmetry, the direction of opening, the maximum or minimum value, the domain and range, and any intercepts.

a) $f(x) = x^2 - 2x - 3$

b) $f(x) = -x^2 + 16$

c) $p(x) = x^2 + 6x$

d) $g(x) = -2x^2 + 8x - 10$

5. Use technology to graph each function. Identify the vertex, the axis of symmetry, the direction of opening, the maximum or minimum value, the domain and range, and any intercepts. Round values to the nearest tenth, if necessary.

a) $y = 3x^2 + 7x - 6$

b) $y = -2x^2 + 5x + 3$

c) $y = 50x - 4x^2$

d) $y = 1.2x^2 + 7.7x + 24.3$

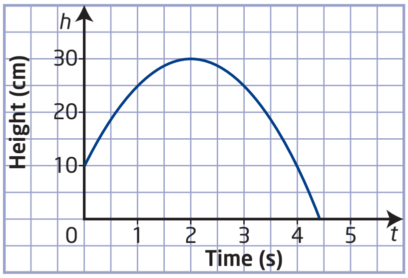
6. The x-coordinate of the vertex is given by $x = \frac{-b}{2a}$. Use this information to determine the vertex of each quadratic function.

a) $y = x^2 + 6x + 2$

b) $y = 3x^2 - 12x + 5$

c) $y = -x^2 + 8x - 11$

7. A siksik, an Arctic ground squirrel, jumps from a rock, travels through the air, and then lands on the tundra. The graph shows the height of its jump as a function of time. Use the graph to answer each of the following, and identify which characteristic(s) of the graph you used in each case.



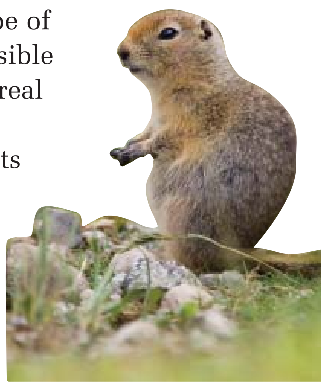
a) What is the height of the rock that the siksik jumped from?

b) What is the maximum height of the siksik? When did it reach that height?

c) How long was the siksik in the air?

d) What are the domain and range in this situation?

e) Would this type of motion be possible for a siksik in real life? Use your answers to parts a) to d) to explain why or why not.



Did You Know?

The *siksik* is named because of the sound it makes.

8. How many x-intercepts does each function have? Explain how you know. Then, determine whether each intercept is positive, negative, or zero.

a) a quadratic function with an axis of symmetry of $x = 0$ and a maximum value of 8

b) a quadratic function with a vertex at (3, 1), passing through the point (1, −3)

c) a quadratic function with a range of $y \geq 1$

d) a quadratic function with a y-intercept of 0 and an axis of symmetry of $x = -1$

9. Consider the function $f(x) = -16x^2 + 64x + 4$.

a) Determine the domain and range of the function.

b) Suppose this function represents the height, in feet, of a football kicked into the air as a function of time, in seconds. What are the domain and range in this case?

c) Explain why the domain and range are different in parts a) and b).

Apply

10. Sketch the graph of a quadratic function that has the characteristics described in each part. Label the coordinates of three points that you know are on the curve.

a) x-intercepts at −1 and 3 and a range of $y \geq -4$

b) one of its x-intercepts at −5 and vertex at (−3, −4)

c) axis of symmetry of $x = 1$, minimum value of 2, and passing through (−1, 6)

d) vertex at (2, 5) and y-intercept of 1

11. Satellite dish antennas have the shape of a parabola. Consider a satellite dish that is 80 cm across. Its cross-sectional shape can be described by the function $d(x) = 0.0125x^2 - x$, where d is the depth, in centimetres, of the dish at a horizontal distance of x centimetres from one edge of the dish.

a) What is the domain of this function?

b) Graph the function to show the cross-sectional shape of the satellite dish.

c) What is the maximum depth of the dish? Does this correspond to the maximum value of the function? Explain.

d) What is the range of the function?

e) How deep is the dish at a point 25 cm from the edge of the dish?

12. A jumping spider jumps from a log onto the ground below. Its height, h, in centimetres, as a function of time, t, in seconds, since it jumped can be modelled by the function $h(t) = -490t^2 + 75t + 12$. Where appropriate, answer the following questions to the nearest tenth.

a) Graph the function.

b) What does the h-intercept represent?

c) When does the spider reach its maximum height? What is its maximum height?

d) When does the spider land on the ground?

e) What domain and range are appropriate in this situation?

f) What is the height of the spider 0.05 s after it jumps?

Did You Know?

There are an estimated 1400 spider species in Canada. About 110 of these are jumping spiders. British Columbia has the greatest diversity of jumping spiders. Although jumping spiders are relatively small (3 mm to 10 mm in length), they can jump horizontal distances of up to 16 cm.

13. A quadratic function can model the relationship between the speed of a moving object and the wind resistance, or drag force, it experiences. For a typical car travelling on a highway, the relationship between speed and drag can be approximated with the function $f(v) = 0.002v^2$, where f is the drag force, in newtons, and v is the speed of the vehicle, in kilometres per hour.

a) What domain do you think is appropriate in this situation?

b) Considering your answer to part a), create a table of values and a graph to represent the function.

c) How can you tell from your graph that the function is not a linear function? How can you tell from your table?

d) What happens to the values of the drag force when the speed of the vehicle doubles? Does the drag force also double?

e) Why do you think a driver might be interested in understanding the relationship between the drag force and the speed of the vehicle?

Did You Know?

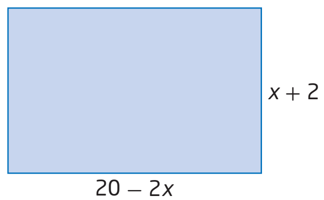
A *newton* (abbreviated N) is a unit of measure of force. One newton is equal to the force required to accelerate a mass of one kilogram at a rate of one metre per second squared.

14. Assume that you are an advisor to business owners who want to analyse their production costs. The production costs, C, to produce n thousand units of their product can be modelled by the function $C(n) = 0.3n^2 - 48.6n + 13\ 500$.

a) Graph the function and identify the important characteristics of the graph.

b) Write a short explanation of what the graph and each piece of information you determined in part a) shows about the production costs.

15. a) Write a function to represent the area of the rectangle. Show that the function fits the definition of a quadratic function.



b) Graph the function.

c) What do the x-intercepts represent in this situation? How do they relate to the dimensions of the rectangle?

d) What information does the vertex give about this situation?

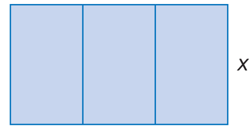
e) What are the domain and range? What do they represent in this situation?

f) Does the function have a maximum value in this situation? Does it have a minimum value?

g) If you graph the function you wrote in part a) with the domain being the set of all real numbers, does it have a minimum value? Explain.

16. Does the function $f(x) = 4x^2 - 3x + 2x(3 - 2x) + 1$ represent a quadratic function? Show why or why not using several different methods.

17. Maria lives on a farm. She is planning to build an enclosure for her animals that is divided into three equal-sized sections, as shown in the diagram. She has 280 m of fencing to use.



a) Write a function that represents the area of the entire enclosure in terms of its width. How do you know that it fits the definition of a quadratic function?

b) Graph the function.

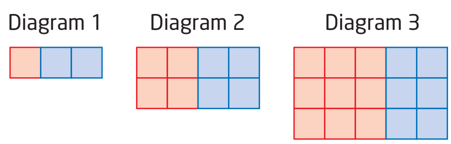
c) What are the coordinates of the vertex? What do they represent?

d) What are the domain and range in this situation?

e) Does the function have a maximum value? Does it have a minimum value? Explain.

f) What assumptions did you make in your analysis of this situation?

18. a) Consider the pattern shown in the sequence of diagrams. The area of each small square is 1 square unit.



a) Draw the next three diagrams in the sequence. What is the total area of each diagram?

b) Write a function to model the total area, A, of each diagram in terms of the diagram number, n.

c) Is the function linear or quadratic? Explain why in terms of the diagrams as well as the function.

d) If the sequence of diagrams continues, what is the domain? Are the values in the domain continuous or discrete? Explain.

e) Considering your answer to part b), graph the function to show the relationship between A and n.

19. a) Write a function for the area, A, of a circle, in terms of its radius, r.

b) What domain and range are appropriate for this function?

c) Considering your answer to part b), graph the function.

d) What are the intercepts of the graph? What meaning do they have in this situation?

e) Does this graph have an axis of symmetry? Explain.

20. The stopping distance of a vehicle is the distance travelled between the time a driver notices a need to stop and the time when the vehicle actually stops. This includes the reaction time before applying the brakes and the time it takes to stop once the brakes are applied. The stopping distance, d, in metres, for a certain vehicle can be approximated using the formula $d(t) = \frac{vt}{3.6} + \frac{v^2}{130}$, where v is the speed of the vehicle, in kilometres per hour, before braking, and t is the reaction time, in seconds, before the driver applies the brakes.

a) Suppose the driver of this vehicle has a reaction time of 1.5 s. Write a function to model the stopping distance, d, for the vehicle and driver as a function of the pre-braking speed, v.

b) Create a table of values and graph the function using a domain of $0 \leq v \leq 200$.

c) When the speed of the vehicle doubles, does the stopping distance also double? Use your table and graph to explain.

d) Assume that you are writing a newspaper or magazine article about safe driving. Write an argument aimed at convincing drivers to slow down. Use your graphs and other results to support your case.

Extend

21. A *family* of functions is a set of functions that are related to each other in some way.

a) Write a set of functions for part of the family defined by $f(x) = k(x^2 + 4x + 3)$ if $k = 1, 2, 3$. Simplify each equation so that it is in standard form.

b) Graph the functions on the same grid using the restricted domain of $\{x \mid 0 \leq x \leq 4, x \in \mathrm{R}\}$.

c) Describe how the graphs are related to each other. How are the values of y related for points on each graph for the same value of x?

d) Predict what the graph would look like if $k = 4$ and if $k = 0.5$. Sketch the graph in each case to test your prediction.

e) Predict what the graphs would look like for negative values of k. Test your prediction.

f) What does the graph look like if $k = 0$?

g) Explain how the members of the family of functions defined by $f(x) = k(x^2 + 4x + 3)$ for all values of k are related.

22. Milos said, "The a in the quadratic function $f(x) = ax^2 + bx + c$ is like the 'steepness' of the graph, just like it is for the a in $f(x) = ax + b$." In what ways might this statement be a reasonable comparison? In what ways is it not completely accurate? Explain, using examples.

23. a) The point $(-2, 1)$ is on the graph of the quadratic function $f(x) = -x^2 + bx + 11$. Determine the value of b.

b) If the points $(-1, 6)$ and $(2, 3)$ are on the graph of the quadratic function $f(x) = 2x^2 + bx + c$, determine the values of b and c.

24. How would projectile motion be different on the moon? Consider the following situations:

- an object launched from an initial height of 35 m above ground with an initial vertical velocity of 20 m/s
- a flare that is shot into the air with an initial velocity of 800 ft/s from ground level
- a rock that breaks loose from the top of a 100-m-high cliff and starts to fall straight down

a) Write a pair of functions for each situation, one representing the motion if the situation occurred on Earth and one if on the moon.

b) Graph each pair of functions.

c) Identify the similarities and differences in the various characteristics for each pair of graphs.

d) What do your graphs show about the differences between projectile motion on Earth and the moon? Explain.

Did You Know?

You can create a function representing the height of any projectile over time using the formula $h(t) = -0.5gt^2 + v_0t + h_0$, where g is the acceleration due to gravity, v_0 is the initial vertical velocity, and h_0 is the initial height.

The *acceleration due to gravity* is a measure of how much gravity slows down an object fired upward or speeds up an object dropped or thrown downward. On the surface of Earth, the acceleration due to gravity is 9.81 m/s^2 in metric units or 32 ft/s^2 in imperial units. On the surface of the moon, the acceleration due to gravity is much less than on Earth, only 1.63 m/s^2 or 5.38 ft/s^2.

25. Determine an expression for the coordinates of each missing point described below. Explain your reasoning.

a) A quadratic function has a vertex at (m, n) and a y-intercept of r. Identify one other point on the graph.

b) A quadratic function has an axis of symmetry of $x = j$ and passes through the point $(4j, k)$. Identify one other point on the graph.

c) A quadratic function has a range of $y \geq d$ and x-intercepts of s and t. What are the coordinates of the vertex?

Create Connections

26. For the graph of a given quadratic function, how are the range, direction of opening, and location of the vertex, axis of symmetry, and x-intercepts connected?

27. MINI LAB Use computer or graphing calculator technology to investigate how the values of a, b, and c affect the graph of the function that corresponds to $y = ax^2 + bx + c$. If you are using graphing/geometry software, you may be able to use it to make sliders to change the values of a, b, and c. The will allow you to dynamically see the effect each parameter has.

For each step below, sketch one or more graphs to illustrate your findings.

Step 1 Change the values of a, b, and c, one at a time. Observe any changes that occur in the graph as a, b, or c is
- increased or decreased
- made positive or negative

Step 2 How is the y-intercept affected by the values of a, b, and c? Do all the values affect its location?

Step 3 Explore how the location of the axis of symmetry is affected by the values. Which values are involved, and how?

Step 4 Explore how the values affect the steepness of the curve as it crosses the y-axis. Do they all have an effect on this aspect?

Step 5 Observe whether any other aspects of the graph are affected by changes in a, b, and c. Explain your findings.

Your Turn

A sporting goods store sells reusable sports water bottles for \$8. At this price their weekly sales are approximately 100 items. Research says that for every \$2 increase in price, the manager can expect the store to sell five fewer water bottles.

a) Represent this situation with a quadratic function.
b) Determine the maximum revenue the manager can expect based on these estimates. What selling price will give that maximum revenue?
c) Verify your solution.
d) Explain any assumptions you made in using a quadratic function in this situation.

Key Ideas

- You can convert a quadratic function from standard form to vertex form by completing the square.

$y = 5x^2 - 30x + 7$	← standard form
$y = 5(x^2 - 6x) + 7$	Group the first two terms. Factor out the leading coefficient if $a \neq 1$.
$y = 5(x^2 - 6x + 9 - 9) + 7$	Add and then subtract the square of half the coefficient of the x-term.
$y = 5[(x^2 - 6x + 9) - 9] + 7$	Group the perfect square trinomial.
$y = 5[(x - 3)^2 - 9] + 7$	Rewrite using the square of a binomial.
$y = 5(x - 3)^2 - 45 + 7$	Simplify.
$y = 5(x - 3)^2 - 38$	← vertex form

- Converting a quadratic function to vertex form, $y = a(x - p)^2 + q$, reveals the coordinates of the vertex, (p, q).
- You can use information derived from the vertex form to solve problems such as those involving maximum and minimum values.

Check Your Understanding

Practise

1. Use a model to determine the value of c that makes each trinomial expression a perfect square. What is the equivalent binomial square expression for each?

a) $x^2 + 6x + c$
b) $x^2 - 4x + c$
c) $x^2 + 14x + c$
d) $x^2 - 2x + c$

2. Write each function in vertex form by completing the square. Use your answer to identify the vertex of the function.

a) $y = x^2 + 8x$
b) $y = x^2 - 18x - 59$
c) $y = x^2 - 10x + 31$
d) $y = x^2 + 32x - 120$

3. Convert each function to the form $y = a(x - p)^2 + q$ by completing the square. Verify each answer with or without technology.

a) $y = 2x^2 - 12x$

b) $y = 6x^2 + 24x + 17$

c) $y = 10x^2 - 160x + 80$

d) $y = 3x^2 + 42x - 96$

4. Convert each function to vertex form algebraically, and verify your answer.

a) $f(x) = -4x^2 + 16x$

b) $f(x) = -20x^2 - 400x - 243$

c) $f(x) = -x^2 - 42x + 500$

d) $f(x) = -7x^2 + 182x - 70$

5. Verify, in at least two different ways, that the two algebraic forms in each pair represent the same function.

a) $y = x^2 - 22x + 13$
and
$y = (x - 11)^2 - 108$

b) $y = 4x^2 + 120x$
and
$y = 4(x + 15)^2 - 900$

c) $y = 9x^2 - 54x - 10$
and
$y = 9(x - 3)^2 - 91$

d) $y = -4x^2 - 8x + 2$
and
$y = -4(x + 1)^2 + 6$

6. Determine the maximum or minimum value of each function and the value of x at which it occurs.

a) $y = x^2 + 6x - 2$

b) $y = 3x^2 - 12x + 1$

c) $y = -x^2 - 10x$

d) $y = -2x^2 + 8x - 3$

7. For each quadratic function, determine the maximum or minimum value.

a) $f(x) = x^2 + 5x + 3$

b) $f(x) = 2x^2 - 2x + 1$

c) $f(x) = -0.5x^2 + 10x - 3$

d) $f(x) = 3x^2 - 4.8x$

e) $f(x) = -0.2x^2 + 3.4x + 4.5$

f) $f(x) = -2x^2 + 5.8x - 3$

8. Convert each function to vertex form.

a) $y = x^2 + \frac{3}{2}x - 7$

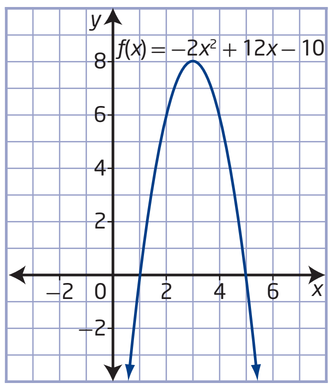
b) $y = -x^2 - \frac{3}{8}x$

c) $y = 2x^2 - \frac{5}{6}x + 1$

Apply

9. a) Convert the quadratic function $f(x) = -2x^2 + 12x - 10$ to vertex form by completing the square.

b) The graph of $f(x) = -2x^2 + 12x - 10$ is shown. Explain how you can use the graph to verify your answer.



10. a) For the quadratic function $y = -4x^2 + 20x + 37$, determine the maximum or minimum value and domain and range without making a table of values or graphing.

b) Explain the strategy you used in part a).

11. Determine the vertex of the graph of $f(x) = 12x^2 - 78x + 126$. Explain the method you used.

12. Identify, explain, and correct the error(s) in the following examples of completing the square.

a) $y = x^2 + 8x + 30$
$y = (x^2 + 4x + 4) + 30$
$y = (x + 2)^2 + 30$

b) $f(x) = 2x^2 - 9x - 55$
$f(x) = 2(x^2 - 4.5x + 20.25 - 20.25) - 55$
$f(x) = 2[(x^2 - 4.5x + 20.25) - 20.25] - 55$
$f(x) = 2[(x - 4.5)^2 - 20.25] - 55$
$f(x) = 2(x - 4.5)^2 - 40.5 - 55$
$f(x) = (x - 4.5)^2 - 95.5$

c) $y = 8x^2 + 16x - 13$
$y = 8(x^2 + 2x) - 13$
$y = 8(x^2 + 2x + 4 - 4) - 13$
$y = 8[(x^2 + 2x + 4) - 4] - 13$
$y = 8[(x + 2)^2 - 4] - 13$
$y = 8(x + 2)^2 - 32 - 13$
$y = 8(x + 2)^2 - 45$

d) $f(x) = -3x^2 - 6x$
$f(x) = -3(x^2 - 6x - 9 + 9)$
$f(x) = -3[(x^2 - 6x - 9) + 9]$
$f(x) = -3[(x - 3)^2 + 9]$
$f(x) = -3(x - 3)^2 + 27$

13. The managers of a business are examining costs. It is more cost-effective for them to produce more items. However, if too many items are produced, their costs will rise because of factors such as storage and overstock. Suppose that they model the cost, C, of producing n thousand items with the function $C(n) = 75n^2 - 1800n + 60\,000$. Determine the number of items produced that will minimize their costs.

14. A gymnast is jumping on a trampoline. His height, h, in metres, above the floor on each jump is roughly approximated by the function $h(t) = -5t^2 + 10t + 4$, where t represents the time, in seconds, since he left the trampoline. Determine algebraically his maximum height on each jump.

15. Sandra is practising at an archery club. The height, h, in feet, of the arrow on one of her shots can be modelled as a function of time, t, in seconds, since it was fired using the function $h(t) = -16t^2 + 10t + 4$.

a) What is the maximum height of the arrow, in feet, and when does it reach that height?

b) Verify your solution in two different ways.

Did You Know?

The use of the bow and arrow dates back before recorded history and appears to have connections with most cultures worldwide. Archaeologists can learn great deal about the history of the ancestors of today's First Nations and Inuit populations in Canada through the study of various forms of spearheads and arrowheads, also referred to as *projectile points*.

16. Austin and Yuri were asked to convert the function $y = -6x^2 + 72x - 20$ to vertex form. Their solutions are shown.

Austin's solution:
$y = -6x^2 + 72x - 20$
$y = -6(x^2 + 12x) - 20$
$y = -6(x^2 + 12x + 36 - 36) - 20$
$y = -6[(x^2 + 12x + 36) - 36] - 20$
$y = -6[(x + 6) - 36] - 20$
$y = -6(x + 6) + 216 - 20$
$y = -6(x + 6) + 196$

Yuri's solution:

$y = -6x^2 + 72x - 20$
$y = -6(x^2 - 12x) - 20$
$y = -6(x^2 - 12x + 36 - 36) - 20$
$y = -6[(x^2 - 12x + 36) - 36] - 20$
$y = -6[(x - 6)^2 - 36] - 20$
$y = -6(x - 6)^2 - 216 - 20$
$y = -6(x - 6)^2 + 236$

a) Identify, explain, and correct any errors in their solutions.

b) Neither Austin nor Yuri verified their answers. Show several methods that they could have used to verify their solutions. Identify how each method would have pointed out if their solutions were incorrect.

17. A parabolic microphone collects and focuses sound waves to detect sounds from a distance. This type of microphone is useful in situations such as nature audio recording and sports broadcasting. Suppose a particular parabolic microphone has a cross-sectional shape that can be described by the function $d(x) = 0.03125x^2 - 1.5x$, where d is the depth, in centimetres, of the microphone's dish at a horizontal distance of x centimetres from one edge of the dish. Use an algebraic method to determine the depth of the dish, in centimetres, at its centre.

18. A concert promoter is planning the ticket price for an upcoming concert for a certain band. At the last concert, she charged \$70 per ticket and sold 2000 tickets. After conducting a survey, the promoter has determined that for every \$1 decrease in ticket price, she might expect to sell 50 more tickets.

a) What maximum revenue can the promoter expect? What ticket price will give that revenue?

b) How many tickets can the promoter expect to sell at that price?

c) Explain any assumptions the concert promoter is making in using this quadratic function to predict revenues.

Digging Roots, a First Nations band, from Barriere, British Columbia

19. The manager of a bike store is setting the price for a new model. Based on past sales history, he predicts that if he sets the price at \$360, he can expect to sell 280 bikes this season. He also predicts that for every \$10 increase in the price, he expects to sell five fewer bikes.

a) Write a function to model this situation.

b) What maximum revenue can the manager expect? What price will give that maximum?

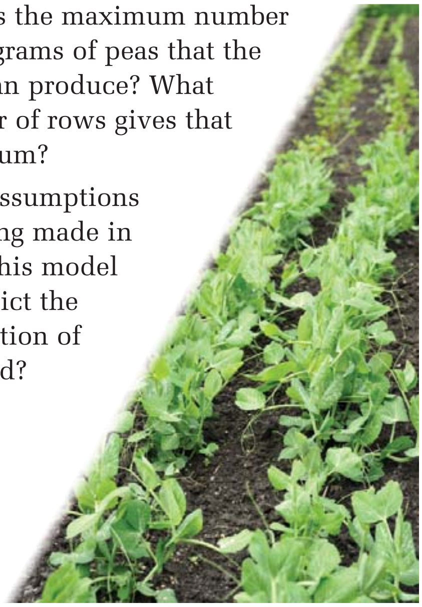
c) Explain any assumptions involved in using this model.

20. A gardener is planting peas in a field. He knows that if he spaces the rows of pea plants closer together, he will have more rows in the field, but fewer peas will be produced by the plants in each row. Last year he planted the field with 30 rows of plants. At this spacing. he got an average of 4000 g of peas per row. He estimates that for every additional row, he will get 100 g less per row.

a) Write a quadratic function to model this situation.

b) What is the maximum number of kilograms of peas that the field can produce? What number of rows gives that maximum?

c) What assumptions are being made in using this model to predict the production of the field?



21. A holding pen is being built alongside a long building. The pen requires only three fenced sides, with the building forming the fourth side. There is enough material for 90 m of fencing.

a) Predict what dimensions will give the maximum area of the pen.

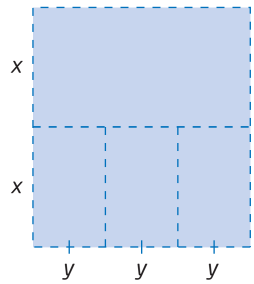
b) Write a function to model the area.

c) Determine the maximum possible area.

d) Verify your solution in several ways, with or without technology. How does the solution compare to your prediction?

e) Identify any assumptions you made in using the model function that you wrote.

22. A set of fenced-in areas, as shown in the diagram, is being planned on an open field. A total of 900 m of fencing is available. What measurements will maximize the overall area of the entire enclosure?

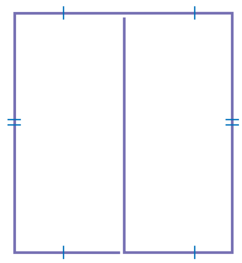


23. Use a quadratic function model to solve each problem.

a) Two numbers have a sum of 29 and a product that is a maximum. Determine the two numbers and the maximum product.

b) Two numbers have a difference of 13 and a product that is a minimum. Determine the two numbers and the minimum product.

24. What is the maximum total area that 450 cm of string can enclose if it is used to form the perimeters of two adjoining rectangles as shown?



Extend

25. Write $f(x) = -\frac{3}{4}x^2 + \frac{9}{8}x + \frac{5}{16}$ in vertex form.

26. a) Show the process of completing the square for the function $y = ax^2 + bx + c$.

b) Express the coordinates of the vertex in terms of a, b, and c.

c) How can you use this information to solve problems involving quadratic functions in standard form?

27. The vertex of a quadratic function in standard form is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

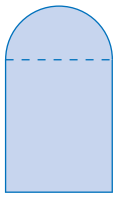
a) Given the function $f(x) = 2x^2 - 12x + 22$ in standard form, determine the vertex.

b) Determine the vertex by converting the function to vertex form.

c) Show the relationship between the parameters a, b, and c in standard form and the parameters a, p, and q in vertex form.

28. A Norman window has the shape of a rectangle with a semicircle on the top. Consider a Norman window with a perimeter of 6 m.

a) Write a function to approximate the area of the window as a function of its width.



b) Complete the square to approximate the maximum possible area of the window and the width that gives that area.

c) Verify your answer to part b) using technology.

d) Determine the other dimensions and draw a scale diagram of the window. Does its appearance match your expectations?

Create Connections

29. a) Is the quadratic function $f(x) = 4x^2 + 24$ written in vertex or in standard form? Discuss with a partner.

b) Could you complete the square for this function? Explain.

30. Martine's teacher asks her to complete the square for the function $y = -4x^2 + 24x + 5$. After looking at her solution, the teacher says that she made four errors in her work. Identify, explain, and correct her errors.

Martine's solution:

$y = -4x^2 + 24x + 5$
$y = -4(x^2 + 6x) + 5$
$y = -4(x^2 + 6x + 36 - 36) + 5$
$y = -4[(x^2 + 6x + 36) - 36] + 5$
$y = -4[(x + 6)^2 - 36] + 5$
$y = -4(x + 6)^2 - 216 + 5$
$y = -4(x + 6)^2 - 211$

31. A local store sells T-shirts for \$10. At this price, the store sells an average of 100 shirts each month. Market research says that for every \$1 increase in the price, the manager of the store can expect to sell five fewer shirts each month.

a) Write a quadratic function to model the revenue in terms of the increase in price.

b) What information can you determine about this situation by completing the square?

c) What assumptions have you made in using this quadratic function to predict revenue?

Project Corner — Quadratic Functions in Motion

- Quadratic functions appear in the shapes of various types of stationary objects, along with situations involving moving ones. You can use a video clip to show the motion of a person, animal, or object that appears to create a quadratic model function using a suitably placed set of coordinate axes.
- What situations involving motion could you model using quadratic functions?

Chapter 3 Review

3.1 Investigating Quadratic Functions in Vertex Form, pages 142–162

1. Use transformations to explain how the graph of each quadratic function compares to the graph of $f(x) = x^2$. Identify the vertex, the axis of symmetry, the direction of opening, the maximum or minimum value, and the domain and range without graphing.

a) $f(x) = (x + 6)^2 - 14$

b) $f(x) = -2x^2 + 19$

c) $f(x) = \frac{1}{5}(x - 10)^2 + 100$

d) $f(x) = -6(x - 4)^2$

2. Sketch the graph of each quadratic function using transformations. Identify the vertex, the axis of symmetry, the maximum or minimum value, the domain and range, and any intercepts.

a) $f(x) = 2(x + 1)^2 - 8$

b) $f(x) = -0.5(x - 2)^2 + 2$

3. Is it possible to determine the number of x-intercepts in each case without graphing? Explain why or why not.

a) $y = -3(x - 5)^2 + 20$

b) a parabola with a domain of all real numbers and a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$

c) $y = 9 + 3x^2$

d) a parabola with a vertex at $(-4, -6)$

4. Determine a quadratic function with each set of characteristics.

a) vertex at (0, 0), passing through the point (20, −150)

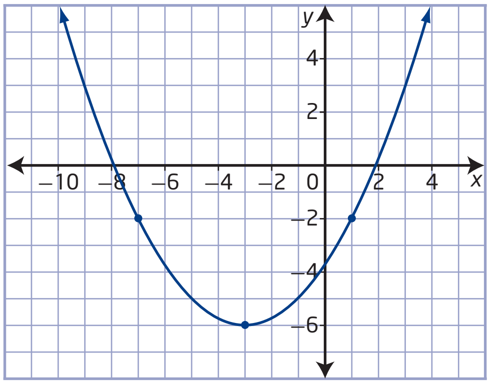
b) vertex at (8, 0), passing through the point (2, 54)

c) minimum value of 12 at $x = -4$ and y-intercept of 60

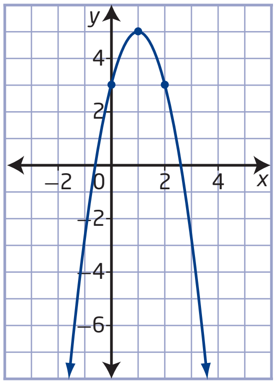
d) x-intercepts of 2 and 7 and maximum value of 25

5. Write a quadratic function in vertex form for each graph.

a)



b)



6. A parabolic trough is a solar-energy collector. It consists of a long mirror with a cross-section in the shape of a parabola. It works by focusing the Sun's rays onto a central axis running down the length of the trough. Suppose a particular solar trough has width 180 cm and depth 56 cm. Determine the quadratic function that represents the cross-sectional shape of the mirror.

7. The main span of the suspension bridge over the Peace River in Dunvegan, Alberta, has supporting cables in the shape of a parabola. The distance between the towers is 274 m. Suppose that the ends of the cables are attached to the tops of the two supporting towers at a height of 52 m above the surface of the water, and the lowest point of the cables is 30 m above the water's surface.

   a) Determine a quadratic function that represents the shape of the cables if the origin is at

   i) the minimum point on the cables

   ii) a point on the water's surface directly below the minimum point of the cables

   iii) the base of the tower on the left

   b) Would the quadratic function change over the course of the year as the seasons change? Explain.

8. A flea jumps from the ground to a height of 30 cm and travels 15 cm horizontally from where it started. Suppose the origin is located at the point from which the flea jumped. Determine a quadratic function in vertex form to model the height of the flea compared to the horizontal distance travelled.

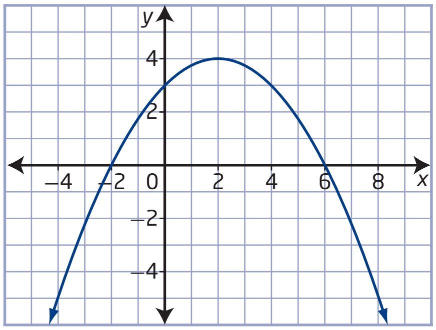
Did You Know?

The average flea can pull 160 000 times its own mass and can jump 200 times its own length. This is equivalent to a human being pulling 24 million pounds and jumping nearly 1000 ft!

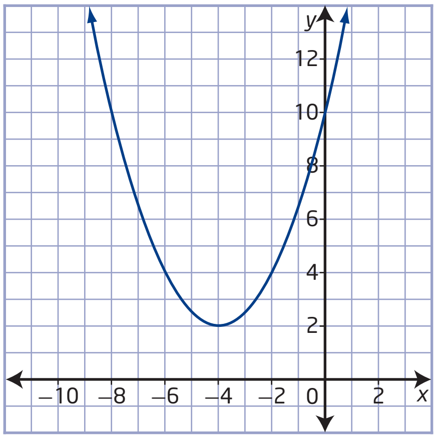
3.2 Investigating Quadratic Functions in Standard Form, pages 163–179

9. For each graph, identify the vertex, axis of symmetry, maximum or minimum value, direction of opening, domain and range, and any intercepts.

   a)



   b)



10. Show why each function fits the definition of a quadratic function.

   a) $y = 7(x + 3)^2 - 41$

   b) $y = (2x + 7)(10 - 3x)$

11. **a)** Sketch the graph of the function $f(x) = -2x^2 + 3x + 5$. Identify the vertex, the axis of symmetry, the direction of opening, the maximum or minimum value, the domain and range, and any intercepts.

   b) Explain how each feature can be identified from the graph.

12. A goaltender kicks a soccer ball through the air to players downfield. The trajectory of the ball can be modelled by the function $h(d) = -0.032d^2 + 1.6d$, where d is the horizontal distance, in metres, from the person kicking the ball and h is the height at that distance, in metres.

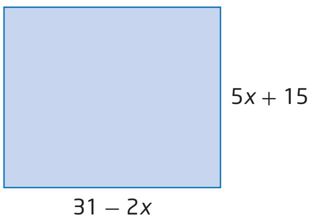
a) Represent the function with a graph, showing all important characteristics.

b) What is the maximum height of the ball? How far downfield is the ball when it reaches that height?

c) How far downfield does the ball hit the ground?

d) What are the domain and range in this situation?

13. a) Write a function to represent the area of the rectangle.



b) Graph the function.

c) What do the x-intercepts represent in this situation?

d) Does the function have a maximum value in this situation? Does it have a minimum value?

e) What information does the vertex give about this situation?

f) What are the domain and range?

3.3 Completing the Square, pages 180–197

14. Write each function in vertex form, and verify your answer.

a) $y = x^2 - 24x + 10$

b) $y = 5x^2 + 40x - 27$

c) $y = -2x^2 + 8x$

d) $y = -30x^2 - 60x + 105$

15. Without graphing, state the vertex, the axis of symmetry, the maximum or minimum value, and the domain and range of the function $f(x) = 4x^2 - 10x + 3$.

16. Amy tried to convert the function $y = -22x^2 - 77x + 132$ to vertex form.

Amy's solution:
$y = -22x^2 - 77x + 132$
$y = -22(x^2 - 3.5x) + 132$
$y = -22(x^2 - 3.5x - 12.25 + 12.25) + 132$
$y = -22(x^2 - 3.5x - 12.25) - 269.5 + 132$
$y = -22(x - 3.5)^2 - 137.5$

a) Identify, explain, and correct the errors.

b) Verify your correct solution in several different ways, both with and without technology.

17. The manager of a clothing company is analysing its costs, revenues, and profits to plan for the upcoming year. Last year, a certain type of children's winter coat was priced at \$40, and the company sold 10 000 of them. Market research says that for every \$2 decrease in the price, the manager can expect the company to sell 500 more coats.

a) Model the expected revenue as a function of the number of price decreases.

b) Without graphing, determine the maximum revenue and the price that will achieve that revenue.

c) Graph the function to confirm your answer.

d) What does the y-intercept represent in this situation? What do the x-intercepts represent?

e) What are the domain and range in this situation?

f) Explain some of the assumptions that the manager is making in using this function to model the expected revenue.

Chapter 3 Practice Test

Multiple Choice

For #1 to #6, choose the best answer.

1. Which function is NOT a quadratic function?

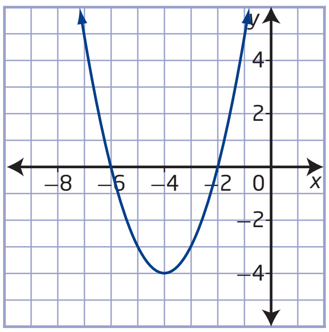
A $f(x) = 2(x + 1)^2 - 7$

B $f(x) = (x - 3)(2x + 5)$

C $f(x) = 5x^2 - 20$

D $f(x) = 3(x - 9) + 6$

2. Which quadratic function represents the parabola shown?



A $y = (x + 4)^2 + 4$

B $y = (x - 4)^2 + 4$

C $y = (x + 4)^2 - 4$

D $y = (x - 4)^2 - 4$

3. Identify the range for the function $y = -6(x - 6)^2 + 6$.

A $\{y \mid y \le 6, y \in \mathrm{R}\}$

B $\{y \mid y \ge 6, y \in \mathrm{R}\}$

C $\{y \mid y \le -6, y \in \mathrm{R}\}$

D $\{y \mid y \ge -6, y \in \mathrm{R}\}$

4. Which quadratic function in vertex form is equivalent to $y = x^2 - 2x - 5$?

A $y = (x - 2)^2 - 1$

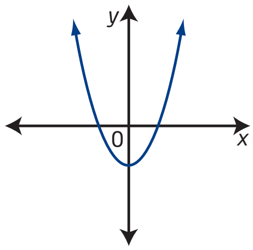
B $y = (x - 2)^2 - 9$

C $y = (x - 1)^2 - 4$

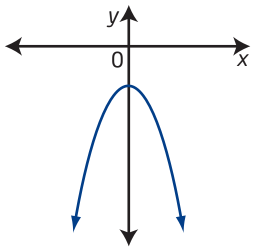
D $y = (x - 1)^2 - 6$

5. Which graph shows the function $y = 1 + ax^2$ if $a < 0$?

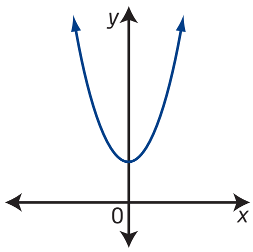
A



B



C



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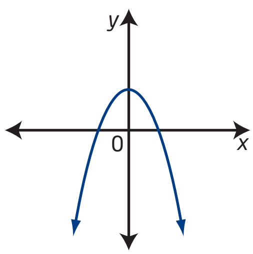


6. What conditions on a and q will give the function $f(x) = a(x - p)^2 + q$ no x-intercepts?

A $a > 0$ and $q > 0$

B $a < 0$ and $q > 0$

C $a > 0$ and $q = 0$

D $a < 0$ and $q = 0$

Short Answer

7. Write each quadratic function in vertex form by completing the square.

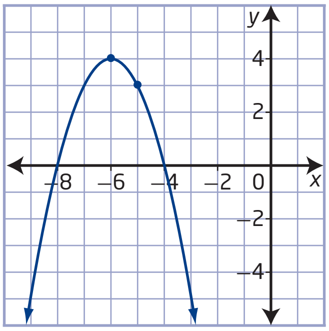
a) $y = x^2 - 18x - 27$

b) $y = 3x^2 + 36x + 13$

c) $y = -10x^2 - 40x$

8. a) For the graph shown, give the coordinates of the vertex, the equation of the axis of symmetry, the minimum or maximum value, the domain and range, and the x-intercepts.

b) Determine a quadratic function in vertex form for the graph.



9. a) Identify the transformation(s) on the graph of $f(x) = x^2$ that could be used to graph each function.

i) $f(x) = 5x^2$

ii) $f(x) = x^2 - 20$

iii) $f(x) = (x + 11)^2$

iv) $f(x) = -\frac{1}{7}x^2$

b) For each function in part a), state which of the following would be different as compared to $f(x) = x^2$ as a result of the transformation(s) involved, and explain why.

i) vertex

ii) axis of symmetry

iii) range

10. Sketch the graph of the function $y = 2(x - 1)^2 - 8$ using transformations. Then, copy and complete the table.

Vertex	
Axis of Symmetry	
Direction of Opening	
Domain	
Range	
x-Intercepts	
y-Intercept	

11. The first three steps in completing the square below contain one or more errors.

$y = 2x^2 - 8x + 9$
$y = 2(x^2 - 8x) + 9$
$y = 2(x^2 - 8x - 64 + 64) + 9$

a) Identify and correct the errors.

b) Complete the process to determine the vertex form of the function.

c) Verify your correct solution in several different ways.

12. The fuel consumption for a vehicle is related to the speed that it is driven and is usually given in litres per one hundred kilometres. Engines are generally more efficient at higher speeds than at lower speeds. For a particular type of car driving at a constant speed, the fuel consumption, C, in litres per one hundred kilometres, is related to the average driving speed, v, in kilometres per hour, by the function $C(v) = 0.004v^2 - 0.62v + 30$.

a) Without graphing, determine the most efficient speed at which this car should be driven. Explain/show the strategy you use.

b) Describe any characteristics of the graph that you can identify without actually graphing, and explain how you know.

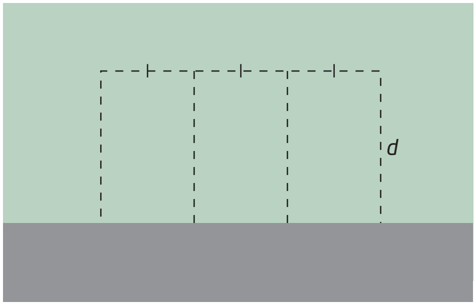
13. The height, h, in metres, of a flare t seconds after it is fired into the air can be modelled by the function $h(t) = -4.9t^2 + 61.25t$.

a) At what height is the flare at its maximum? How many seconds after being shot does this occur?

b) Verify your solution both with and without technology.

Extended Response

14. Three rectangular areas are being enclosed along the side of a building, as shown. There is enough material to make 24 m of fencing.



a) Write the function that represents the total area in terms of the distance from the wall.

b) Show that the function fits the definition of a quadratic function.

c) Graph the function. Explain the strategy you used.

d) What are the coordinates of the vertex? What do they represent?

e) What domain and range does the function have in this situation? Explain.

f) Does the function have a maximum value? Does it have a minimum value? Explain.

g) What assumptions are made in using this quadratic function model?

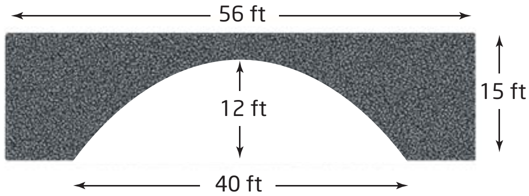
15. A stone bridge has the shape of a parabolic arch, as shown. Determine a quadratic function to represent the shape of the arch if the origin

a) is at the top of the opening under the bridge

b) is on the ground at the midpoint of the opening

c) is at the base of the bridge on the right side of the opening

d) is on the left side at the top surface of the bridge



16. A store sells energy bars for \$2.25. At this price, the store sold an average of 120 bars per month last year. The manager has been told that for every 5¢ decrease in price, he can expect the store to sell eight more bars monthly.

a) What quadratic function can you use to model this situation?

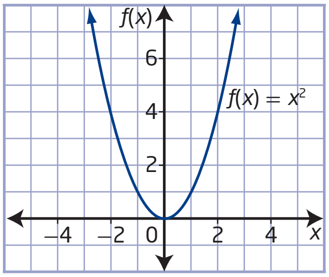
b) Use an algebraic method to determine the maximum revenue the manager can expect the store to achieve. What price will give that maximum?

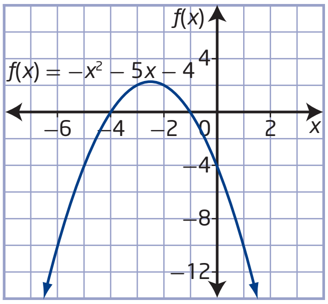
c) What assumptions are made in this situation?

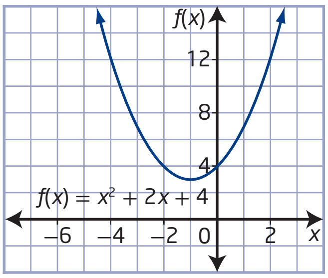
Check Your Understanding

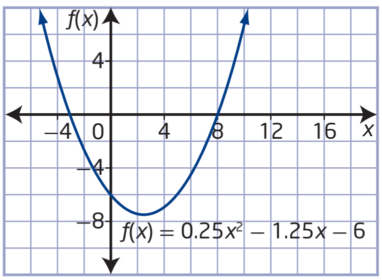
Practise

1. How many x-intercepts does each quadratic function graph have?

a) 

b) 

c) 

d) 

2. What are the roots of the corresponding quadratic equations represented by the graphs of the functions shown in #1? Verify your answers.

3. Solve each equation by graphing the corresponding function.

a) $0 = x^2 - 5x - 24$

b) $0 = -2r^2 - 6r$

c) $h^2 + 2h + 5 = 0$

d) $5x^2 - 5x = 30$

e) $-z^2 + 4z = 4$

f) $0 = t^2 + 4t + 10$

4. What are the roots of each quadratic equation? Where integral roots cannot be found, estimate the roots to the nearest tenth.

a) $n^2 - 10 = 0$

b) $0 = 3x^2 + 9x - 12$

c) $0 = -w^2 + 4w - 3$

d) $0 = 2d^2 + 20d + 32$

e) $0 = v^2 + 6v + 6$

f) $m^2 - 10m = -21$

Apply

5. In a Canadian Football League game, the path of the football at one particular kick-off can be modelled using the function $h(d) = -0.02d^2 + 2.6d - 66.5$, where h is the height of the ball and d is the horizontal distance from the kicking team's goal line, both in yards. A value of $h(d) = 0$ represents the height of the ball at ground level. What horizontal distance does the ball travel before it hits the ground?

6. Two numbers have a sum of 9 and a product of 20.

a) What single-variable quadratic equation in the form $ax^2 + bx + c = 0$ can be used to represent the product of the two numbers?

b) Determine the two numbers by graphing the corresponding quadratic function.

7. Two consecutive even integers have a product of 168.

a) What single-variable quadratic equation in the form $ax^2 + bx + c = 0$ can be used to represent the product of the two numbers?

b) Determine the two numbers by graphing the corresponding quadratic function.

8. The path of the stream of water coming out of a fire hose can be approximated using the function $h(x) = -0.09x^2 + x + 1.2$, where h is the height of the water stream and x is the horizontal distance from the firefighter holding the nozzle, both in metres.

a) What does the equation $-0.09x^2 + x + 1.2 = 0$ represent in this situation?

b) At what maximum distance from the building could a firefighter stand and still reach the base of the fire with the water? Express your answer to the nearest tenth of a metre.

c) What assumptions did you make when solving this problem?

9. The HSBC Celebration of Light is an annual pyro-musical fireworks competition that takes place over English Bay in Vancouver. The fireworks are set off from a barge so they land on the water. The path of a particular fireworks rocket is modelled by the function $h(t) = -4.9(t - 3)^2 + 47$, where h is the rocket's height above the water, in metres, at time, t, in seconds.

a) What does the equation $0 = -4.9(t - 3)^2 + 47$ represent in this situation?

b) The fireworks rocket stays lit until it hits the water. For how long is it lit, to the nearest tenth of a second?

10. A skateboarder jumps off a ledge at a skateboard park. His path is modelled by the function $h(d) = -0.75d^2 + 0.9d + 1.5$, where h is the height above ground and d is the horizontal distance the skateboarder travels from the ledge, both in metres.

a) Write a quadratic equation to represent the situation when the skateboarder lands.

b) At what distance from the base of the ledge will the skateboarder land? Express your answer to the nearest tenth of a metre.

11. Émilie Heymans is a three-time Canadian Olympic diving medallist. Suppose that for a dive off the 10-m tower, her height, h, in metres, above the surface of the water is given by the function $h(d) = -2d^2 + 3d + 10$, where d is the horizontal distance from the end of the tower platform, in metres.

a) Write a quadratic equation to represent the situation when Émilie enters the water.

b) What is Émilie's horizontal distance from the end of the tower platform when she enters the water? Express your answer to the nearest tenth of a metre.

Did You Know?

Émilie Heymans, from Montréal, Québec, is only the fifth Canadian to win medals at three consecutive Olympic Games.

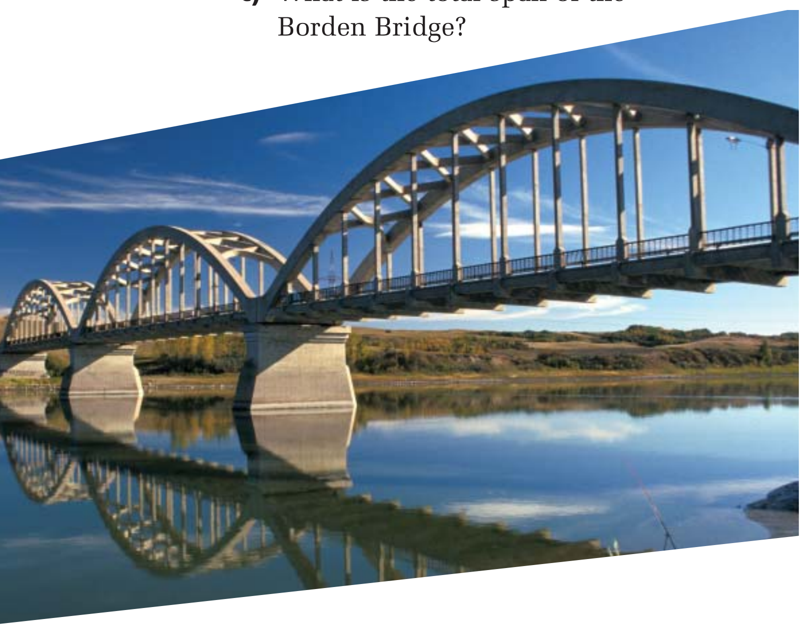
12. Matthew is investigating the old Borden Bridge, which spans the North Saskatchewan River about 50 km west of Saskatoon. The three parabolic arches of the bridge can be modelled using quadratic functions, where h is the height of the arch above the bridge deck and x is the horizontal distance of the bridge deck from the beginning of the first arch, both in metres.

First arch:
$h(x) = -0.01x^2 + 0.84x$
Second arch:
$h(x) = -0.01x^2 + 2.52x - 141.12$
Third arch:
$h(x) = -0.01x^2 + 4.2x - 423.36$

a) What are the zeros of each quadratic function?

b) What is the significance of the zeros in this situation?

c) What is the total span of the Borden Bridge?



Extend

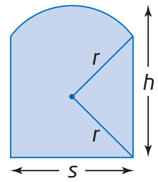
13. For what values of k does the equation $x^2 + 6x + k = 0$ have

a) one real root?

b) two distinct real roots?

c) no real roots?

14. The height of a circular arch is represented by $4h^2 - 8hr + s^2 = 0$, where h is the height, r is the radius, and s is the span of the arch, all in feet.



a) How high must an arch be to have a span of 64 ft and a radius of 40 ft?

b) How would this equation change if all the measurements were in metres? Explain.

15. Two new hybrid vehicles accelerate at different rates. The Ultra Range's acceleration can be modelled by the function $d(t) = 1.5t^2$, while the Edison's can be modelled by the function $d(t) = 5.4t^2$, where d is the distance, in metres, and t is the time, in seconds. The Ultra Range starts the race at 0 s. At what time should the Edison start so that both cars are at the same point 5 s after the race starts? Express your answer to the nearest tenth of a second.

Did You Know?

A hybrid vehicle uses two or more distinct power sources. The most common hybrid uses a combination of an internal combustion engine and an electric motor. These are called hybrid electric vehicles or HEVs.

Create Connections

16. Suppose the value of a quadratic function is negative when $x = 1$ and positive when $x = 2$. Explain why it is reasonable to assume that the related equation has a root between 1 and 2.

17. The equation of the axis of symmetry of a quadratic function is $x = 0$ and one of the x-intercepts is -4. What is the other x-intercept? Explain using a diagram.

18. The roots of the quadratic equation $0 = x^2 - 4x - 12$ are 6 and -2. How can you use the roots to determine the vertex of the graph of the corresponding function?

Key Ideas

- You can solve some quadratic equations by factoring.
- If two factors of a quadratic equation have a product of zero, then by the zero product property one of the factors must be equal to zero.
- To solve a quadratic equation by factoring, first write the equation in the form $ax^2 + bx + c = 0$, and then factor the left side. Next, set each factor equal to zero, and solve for the unknown.

  For example,

  $$\begin{aligned} x^2 + 8x &= -12 \\ x^2 + 8x + 12 &= 0 \\ (x + 2)(x + 6) &= 0 \end{aligned}$$

  $$\begin{aligned} x + 2 &= 0 \quad \text{or} \quad & x + 6 &= 0 \\ x &= -2 & x &= -6 \end{aligned}$$

- The solutions to a quadratic equation are called the roots of the equation.
- You can factor polynomials in quadratic form.
  - Factor trinomials of the form $a(P)^2 + b(P) + c$, where $a \neq 0$ and P is any expression, by replacing the expression for P with a single variable. Then substitute the expression for P back into the factored expression. Simplify the final factors, if possible.

    For example, factor $2(x + 3)^2 - 11(x + 3) + 15$ by letting $r = x + 3$.

    $$\begin{aligned} 2(x + 3)^2 - 11(x + 3) + 15 &= 2r^2 - 11r + 15 \\ &= 2r^2 - 5r - 6r + 15 \\ &= (2r^2 - 5r) + (-6r + 15) \\ &= r(2r - 5) - 3(2r - 5) \\ &= (2r - 5)(r - 3) \\ &= [2(x + 3) - 5][(x + 3) - 3] \\ &= (2x + 1)(x) \\ &= x(2x + 1) \end{aligned}$$

  - Factor a difference of squares, $P^2 - Q^2$, where P and Q are any expressions, as $[P - Q][P + Q]$.

Check Your Understanding

Practise

1. Factor completely.

a) $x^2 + 7x + 10$

b) $5z^2 + 40z + 60$

c) $0.2d^2 - 2.2d + 5.6$

2. Factor completely.

a) $3y^2 + 4y - 7$

b) $8k^2 - 6k - 5$

c) $0.4m^2 + 0.6m - 1.8$

3. Factor completely.

a) $x^2 + x - 20$

b) $x^2 - 12x + 36$

c) $\frac{1}{4}x^2 + 2x + 3$

d) $2x^2 + 12x + 18$

4. Factor each expression.

a) $4y^2 - 9x^2$

b) $0.36p^2 - 0.49q^2$

c) $\frac{1}{4}s^2 - \frac{9}{25}t^2$

d) $0.16t^2 - 16s^2$

5. Factor each expression.

a) $(x + 2)^2 - (x + 2) - 42$

b) $6(x^2 - 4x + 4)^2 + (x^2 - 4x + 4) - 1$

c) $(4j - 2)^2 - (2 + 4j)^2$

6. What are the factors of each expression?

a) $4(5b - 3)^2 + 10(5b - 3) - 6$

b) $16(x^2 + 1)^2 - 4(2x)^2$

c) $-\frac{1}{4}(2x)^2 + 25(2y^3)^2$

7. Solve each factored equation.

a) $(x + 3)(x + 4) = 0$

b) $(x - 2)\left(x + \frac{1}{2}\right) = 0$

c) $(x + 7)(x - 8) = 0$

d) $x(x + 5) = 0$

e) $(3x + 1)(5x - 4) = 0$

f) $2(x - 4)(7 - 2x) = 0$

8. Solve each quadratic equation by factoring. Check your answers.

a) $10n^2 - 40 = 0$

b) $\frac{1}{4}x^2 + \frac{5}{4}x + 1 = 0$

c) $3w^2 + 28w + 9 = 0$

d) $8y^2 - 22y + 15 = 0$

e) $d^2 + \frac{5}{2}d + \frac{3}{2} = 0$

f) $4x^2 - 12x + 9 = 0$

9. Determine the roots of each quadratic equation. Verify your answers.

a) $k^2 - 5k = 0$

b) $9x^2 = x + 8$

c) $\frac{8}{3}t + 5 = -\frac{1}{3}t^2$

d) $\frac{25}{49}y^2 - 9 = 0$

e) $2s^2 - 4s = 70$

f) $4q^2 - 28q = -49$

10. Solve each equation.

a) $42 = x^2 - x$

b) $g^2 = 30 - 7g$

c) $y^2 + 4y = 21$

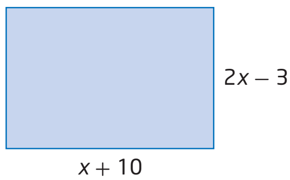
d) $3 = 6p^2 - 7p$

e) $3x^2 + 9x = 30$

f) $2z^2 = 3 - 5z$

Apply

11. A rectangle has dimensions $x + 10$ and $2x - 3$, where x is in centimetres. The area of the rectangle is 54 cm^2.



a) What equation could you use to determine the value of x?

b) What is the value of x?

12. An osprey, a fish-eating bird of prey, dives toward the water to catch a salmon. The height, h, in metres, of the osprey above the water t seconds after it begins its dive can be approximated by the function $h(t) = 5t^2 - 30t + 45$.

a) Determine the time it takes for the osprey to reach a height of 20 m.

b) What assumptions did you make? Are your assumptions reasonable? Explain.

13. A flare is launched from a boat. The height, h, in metres, of the flare above the water is approximately modelled by the function $h(t) = 150t - 5t^2$, where t is the number of seconds after the flare is launched.

a) What equation could you use to determine the time it takes for the flare to return to the water?

b) How many seconds will it take for the flare to return to the water?

14. The product of two consecutive even integers is 16 more than 8 times the smaller integer. Determine the integers.

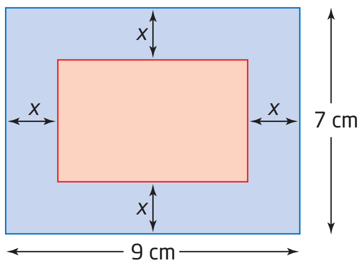
15. The area of a square is tripled by adding 10 cm to one dimension and 12 cm to the other. Determine the side length of the square.

16. Ted popped a baseball straight up with an initial upward velocity of 48 ft/s. The height, h, in feet, of the ball above the ground is modelled by the function $h(t) = 3 + 48t - 16t^2$. How long was the ball in the air if the catcher catches the ball 3 ft above the ground? Is your answer reasonable in this situation? Explain.

Did You Know?

Many Canadians have made a positive impact on Major League Baseball. Players such as Larry Walker of Maple Ridge, British Columbia, Jason Bay of Trail, British Columbia, and Justin Morneau of Westminster, British Columbia have had very successful careers in baseball's highest league.

17. A rectangle with area of 35 cm^2 is formed by cutting off strips of equal width from a rectangular piece of paper.



a) What is the width of each strip?

b) What are the dimensions of the new rectangle?

18. Without factoring, state if the binomial is a factor of the trinomial. Explain why or why not.

a) $x^2 - 5x - 36$, $x - 5$

b) $x^2 - 2x - 15$, $x + 3$

c) $6x^2 + 11x + 4$, $4x + 1$

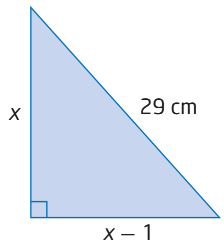
d) $4x^2 + 4x - 3$, $2x - 1$

19. Solve each equation.

a) $x(2x - 3) - 2(3 + 2x) = -4(x + 1)$

b) $3(x - 2)(x + 1) - 4 = 2(x - 1)^2$

20. The hypotenuse of a right triangle measures 29 cm. One leg is 1 cm shorter than the other. What are the lengths of the legs?



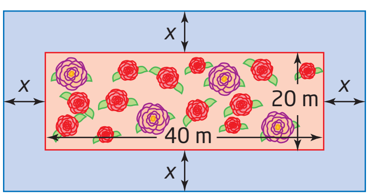
21. A field is in the shape of a right triangle. The fence around the perimeter of the field measures 40 m. If the length of the hypotenuse is 17 m, find the length of the other two sides.

22. The width of the top of a notebook computer is 7 cm less than the length. The surface area of the top of the notebook is 690 cm^2.

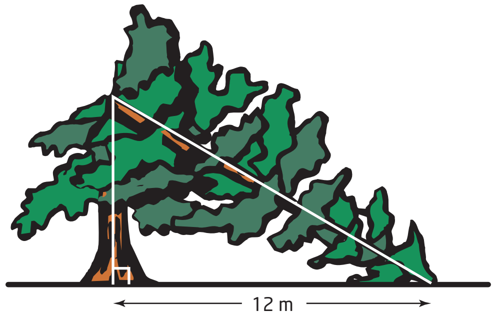
a) Write an equation to represent the surface area of the top of the notebook computer.

b) What are the dimensions of the top of the computer?

23. Stephan plans to build a uniform walkway around a rectangular flower bed that is 20 m by 40 m. There is enough material to make a walkway that has a total area of 700 m^2. What is the width of the walkway?



24. An 18-m-tall tree is broken during a severe storm, as shown. The distance from the base of the trunk to the point where the tip touches the ground is 12 m. At what height did the tree break?



25. The pressure difference, P, in newtons per square metre, above and below an airplane wing is described by the formula $P = \left(\frac{1}{2}d\right)(v_1)^2 - \left(\frac{1}{2}d\right)(v_2)^2$, where d is the density of the air, in kilograms per cubic metre; v_1 is the velocity, in metres per second, of the air passing above; and v_2 is the velocity, in metres per second, of the air passing below. Write this formula in factored form.

26. Carlos was asked to factor the trinomial $6x^2 - 16x + 8$ completely. His work is shown below.

Carlos's solution:

$$\begin{aligned} &6x^2 - 16x + 8 \\ &= 6x^2 - 12x - 4x + 8 \\ &= 6x(x - 2) - 4(x - 2) \\ &= (x - 2)(6x - 4) \end{aligned}$$

Is Carlos correct? Explain.

27. Factor each expression.

a) $3(2z + 3)^2 - 9(2z + 3) - 30$

b) $16(m^2 - 4)^2 - 4(3n)^2$

c) $\frac{1}{9}y^2 - \frac{1}{3}yx + \frac{1}{4}x^2$

d) $-28\left(w + \frac{2}{3}\right)^2 + 7\left(3w - \frac{1}{3}\right)^2$

Extend

28. A square has an area of $(9x^2 + 30xy + 25y^2)$ square centimetres. What is an expression for the perimeter of the square?

29. Angela opened a surf shop in Tofino, British Columbia. Her accountant models her profit, P, in dollars, with the function $P(t) = 1125(t - 1)^2 - 10\,125$, where t is the number of years of operation. Use graphing or factoring to determine how long it will take for the shop to start making a profit.

Pete Devries

Did You Know?

Pete Devries was the first Canadian to win an international surfing competition. In 2009, he outperformed over 110 world-class surfers to win the O'Neill Cold Water Classic Canada held in Tofino, British Columbia.

Create Connections

30. Write a quadratic equation in standard form with the given root(s).

a) -3 and 3

b) 2

c) $\frac{2}{3}$ and 4

d) $\frac{3}{5}$ and $-\frac{1}{2}$

31. Create an example of a quadratic equation that cannot be solved by factoring. Explain why it cannot be factored. Show the graph of the corresponding quadratic function and show where the roots are located.

32. You can use the difference of squares pattern to perform certain mental math shortcuts. For example,

$$\begin{aligned} 81 - 36 &= (9 - 6)(9 + 6) \\ &= (3)(15) \\ &= 45 \end{aligned}$$

a) Explain how this strategy works. When can you use it?

b) Create two examples to illustrate the strategy.

Project Corner — Avalanche Safety

- Experts use avalanche control all over the world above highways, ski resorts, railroads, mining operations, and utility companies, and anywhere else that may be threatened by avalanches.
- Avalanche control is the intentional triggering of avalanches. People are cleared away to a safe distance, then experts produce more frequent, but smaller, avalanches at controlled times.
- Because avalanches tend to occur in the same zones and under certain conditions, avalanche experts can predict when avalanches are likely to occur.
- Charges are delivered by launchers, thrown out of helicopters, or delivered above the avalanche starting zones by an avalanche control expert on skis.
- What precautions would avalanche control experts need to take to ensure public safety?

Key Ideas

- Completing the square is the process of rewriting a quadratic polynomial from the standard form, $ax^2 + bx + c$, to the vertex form, $a(x - p)^2 + q$.
- You can use completing the square to determine the roots of a quadratic equation in standard form.

For example,

$$
\begin{aligned}
2x^2 - 4x - 2 &= 0 \\
x^2 - 2x - 1 &= 0 && \text{Divide both sides by a common factor of 2.} \\
x^2 - 2x &= 1 && \text{Isolate the variable terms on the left side.} \\
x^2 - 2x + 1 &= 1 + 1 && \text{Complete the square on the left side.} \\
(x - 1)^2 &= 2 \\
x - 1 &= \pm\sqrt{2} && \text{Take the square root of both sides.}
\end{aligned}
$$

$$
\begin{aligned}
x - 1 &= \sqrt{2} \quad \text{or} \quad & x - 1 &= -\sqrt{2} && \text{Solve for } x. \\
x &= 1 + \sqrt{2} & x &= 1 - \sqrt{2} \\
x &\approx 2.41 & x &\approx -0.41
\end{aligned}
$$

- Express roots of quadratic equations as exact roots or as decimal approximations.

Check Your Understanding

Practise

1. What value of c makes each expression a perfect square?

a) $x^2 + x + c$

b) $x^2 - 5x + c$

c) $x^2 - 0.5x + c$

d) $x^2 + 0.2x + c$

e) $x^2 + 15x + c$

f) $x^2 - 9x + c$

2. Complete the square to write each quadratic equation in the form $(x + p)^2 = q$.

a) $2x^2 + 8x + 4 = 0$

b) $-3x^2 - 12x + 5 = 0$

c) $\frac{1}{2}x^2 - 3x + 5 = 0$

3. Write each equation in the form $a(x - p)^2 + q = 0$.

a) $x^2 - 12x + 9 = 0$

b) $5x^2 - 20x - 1 = 0$

c) $-2x^2 + x - 1 = 0$

d) $0.5x^2 + 2.1x + 3.6 = 0$

e) $-1.2x^2 - 5.1x - 7.4 = 0$

f) $\frac{1}{2}x^2 + 3x - 6 = 0$

4. Solve each quadratic equation. Express your answers as exact roots.

a) $x^2 = 64$

b) $2s^2 - 8 = 0$

c) $\frac{1}{3}t^2 - 1 = 11$

d) $-y^2 + 5 = -6$

5. Solve. Express your answers as exact roots.

a) $(x - 3)^2 = 4$

b) $(x + 2)^2 = 9$

c) $\left(d + \frac{1}{2}\right)^2 = 1$

d) $\left(h - \frac{3}{4}\right)^2 = \frac{7}{16}$

e) $(s + 6)^2 = \frac{3}{4}$

f) $(x + 4)^2 = 18$

6. Solve each quadratic equation by completing the square. Express your answers as exact roots.

a) $x^2 + 10x + 4 = 0$

b) $x^2 - 8x + 13 = 0$

c) $3x^2 + 6x + 1 = 0$

d) $-2x^2 + 4x + 3 = 0$

e) $-0.1x^2 - 0.6x + 0.4 = 0$

f) $0.5x^2 - 4x - 6 = 0$

7. Solve each quadratic equation by completing the square. Express your answers to the nearest tenth.

a) $x^2 - 8x - 4 = 0$

b) $-3x^2 + 4x + 5 = 0$

c) $\frac{1}{2}x^2 - 6x - 5 = 0$

d) $0.2x^2 + 0.12x - 11 = 0$

e) $-\frac{2}{3}x^2 - x + 2 = 0$

f) $\frac{3}{4}x^2 + 6x + 1 = 0$

Apply

8. Dinahi's rectangular dog kennel measures 4 ft by 10 ft. She plans to double the area of the kennel by extending each side by an equal amount.

a) Sketch and label a diagram to represent this situation.

b) Write the equation to model the new area.

c) What are the dimensions of the new dog kennel, to the nearest tenth of a foot?

9. Evan passes a flying disc to a teammate during a competition at the Flatland Ultimate and Cups Tournament in Winnipeg. The flying disc follows the path $h(d) = -0.02d^2 + 0.4d + 1$, where h is the height, in metres, and d is the horizontal distance, in metres, that the flying disc has travelled from the thrower. If no one catches the flying disc, the height of the disc above the ground when it lands can be modelled by $h(d) = 0$.

a) What quadratic equation can you use to determine how far the disc will travel if no one catches it?

b) How far will the disc travel if no one catches it? Express your answer to the nearest tenth of a metre.

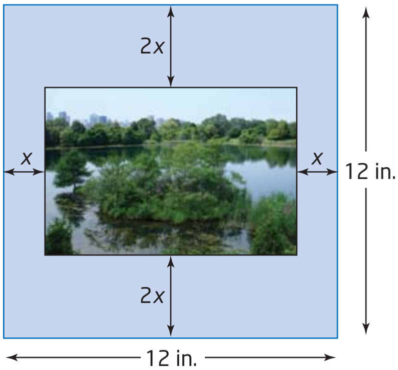
Did You Know?

Each August, teams compete in the Canadian Ultimate Championships for the national title in five different divisions: juniors, masters, mixed, open, and women's. This tournament also determines who will represent Canada at the next world championships.

10. A model rocket is launched from a platform. Its trajectory can be approximated by the function $h(d) = -0.01d^2 + 2d + 1$, where h is the height, in metres, of the rocket and d is the horizontal distance, in metres, the rocket travels. How far does the rocket land from its launch position? Express your answer to the nearest tenth of a metre.

11. Brian is placing a photograph behind a 12-in. by 12-in. piece of matting. He positions the photograph so the matting is twice as wide at the top and bottom as it is at the sides.

The visible area of the photograph is 54 sq. in. What are the dimensions of the photograph?



12. The path of debris from fireworks when the wind is about 25 km/h can be modelled by the quadratic function $h(x) = -0.04x^2 + 2x + 8$, where h is the height and x is the horizontal distance travelled, both measured in metres. How far away from the launch site will the debris land? Express your answer to the nearest tenth of a metre.

Extend

13. Write a quadratic equation with the given roots.

a) $\sqrt{7}$ and $-\sqrt{7}$

b) $1 + \sqrt{3}$ and $1 - \sqrt{3}$

c) $\dfrac{5 + \sqrt{11}}{2}$ and $\dfrac{5 - \sqrt{11}}{2}$

14. Solve each equation for x by completing the square.

a) $x^2 + 2x = k$

b) $kx^2 - 2x = k$

c) $x^2 = kx + 1$

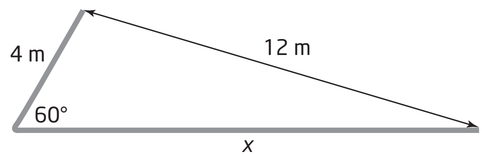
15. Determine the roots of $ax^2 + bx + c = 0$ by completing the square. Can you use this result to solve any quadratic equation? Explain.

16. The sum of the first n terms, S_n, of an arithmetic series can be found using the formula $S_n = \frac{n}{2}[2t_1 + (n - 1)d]$, where t_1 is the first term and d is the common difference.

a) The sum of the first n terms in the arithmetic series $6 + 10 + 14 + \cdots$ is 3870. Determine the value of n.

b) The sum of the first n consecutive natural numbers is 780. Determine the value of n.

17. A machinist in a fabrication shop needs to bend a metal rod at an angle of 60° at a point 4 m from one end of the rod so that the ends of the rod are 12 m apart, as shown.



a) Using the cosine law, write a quadratic equation to represent this situation.

b) Solve the quadratic equation. How long is the rod, to the nearest tenth of a metre?

Create Connections

18. The solution to $x^2 = 9$ is $x = \pm 3$. The solution to the equation $x = \sqrt{9}$ is $x = 3$. Explain why the solutions to the two equations are different.

19. Allison completed the square to determine the vertex form of the quadratic function $y = x^2 - 6x - 27$. Her method is shown.

Allison's method:

$y = x^2 - 6x - 27$
$y = (x^2 - 6x) - 27$
$y = (x^2 - 6x + 9 - 9) - 27$
$y = [(x - 3)^2 - 9] - 27$
$y = (x - 3)^2 - 36$

Riley completed the square to begin to solve the quadratic equation $0 = x^2 - 6x - 27$. His method is shown.

Riley's method:

$$\begin{aligned} 0 &= x^2 - 6x - 27 \\ 27 &= x^2 - 6x \\ 27 + 9 &= x^2 - 6x + 9 \\ 36 &= (x - 3)^2 \\ \pm 6 &= x - 3 \end{aligned}$$

Describe the similarities and differences between the two uses of the method of completing the square.

20. Compare and contrast the following strategies for solving $x^2 - 5x - 6 = 0$.
- completing the square
- graphing the corresponding function
- factoring

21. Write a quadratic function in the form $y = a(x - p)^2 + q$ satisfying each of the following descriptions. Then, write the corresponding quadratic equation in the form $0 = ax^2 + bx + c$. Use graphing technology to verify that your equation also satisfies the description.

a) two distinct real roots

b) one distinct real root, or two equal real roots

c) no real roots

Project Corner — Avalanche Blasting

- An avalauncher is a two-chambered compressed-gas cannon used in avalanche control work. It fires projectiles with trajectories that can be varied by altering the firing angle and the nitrogen pressure.
- The main disadvantages of avalaunchers, compared to powerful artillery such as the howitzer, are that they have a short range and poor accuracy in strong winds.
- Which would you use if you were an expert initiating a controlled avalanche near a ski resort, a howitzer or an avalauncher? Why?

Howitzer

Avalauncher

Check Your Understanding

Practise

1. Use the discriminant to determine the nature of the roots for each equation. Do not solve the equations. Check your answers graphically.

a) $x^2 - 7x + 4 = 0$

b) $s^2 + 3s - 2 = 0$

c) $r^2 + 9r + 6 = 0$

d) $n^2 - 2n + 1 = 0$

e) $7y^2 + 3y + 2 = 0$

f) $4t^2 + 12t + 9 = 0$

2. Without graphing, determine the number of zeros for each function.

a) $f(x) = x^2 - 2x - 14$

b) $g(x) = -3x^2 + 0.06x + 4$

c) $f(x) = \frac{1}{4}x^2 - 3x + 9$

d) $f(v) = -v^2 + 2v - 1$

e) $f(x) = \frac{1}{2}x^2 - x + \frac{5}{2}$

f) $g(y) = -6y^2 + 5y - 1$

3. Use the quadratic formula to solve each quadratic equation. Express your answers as exact roots.

a) $7x^2 + 24x + 9 = 0$

b) $4p^2 - 12p - 9 = 0$

c) $3q^2 + 5q = 1$

d) $2m^2 + 4m - 7 = 0$

e) $2j^2 - 7j = -4$

f) $16g^2 + 24g = -9$

4. Use the quadratic formula to solve each equation. Express your answers to the nearest hundredth.

a) $3z^2 + 14z + 5 = 0$

b) $4c^2 - 7c - 1 = 0$

c) $-5u^2 + 16u - 2 = 0$

d) $8b^2 + 12b = -1$

e) $10w^2 - 45w = 7$

f) $-6k^2 + 17k + 5 = 0$

5. Determine the roots of each quadratic equation. Express your answers as exact values and to the nearest hundredth.

a) $3x^2 + 6x + 1 = 0$

b) $h^2 + \frac{h}{6} - \frac{1}{2} = 0$

c) $0.2m^2 = -0.3m + 0.1$

d) $4y^2 + 7 - 12y = 0$

e) $\frac{x}{2} + 1 = \frac{7x^2}{2}$

f) $2z^2 = 6z - 1$

6. Marge claims that the most efficient way to solve all quadratic equations is to use the quadratic formula. Do you agree with her? Explain with examples.

7. Solve using an appropriate method. Justify your choice of method.

a) $n^2 + 2n - 2 = 0$

b) $-y^2 + 6y - 9 = 0$

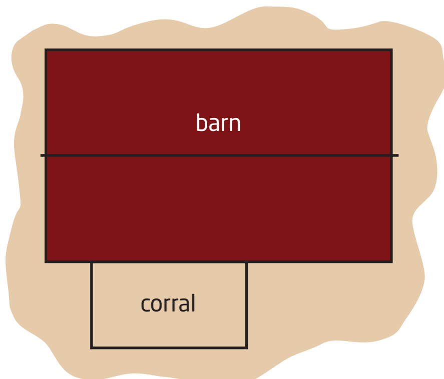
c) $-2u^2 + 16 = 0$

d) $\frac{x^2}{2} - \frac{x}{3} = 1$

e) $x^2 - 4x + 8 = 0$

Apply

8. To save materials, Choma decides to build a horse corral using the barn for one side. He has 30 m of fencing materials and wants the corral to have an area of 100 m². What are the dimensions of the corral?



9. A mural is being painted on an outside wall that is 15 m wide and 12 m tall. A border of uniform width surrounds the mural. The mural covers 75% of the area of the wall. How wide is the border? Express your answer to the nearest hundredth of a metre.

10. Subtracting a number from half its square gives a result of 11. What is the number? Express your answers as exact values and to the nearest hundredth.

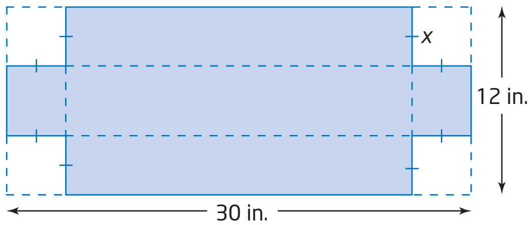
11. The mural *Northern Tradition and Transition*, located in the Saskatchewan Legislature, was painted by Métis artist Roger Jerome to honour the province of Saskatchewan's 100th anniversary in 2005. The mural includes a parabolic arch. The approximate shape of the arch can be modelled by the function $h(d) = -0.4(d - 2.5)^2 + 2.5$, where h is the height of the arch, in metres, and d is the distance, in metres, from one end of the arch. How wide is the arch at its base?

Did You Know?

Roger Jerome included the arch shape to symbolize the unity of northern and southern Saskatchewan.

Northern Tradition and Transition by Roger Jerome

12. An open-topped box is being made from a piece of cardboard measuring 12 in. by 30 in. The sides of the box are formed when four congruent squares are cut from the corners, as shown in the diagram. The base of the box has an area of 208 sq. in..



a) What equation represents the surface area of the base of the box?

b) What is the side length, x, of the square cut from each corner?

c) What are the dimensions of the box?

13. A car travelling at a speed of v kilometres per hour needs a stopping distance of d metres to stop without skidding. This relationship can be modelled by the function $d(v) = 0.0067v^2 + 0.15v$. At what speed can a car be travelling to be able to stop in each distance? Express your answer to the nearest tenth of a kilometre per hour.

a) 42 m

b) 75 m

c) 135 m

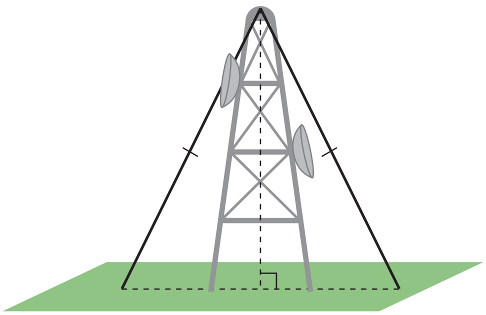
14. A study of the air quality in a particular city suggests that t years from now, the level of carbon monoxide in the air, A, in parts per million, can be modelled by the function $A(t) = 0.3t^2 + 0.1t + 4.2$.

a) What is the level, in parts per million, of carbon monoxide in the air now, at $t = 0$?

b) In how many years from now will the carbon monoxide level be 8 parts per million? Express your answer to the nearest tenth of a year.

15. A sporting goods store sells 90 ski jackets in a season for \$275 each. Each \$15 decrease in the price results in five more jackets being sold. What is the lowest price that would produce revenues of at least \$19 600? How many jackets would be sold at this price?

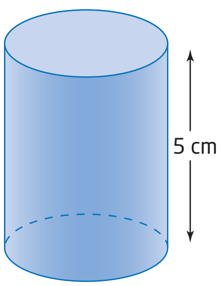
16. Two guy wires are attached to the top of a telecommunications tower and anchored to the ground on opposite sides of the tower, as shown. The length of the guy wire is 20 m more than the height of the tower. The horizontal distance from the base of the tower to where the guy wire is anchored to the ground is one-half the height of the tower. How tall is the tower, to the nearest tenth of a metre?



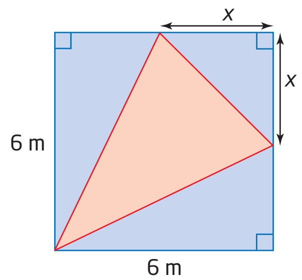
Extend

17. One root of the equation $2x^2 + bx - 24 = 0$ is -8. What are the possible values of b and the other root?

18. A cylinder has a height of 5 cm and a surface area of 100 cm². What is the radius of the cylinder, to the nearest tenth of a centimetre?



19. In the diagram, the square has side lengths of 6 m. The square is divided into three right triangles and one acute isosceles triangle. The areas of the three right triangles are equal.



a) Determine the exact value of x.

b) What is the exact area of the acute isosceles triangle?

20. Two small private planes take off from the same airport. One plane flies north at 150 km/h. Two hours later, the second plane flies west at 200 km/h. How long after the first plane takes off will the two planes be 600 km apart? Express your answer to the nearest tenth of an hour.

Create Connections

21. Determine the error(s) in the following solution. Explain how to correct the solution.

Solve $-3x^2 - 7x + 2 = 0$.

Line 1: $x = \dfrac{-7 \pm \sqrt{(-7)^2 - 4(-3)(2)}}{2(-3)}$

Line 2: $x = \dfrac{-7 \pm \sqrt{49 - 24}}{-6}$

Line 3: $x = \dfrac{-7 \pm \sqrt{25}}{-6}$

Line 4: $x = \dfrac{-7 \pm 5}{-6}$

Line 5: So, $x = 2$ or $x = \dfrac{1}{3}$.

22. Pierre calculated the roots of a quadratic equation as $x = \dfrac{3 \pm \sqrt{25}}{2}$.

a) What are the x-intercepts of the graph of the corresponding quadratic function?

b) Describe how to use the x-intercepts to determine the equation of the axis of symmetry.

Chapter 4 Review

4.1 Graphical Solutions of Quadratic Equations, pages 206–217

1. Solve each quadratic equation by graphing the corresponding quadratic function.

a) $0 = x^2 + 8x + 12$

b) $0 = x^2 - 4x - 5$

c) $0 = 3x^2 + 10x + 8$

d) $0 = -x^2 - 3x$

e) $0 = x^2 - 25$

2. Use graphing technology to determine which of the following quadratic equations has different roots from the other three.

A $0 = 3 - 3x - 3x^2$

B $0 = x^2 + x - 1$

C $0 = 2(x - 1)^2 + 6x - 4$

D $0 = 2x + 2 + 2x^2$

3. Explain what must be true about the graph of the corresponding function for a quadratic equation to have no real roots.

4. A manufacturing company produces key rings. Last year, the company collected data about the number of key rings produced per day and the corresponding profit. The data can be modelled by the function $P(k) = -2k^2 + 12k - 10$, where P is the profit, in thousands of dollars, and k is the number of key rings, in thousands.

a) Sketch a graph of the function.

b) Using the equation $-2k^2 + 12k - 10 = 0$, determine the number of key rings that must be produced so that there is no profit or loss. Justify your answer.

5. The path of a soccer ball can be modelled by the function $h(d) = -0.1d^2 + 0.5d + 0.6$, where h is the height of the ball and d is the horizontal distance from the kicker, both in metres.

a) What are the zeros of the function?

b) You can use the quadratic equation $0 = -0.1d^2 + 0.5d + 0.6$ to determine the horizontal distance that a ball travels after being kicked. How far did the ball travel downfield before it hit the ground?

4.2 Factoring Quadratic Equations, pages 218–233

6. Factor.

a) $4x^2 - 13x + 9$

b) $\frac{1}{2}x^2 - \frac{3}{2}x - 2$

c) $3(v + 1)^2 + 10(v + 1) + 7$

d) $9(a^2 - 4)^2 - 25(7b)^2$

7. Solve by factoring. Check your solutions.

a) $0 = x^2 + 10x + 21$

b) $\frac{1}{4}m^2 + 2m - 5 = 0$

c) $5p^2 + 13p - 6 = 0$

d) $0 = 6z^2 - 21z + 9$

8. Solve.

a) $-4g^2 + 6 = -10g$

b) $8y^2 = -5 + 14y$

c) $30k - 25k^2 = 9$

d) $0 = 2x^2 - 9x - 18$

9. Write a quadratic equation in standard form with the given roots.

a) 2 and 3

b) -1 and -5

c) $\frac{3}{2}$ and -4

10. The path of a paper airplane can be modelled approximately by the function $h(t) = -\frac{1}{4}t^2 + t + 3$, where h is the height above the ground, in metres, and t is the time of flight, in seconds. Determine how long it takes for the paper airplane to hit the ground, $h(t) = 0$.

11. The length of the base of a rectangular prism is 2 m more than its width, and the height of the prism is 15 m.

a) Write an algebraic expression for the volume of the rectangular prism.

b) The volume of the prism is 2145 m^3. Write an equation to model the situation.

c) Solve the equation in part b) by factoring. What are the dimensions of the base of the rectangular prism?

12. Solve the quadratic equation $x^2 - 2x - 24 = 0$ by factoring and by graphing. Which method do you prefer to use? Explain.

4.3 Solving Quadratic Equations by Completing the Square, pages 234–243

13. Determine the value of k that makes each expression a perfect square trinomial.

a) $x^2 + 4x + k$

b) $x^2 + 3x + k$

14. Solve. Express your answers as exact values.

a) $2x^2 - 98 = 0$

b) $(x + 3)^2 = 25$

c) $(x - 5)^2 = 24$

d) $(x - 1)^2 = \frac{5}{9}$

15. Complete the square to determine the roots of each quadratic equation. Express your answers as exact values.

a) $-2x^2 + 16x - 3 = 0$

b) $5y^2 + 20y + 1 = 0$

c) $4p^2 + 2p = -5$

16. In a simulation, the path of a new aircraft after it has achieved weightlessness can be modelled approximately by $h(t) = -5t^2 + 200t + 9750$, where h is the altitude of the aircraft, in metres, and t is the time, in seconds, after weightlessness is achieved. How long does the aircraft take to return to the ground, $h(t) = 0$? Express your answer to the nearest tenth of a second.

17. The path of a snowboarder after jumping from a ramp can be modelled by the function $h(d) = -\frac{1}{2}d^2 + 2d + 1$, where h is the height above the ground and d is the horizontal distance the snowboarder travels, both in metres.

a) Write a quadratic equation you would solve to determine the horizontal distance the snowboarder has travelled when she lands.

b) What horizontal distance does the snowboarder travel? Express your answer to the nearest tenth of a metre.

4.4 The Quadratic Formula, pages 244–257

18. Use the discriminant to determine the nature of the roots for each quadratic equation. Do not solve the equation.

a) $2x^2 + 11x + 5 = 0$

b) $4x^2 - 4x + 1 = 0$

c) $3p^2 + 6p + 24 = 0$

d) $4x^2 + 4x - 7 = 0$

19. Use the quadratic formula to determine the roots for each quadratic equation. Express your answers as exact values.

a) $-3x^2 - 2x + 5 = 0$

b) $5x^2 + 7x + 1 = 0$

c) $3x^2 - 4x - 1 = 0$

d) $25x^2 + 90x + 81 = 0$

20. A large fountain in a park has 35 water jets. One of the streams of water shoots out of a metal rod and follows a parabolic path. The path of the stream of water can be modelled by the function $h(x) = -2x^2 + 6x + 1$, where h is the height, in metres, at any horizontal distance x metres from its jet.

a) What quadratic equation would you solve to determine the maximum horizontal distance the water jet can reach?

b) What is the maximum horizontal distance the water jet can reach? Express your answer to the nearest tenth of a metre.

21. A ferry carries people to an island airport. It carries 2480 people per day at a cost of \$3.70 per person. Surveys have indicated that for every \$0.05 decrease in the fare, 40 more people will use the ferry. Use x to represent the number of decreases in the fare.

a) Write an expression to model the fare per person.

b) Write an expression to model the number of people that would use the ferry per day.

c) Determine the expression that models the revenue, R, for the ferry, which is the product of the number of people using the ferry per day and the fare per person.

d) Determine the number of fare decreases that result in a revenue of \$9246.

22. Given the quadratic equation in standard form, $ax^2 + bx + c = 0$, arrange the following algebraic steps and explanations in the order necessary to derive the quadratic formula.

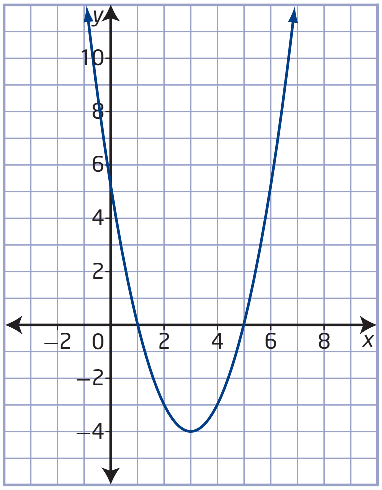
Algebraic Steps	Explanations
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	Complete the square.
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Solve for x.
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Subtract c from both sides.
$ax^2 + bx = -c$	Take the square root of both side.
$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$	Divide both sides by a.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Factor the perfect square trinomial.

Chapter 4 Practice Test

Multiple Choice

For #1 to #5, choose the best answer.

1. What points on the graph of this quadratic function represent the locations of the zeros of the function?



A (0, 5) and (1, 0)

B (0, 1) and (0, 5)

C (1, 0) and (5, 0)

D (5, 0) and (0, 1)

2. What is one of the factors of $x^2 - 3x - 10$?

A $x + 5$ **B** $x - 5$

C $x - 10$ **D** $x + 10$

3. What integral values of k will make $2x^2 + kx - 1$ factorable?

A -1 and 2 **B** -2 and 2

C -2 and 1 **D** -1 and 1

4. The roots, to the nearest hundredth, of $0 = -\frac{1}{2}x^2 + x + \frac{7}{2}$ are

A 1.83 and 3.83

B -1.83 and 3.83

C 1.83 and -3.83

D -1.83 and -3.83

5. The number of baseball games, G, that must be scheduled in a league with n teams can be modelled by the function $G(n) = \frac{n^2 - n}{2}$, where each team plays every other team exactly once. Suppose a league schedules 15 games. How many teams are in the league?

A 5

B 6

C 7

D 8

Short Answer

6. Determine the roots of each quadratic equation. If the quadratic equation does not have real roots, use a graph of the corresponding function to explain.

a) $0 = x^2 - 4x + 3$

b) $0 = 2x^2 - 7x - 15$

c) $0 = -x^2 - 2x + 3$

7. Solve the quadratic equation $0 = 3x^2 + 5x - 1$ by completing the square. Express your answers as exact roots.

8. Use the quadratic formula to determine the roots of the equation $x^2 + 4x - 7 = 0$. Express your answers as exact roots in simplest radical form.

9. Without solving, determine the nature of the roots for each quadratic equation.

a) $x^2 + 10x + 25 = 0$

b) $2x^2 + x = 5$

c) $2x^2 + 6 = 4x$

d) $\frac{2}{3}x^2 + \frac{1}{2}x - 3 = 0$

10. The length of the hypotenuse of a right triangle is 1 cm more than triple that of the shorter leg. The length of the longer leg is 1 cm less than triple that of the shorter leg.

a) Sketch and label a diagram with expressions for the side lengths.

b) Write an equation to model the situation.

c) Determine the lengths of the sides of the triangle.

Extended Response

11. A pebble is tossed upward from a scenic lookout and falls to the river below. The approximate height, h, in metres, of the pebble above the river t seconds after being tossed is modelled by the function $h(x) = -5t^2 + 10t + 35$.

a) After how many seconds does the pebble hit the river? Express your answer to the nearest tenth of a second.

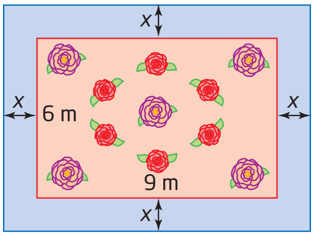
b) How high is the scenic lookout above the river?

c) Which method did you choose to solve the quadratic equation? Justify your choice.

12. Three rods measure 20 cm, 41 cm, and 44 cm. If the same length is cut off each piece, the remaining lengths can be formed into a right triangle. What length is cut off?

13. A rectangular piece of paper has a perimeter of 100 cm and an area of 616 cm². Determine the dimensions of the paper.

14. The parks department is planning a new flower bed. It will be rectangular with dimensions 9 m by 6 m. The flower bed will be surrounded by a grass strip of constant width with the same area as the flower bed.



a) Write a quadratic equation to model the situation.

b) Solve the quadratic equation. Justify your choice of method.

c) Calculate the perimeter of the outside of the path.

Cumulative Review, Chapters 3–4

Chapter 3 Quadratic Functions

1. Match each characteristic with the correct quadratic function.

Characteristic

a) vertex in quadrant III

b) opens downward

c) axis of symmetry: $x = 3$

d) range: $\{y \mid y \geq 5, y \in \mathrm{R}\}$

Quadratic Function

A $y = -5(x - 2)^2 - 3$

B $y = 3(x + 3)^2 + 5$

C $y = 2(x + 2)^2 - 3$

D $y = 3(x - 3)^2 - 5$

2. Classify each as a quadratic function or a function that is not quadratic.

a) $y = (x + 6) - 1$

b) $y = -5(x + 1)^2$

c) $y = \sqrt{(x + 2)^2} + 7$

d) $y + 8 = x^2$

3. Sketch a possible graph for a quadratic function given each set of characteristics.

a) axis of symmetry: $x = -2$
range: $\{y \mid y \leq 4, y \in \mathrm{R}\}$

b) axis of symmetry: $x = 3$
range: $\{y \mid y \geq 2, y \in \mathrm{R}\}$

c) opens upward, vertex at $(1, -3)$, one x-intercept at the point $(3, 0)$

4. Identify the vertex, domain, range, axis of symmetry, x-intercepts, and y-intercept for each quadratic function.

a) $f(x) = (x + 4)^2 - 3$

b) $f(x) = -(x - 2)^2 + 1$

c) $f(x) = -2x^2 - 6$

d) $f(x) = \frac{1}{2}(x + 8)^2 + 6$

5. Rewrite each function in the form $y = a(x - p)^2 + q$. Compare the graph of each function to the graph of $y = x^2$.

a) $y = x^2 - 10x + 18$

b) $y = -x^2 + 4x - 7$

c) $y = 3x^2 - 6x + 5$

d) $y = \frac{1}{4}x^2 + 4x + 20$

6. a) The approximate height, h, in metres, of an arrow shot into the air with an initial velocity of 20 m/s after t seconds can be modelled by the function $h(t) = -5t^2 + 20t + 2$. What is the maximum height reached by the arrow?

b) From what height was the arrow shot?

c) How long did it take for the arrow to hit the ground, to the nearest second?

Chapter 4 Quadratic Equations

7. Copy and complete the sentence, filling in the blanks with the correct terms: zeros, x-intercepts, roots.

When solving quadratic equations, you may consider the relationship among the ______ of a quadratic equation, the ______ of the corresponding quadratic function, and the ______ of the graph of the quadratic function.

8. Factor each polynomial expression.

a) $9x^2 + 6x - 8$

b) $16r^2 - 81s^2$

c) $2(x + 1)^2 + 11(x + 1) + 14$

d) $x^2y^2 - 5xy - 36$

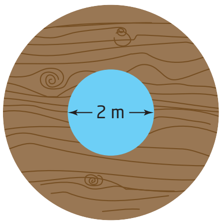
e) $9(3a + b)^2 - 4(2a - b)^2$

f) $121r^2 - 400$

9. The sum of the squares of three consecutive integers is 194. What are the integers?

10. The Empress Theatre, in Fort Macleod, is Alberta's oldest continually operating theatre. Much of the theatre is the same as when it was constructed in 1912, including the 285 original seats on the main floor. The number of rows on the main floor is 4 more than the number of seats in each row. Determine the number of rows and the number of seats in each row.

11. An outdoor hot tub has a diameter of 2 m. The hot tub is surrounded by a circular wooden deck, so that the deck has a uniform width. If the top area of the deck and the hot tub is 63.6 m², how wide is deck, to the nearest tenth of a metre?



12. Dallas solves the quadratic equation $2x^2 - 12x - 7 = 0$ by completing the square. Doug solves the quadratic equation by using the quadratic formula. The solution for each student is shown. Identify the errors, if any, made by the students and determine the correct solution.

Dallas's solution:

$$\begin{aligned} 2(x^2 - 12x) &= 7 \\ 2(x^2 - 12x + 36) &= 7 + 36 \\ 2(x - 6)^2 &= 43 \\ x &= 6 \pm \sqrt{\frac{43}{2}} \end{aligned}$$

Doug's solution:

$$x = \frac{-12 \pm \sqrt{(-12)^2 - 4(2)(-7)}}{2(2)}$$
$$x = \frac{-12 \pm \sqrt{80}}{4}$$
$$x = -3 \pm \sqrt{20}$$
$$x = -3 \pm 2\sqrt{5}$$

13. Name the method you would choose to solve each quadratic equation. Determine the roots of each equation and verify the answer.

a) $3x^2 - 6 = 0$

b) $m^2 - 15m = -26$

c) $s^2 - 2s - 35 = 0$

d) $-16x^2 + 47x + 3 = 0$

14. Use the discriminant to determine the nature of the roots for each quadratic equation.

a) $x^2 - 6x + 3 = 0$

b) $x^2 + 22x + 121 = 0$

c) $-x^2 + 3x = 5$

15. James Michels was raised in Merritt, British Columbia, and is a member of the Penticton Métis Association. James apprenticed with Coast Salish artist Joseph Campbell and now produces intricate bentwood boxes.

For one particular bentwood box, the side length of the top square piece is 1 in. longer than the side length of the bottom. Their combined area is 85 sq. in..

a) Write a quadratic equation to determine the dimensions of each square piece.

b) Select an algebraic method and solve for the roots of the quadratic equation.

c) What are the dimensions of the top and bottom of the box?

d) Explain why one of the roots from part b) is extraneous.

Natural Wolf
by James Michels

Unit 2 Test

Multiple Choice

For #1 to #5, choose the best answer.

1. The graph of which function is congruent to the graph of $f(x) = x^2 + 3$ but translated vertically 2 units down?

A $f(x) = x^2 + 1$

B $f(x) = x^2 - 1$

C $f(x) = x^2 + 5$

D $f(x) = (x - 2)^2 + 3$

2. The equation of the quadratic function in the form $y = a(x - p)^2 + q$ with a vertex at $(-1, -2)$ and passing through the point $(1, 6)$ is

A $y = 4(x + 1)^2 - 2$

B $y = 4(x - 1)^2 - 2$

C $y = 2(x - 1)^2 - 2$

D $y = 2(x + 1)^2 - 2$

3. The graph of $y = ax^2 + q$ intersects the x-axis in two places when

A $a > 0, q > 0$

B $a < 0, q < 0$

C $a > 0, q = 0$

D $a > 0, q < 0$

4. When $y = 2x^2 - 8x + 2$ is written in the form $y = a(x - p)^2 + q$, the values of p and q are

A $p = -2, q = -6$

B $p = 2, q = -6$

C $p = 4, q = 0$

D $p = -2, q = 6$

5. Michelle wants to complete the square to identify the vertex of the graph of the quadratic function $y = -3x^2 + 5x - 2$. Her partial work is shown.

Step 1: $y = -3\left(x^2 + \frac{5}{3}x\right) - 2$

Step 2: $y = -3\left(x^2 + \frac{5}{3}x + \frac{25}{36}\right) - 2 - \frac{25}{36}$

Step 3: $y = -3\left(x + \frac{5}{6}\right)^2 - \frac{97}{36}$

Identify the step where Michelle made her first error, as well as the correct coordinates of the vertex.

A Step 1, vertex $\left(-\frac{5}{6}, -\frac{97}{36}\right)$

B Step 1, vertex $\left(\frac{5}{6}, \frac{1}{12}\right)$

C Step 2, vertex $\left(\frac{5}{6}, \frac{1}{12}\right)$

D Step 2, vertex $\left(-\frac{5}{6}, -\frac{25}{12}\right)$

Numerical Response

Copy and complete the statements in #6 to #8.

6. The value of the discriminant for the quadratic function $y = -3x^2 - 4x + 5$ is ■.

7. The manager of an 80-unit apartment complex is trying to decide what rent to charge. At a rent of \$200 per week, all the units will be full. For each increase in rent of \$20 per week, one more unit will become vacant. The manager should charge ■ per week to maximize the revenue of the apartment complex.

8. The greater solution to the quadratic equation $9x^2 + 4x - 1 = 0$, rounded to the nearest hundredth, is ■.

Written Response

9. You create a circular piece of art for a project in art class. Your initial task is to paint a background circle entirely in blue. Suppose you have a can of blue paint that will cover 9000 cm^2.

a) Determine the radius, to the nearest tenth of a centimetre, of the largest circle you can paint.

b) If you had two cans of paint, what would be the radius of the largest circle you could paint, to the nearest tenth of a centimetre?

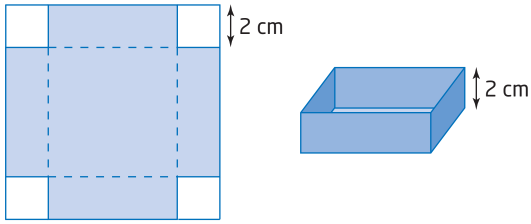
c) Does the radius of the largest circle double when the amount of paint doubles?

10. Suppose Clair hits a high pop-up with an initial upward velocity of 30 m/s from a height of 1.6 m above the ground. The height, h, in metres, of the ball, t seconds after it was hit can be modelled by the function $h(t) = -4.9t^2 + 30t + 1.6$.

a) What is the maximum height the ball reaches? Express your answer to the nearest tenth of a metre.

b) The pitcher caught the ball at a height of 1.1 m. How long was the ball in the air? Express your answer to the nearest tenth of a second.

11. The bottom of a jewellery box is made from a square piece of cardboard by cutting 2-cm squares from each corner and turning up the sides, as shown in the figure below. The volume of the open box is 128 cm^2. What size of cardboard is needed?



12. The side lengths of the tops of three decorative square tables can be described as three consecutive integers. The combined area of the table tops is 677 sq. in..

a) Write a single-variable quadratic equation, in simplified form, to determine the side length of each square tabletop.

b) Algebraically determine the roots of the quadratic equation.

c) What are the side lengths of the tabletops?

d) Why would you not consider both of the roots of the quadratic equation when determining a possible side length?

Key Ideas

- You can compare and order radicals using a variety of strategies:
  - Convert unlike radicals to entire radicals. If the radicals have the same index, the radicands can be compared.
  - Compare the coefficients of like radicals.
  - Compare the indices of radicals with equal radicands.
- When adding or subtracting radicals, combine coefficients of like radicals. In general, $m\sqrt[r]{a} + n\sqrt[r]{a} = (m + n)\sqrt[r]{a}$, where r is a natural number, and m, n, and a are real numbers. If r is even, then $a \geq 0$.
- A radical is in simplest form if the radicand does not contain a fraction or any factor which may be removed, and the radical is not part of the denominator of a fraction.

  For example, $$\begin{aligned} 5\sqrt{40} &= 5\sqrt{4(10)} \\ &= 5\sqrt{4}\left(\sqrt{10}\right) \\ &= 5(2)\sqrt{10} \\ &= 10\sqrt{10} \end{aligned}$$
- When a radicand contains variables, identify the values of the variables that make the radical a real number by considering the index and the radicand:
  - If the index is an even number, the radicand must be non-negative.

    For example, in $\sqrt{3n}$, the index is even. So, the radicand must be non-negative.

    $3n \geq 0$

    $n \geq 0$
  - If the index is an odd number, the radicand may be any real number.

    For example, in $\sqrt[3]{x}$, the index is odd. So, the radicand, x, can be any real number—positive, negative, or zero.

Check Your Understanding

Practise

1. Copy and complete the table.

Mixed Radical Form	Entire Radical Form
$4\sqrt{7}$	
	$\sqrt{50}$
$-11\sqrt{8}$	
	$-\sqrt{200}$

2. Express each radical as a mixed radical in simplest form.

a) $\sqrt{56}$ **b)** $3\sqrt{75}$

c) $\sqrt[3]{24}$ **d)** $\sqrt{c^3d^2}$, $c \geq 0$, $d \geq 0$

3. Write each expression in simplest form. Identify the values of the variable for which the radical represents a real number.

a) $3\sqrt{8m^4}$ **b)** $\sqrt[3]{24q^5}$

c) $-2\sqrt[5]{160s^5t^6}$

4. Copy and complete the table. State the values of the variable for which the radical represents a real number.

Mixed Radical Form	Entire Radical Form
$3n\sqrt{5}$	
	$\sqrt[3]{-432}$
$\frac{1}{2a}\sqrt[3]{7a}$	
	$\sqrt[3]{128x^4}$

5. Express each pair of terms as like radicals. Explain your strategy.
   a) $15\sqrt{5}$ and $8\sqrt{125}$
   b) $8\sqrt{112z^8}$ and $48\sqrt{7z^4}$
   c) $-35\sqrt[4]{w^2}$ and $3\sqrt[4]{81w^{10}}$
   d) $6\sqrt[3]{2}$ and $6\sqrt[3]{54}$
6. Order each set of numbers from least to greatest.
   a) $3\sqrt{6}$, 10, and $7\sqrt{2}$
   b) $-2\sqrt{3}$, -4, $-3\sqrt{2}$, and $-2\sqrt{\frac{7}{2}}$
   c) $\sqrt[3]{21}$, $3\sqrt[3]{2}$, 2.8, $2\sqrt[3]{5}$
7. Verify your answer to #6b) using a different method.
8. Simplify each expression.
   a) $-\sqrt{5} + 9\sqrt{5} - 4\sqrt{5}$
   b) $1.4\sqrt{2} + 9\sqrt{2} - 7$
   c) $\sqrt[4]{11} - 1 - 5\sqrt[4]{11} + 15$
   d) $-\sqrt{6} + \frac{9}{2}\sqrt{10} - \frac{5}{2}\sqrt{10} + \frac{1}{3}\sqrt{6}$
9. Simplify.
   a) $3\sqrt{75} - \sqrt{27}$
   b) $2\sqrt{18} + 9\sqrt{7} - \sqrt{63}$
   c) $-8\sqrt{45} + 5.1 - \sqrt{80} + 17.4$
   d) $\frac{2}{3}\sqrt[3]{81} + \frac{\sqrt[3]{375}}{4} - 4\sqrt{99} + 5\sqrt{11}$
10. Simplify each expression. Identify any restrictions on the values for the variables.
   a) $2\sqrt{a^3} + 6\sqrt{a^3}$
   b) $3\sqrt{2x} + 3\sqrt{8x} - \sqrt{x}$
   c) $-4\sqrt[3]{625r} + \sqrt[3]{40r^4}$
   d) $\frac{w}{5}\sqrt[3]{-64} + \frac{\sqrt[3]{512w^3}}{5} - \frac{2}{5}\sqrt{50w} - 4\sqrt{2w}$

Apply

11. The air pressure, p, in millibars (mbar) at the centre of a hurricane, and wind speed, w, in metres per second, of the hurricane are related by the formula $w = 6.3\sqrt{1013 - p}$. What is the exact wind speed of a hurricane if the air pressure is 965 mbar?

12. Saskatoon artist Jonathan Forrest's painting, *Clincher*, contains geometric shapes. The isosceles right triangle at the top right has legs that measure approximately 12 cm. What is the length of the hypotenuse? Express your answer as a radical in simplest form.

Clincher, by Jonathan Forrest Saskatoon, Saskatchewan

13. The distance, d, in millions of kilometres, between a planet and the Sun is a function of the length, n, in Earth-days, of the planet's year. The formula is $d = \sqrt[3]{25n^2}$. The length of 1 year on Mercury is 88 Earth-days, and the length of 1 year on Mars is 704 Earth-days. Use the subtraction of radicals to determine the difference between the distances of Mercury and Mars from the Sun. Express your answer in exact form.

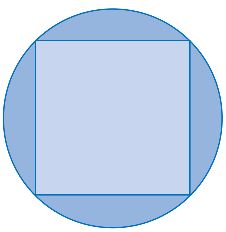
Planet Venus

Did You Know?

Distances in space are frequently measured in astronomical units (AU). The measurement 1 AU represents the average distance between the Sun and Earth during Earth's orbit. According to NASA, the average distance between Venus and Earth is 0.723 AU.

14. The speed, s, in metres per second, of a tsunami is related to the depth, d, in metres, of the water through which it travels. This relationship can be modelled with the formula $s = \sqrt{10d}$, $d \geq 0$. A tsunami has a depth of 12 m. What is the speed as a mixed radical and an approximation to the nearest metre per second?

15. A square is inscribed in a circle. The area of the circle is 38π m².



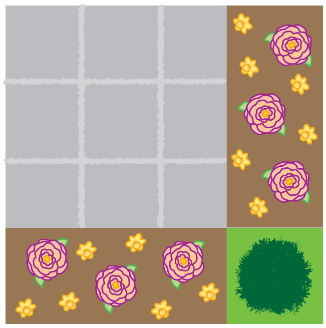
a) What is the exact length of the diagonal of the square?

b) Determine the exact perimeter of the square.

16. You can use Heron's formula to determine the area of a triangle given all three side lengths. The formula is $A = \sqrt{s(s - a)(s - b)(s - c)}$, where s represents the half-perimeter of the triangle and a, b, and c are the three side lengths. What is the exact area of a triangle with sides of 8 mm, 10 mm, and 12 mm? Express your answer as an entire radical and as a mixed radical.

17. Suppose an ant travels in a straight line across the Cartesian plane from (3, 4) to (6, 10). Then, it travels in a straight line from (6, 10) to (10, 18). How far does the ant travel? Express your answer in exact form.

18. Leslie's backyard is in the shape of a square. The area of her entire backyard is 98 m². The green square, which contains a tree, has an area of 8 m². What is the exact perimeter of one of the rectangular flowerbeds?



19. Kristen shows her solution to a radical problem below. Brady says that Kristen's final radical is not in simplest form. Is he correct? Explain your reasoning.

Kristen's Solution

$$\begin{aligned} y\sqrt{4y^3} + \sqrt{64y^5} &= y\sqrt{4y^3} + 4y\sqrt{4y^3} \\ &= 5y\sqrt{4y^3} \end{aligned}$$

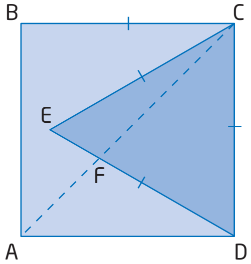
20. Which expression is not equivalent to $12\sqrt{6}$?

$2\sqrt{216}$, $3\sqrt{96}$, $4\sqrt{58}$, $6\sqrt{24}$

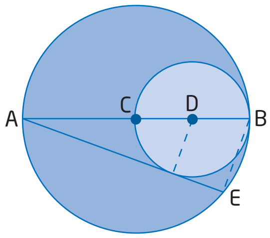
Explain how you know without using technology.

Extend

21. A square, ABCD, has a perimeter of 4 m. $\triangle$CDE is an equilateral triangle inside the square. The intersection of AC and DE occurs at point F. What is the exact length of AF?



22. A large circle has centre C and diameter AB. A smaller circle has centre D and diameter BC. Chord AE is tangent to the smaller circle. If AB = 18 cm, what is the exact length of AE?



Create Connections

23. What are the exact values of the common difference and missing terms in the following arithmetic sequence? Justify your work.
$\sqrt{27}$, ■, ■, $9\sqrt{3}$

24. Consider the following set of radicals:
$-3\sqrt{12}$, $2\sqrt{75}$, $-\sqrt{27}$, $108^{\frac{1}{2}}$

Explain how you could determine the answer to each question without using a calculator.

a) Using only two of the radicals, what is the greatest sum?

b) Using only two of the radicals, what is the greatest difference?

25. Support each equation using examples.

a) $(-x)^2 = x^2$

b) $\sqrt{x^2} \neq \pm x$

Project Corner — The Milky Way Galaxy

- Our solar system is located in the Milky Way galaxy, which is a spiral galaxy.
- A galaxy is a congregation of billions of stars, gases, and dust held together by gravity.
- The solar system consists of the Sun, eight planets and their satellites, and thousands of other smaller heavenly bodies such as asteroids, comets, and meteors.
- The motion of planets can be described by Kepler's three laws.
- Kepler's third law states that the ratio of the squares of the orbital periods of any two planets is equal to the ratio of the cubes of their semi-major axes. How could you express Kepler's third law using radicals? Explain this law using words and diagrams.

Key Ideas

- When multiplying radicals with identical indices, multiply the coefficients and multiply the radicands:
  $$(m\sqrt[k]{a})(n\sqrt[k]{b}) = mn\sqrt[k]{ab}$$
  where k is a natural number, and m, n, a, and b are real numbers. If k is even, then $a \geq 0$ and $b \geq 0$.
- When dividing two radicals with identical indices, divide the coefficients and divide the radicands:
  $$\frac{m\sqrt[k]{a}}{n\sqrt[k]{b}} = \frac{m}{n}\sqrt[k]{\frac{a}{b}}$$
  where k is a natural number, and m, n, a, and b are real numbers. $n \neq 0$ and $b \neq 0$. If k is even, then $a \geq 0$ and b > 0.
- When multiplying radical expressions with more than one term, use the distributive property and then simplify.
- To rationalize a monomial denominator, multiply the numerator and denominator by an expression that produces a rational number in the denominator.
  $$\frac{2}{\sqrt[5]{n}}\left(\frac{(\sqrt[5]{n})^4}{(\sqrt[5]{n})^4}\right) = \frac{2(\sqrt[5]{n})^4}{n}$$
- To simplify an expression with a square-root binomial in the denominator, rationalize the denominator using these steps:
  - Determine a conjugate of the denominator.
  - Multiply the numerator and denominator by this conjugate.
  - Express in simplest form.

Check Your Understanding

Practise

1. Multiply. Express all products in simplest form.
   a) $2\sqrt{5}(7\sqrt{3})$
   b) $-\sqrt{32}(7\sqrt{2})$
   c) $2\sqrt[4]{48}(\sqrt[4]{5})$
   d) $4\sqrt{19x}(\sqrt{2x^2})$, $x \geq 0$
   e) $\sqrt[3]{54y^7}(\sqrt[3]{6y^4})$
   f) $\sqrt{6t}\left(3t^2\sqrt{\frac{t}{4}}\right)$, $t \geq 0$

2. Multiply using the distributive property. Then, simplify.
   a) $\sqrt{11}(3 - 4\sqrt{7})$
   b) $-\sqrt{2}(14\sqrt{5} + 3\sqrt{6} - \sqrt{13})$
   c) $\sqrt{y}(2\sqrt{y} + 1)$, $y \geq 0$
   d) $z\sqrt{3}(z\sqrt{12} - 5z + 2)$

3. Simplify. Identify the values of the variables for which the radicals represent real numbers.
   a) $-3(\sqrt{2} - 4) + 9\sqrt{2}$
   b) $7(-1 - 2\sqrt{6}) + 5\sqrt{6} + 8$
   c) $4\sqrt{5}(\sqrt{3j} + 8) - 3\sqrt{15j} + \sqrt{5}$
   d) $3 - \sqrt[3]{4k}(12 + 2\sqrt[3]{8})$

4. Expand and simplify each expression.

a) $(8\sqrt{7} + 2)(\sqrt{2} - 3)$

b) $(4 - 9\sqrt{5})(4 + 9\sqrt{5})$

c) $(\sqrt{3} + 2\sqrt{15})(\sqrt{3} - \sqrt{15})$

d) $(6\sqrt[3]{2} - 4\sqrt{13})^2$

e) $(-\sqrt{6} + 2)(2\sqrt{2} - 3\sqrt{5} + 1)$

5. Expand and simplify. State any restrictions on the values for the variables.

a) $(15\sqrt{c} + 2)(\sqrt{2c} - 6)$

b) $(1 - 10\sqrt{8x^3})(2 + 7\sqrt{5x})$

c) $(9\sqrt{2m} - 4\sqrt{6m})^2$

d) $(10r - 4\sqrt[3]{4r})(2\sqrt[3]{6r^2} + 3\sqrt[3]{12r})$

6. Divide. Express your answers in simplest form.

a) $\dfrac{\sqrt{80}}{\sqrt{10}}$

b) $\dfrac{-2\sqrt{12}}{4\sqrt{3}}$

c) $\dfrac{3\sqrt{22}}{\sqrt{11}}$

d) $\dfrac{3\sqrt{135m^5}}{\sqrt{21m^3}}$, $m > 0$

7. Simplify.

a) $\dfrac{9\sqrt{432p^5} - 7\sqrt{27p^5}}{\sqrt{33p^4}}$, $p > 0$

b) $\dfrac{6\sqrt[3]{4v^7}}{\sqrt[3]{14v}}$, $v > 0$

8. Rationalize each denominator. Express each radical in simplest form.

a) $\dfrac{20}{\sqrt{10}}$

b) $\dfrac{-\sqrt{21}}{\sqrt{7m}}$, $m > 0$

c) $-\dfrac{2}{3}\sqrt{\dfrac{5}{12u}}$, $u > 0$

d) $20\sqrt[3]{\dfrac{6t}{5}}$

9. Determine a conjugate for each binomial. What is the product of each pair of conjugates?

a) $2\sqrt{3} + 1$

b) $7 - \sqrt{11}$

c) $8\sqrt{z} - 3\sqrt{7}$, $z \geq 0$

d) $19\sqrt{h} + 4\sqrt{2h}$, $h \geq 0$

10. Rationalize each denominator. Simplify.

a) $\dfrac{5}{2 - \sqrt{3}}$

b) $\dfrac{7\sqrt{2}}{\sqrt{6} + 8}$

c) $\dfrac{-\sqrt{7}}{\sqrt{5} - 2\sqrt{2}}$

d) $\dfrac{\sqrt{3} + \sqrt{13}}{\sqrt{3} - \sqrt{13}}$

11. Write each fraction in simplest form. Identify the values of the variables for which each fraction is a real number.

a) $\dfrac{4r}{\sqrt{6r} + 9}$

b) $\dfrac{18\sqrt{3n}}{\sqrt{24n}}$

c) $\dfrac{8}{4 - \sqrt{6t}}$

d) $\dfrac{5\sqrt{3y}}{\sqrt{10} + 2}$

12. Use the distributive property to simplify $(c + c\sqrt{c})(c + 7\sqrt{3c})$, $c \geq 0$.

Apply

13. Malcolm tries to rationalize the denominator in the expression $\dfrac{4}{3 - 2\sqrt{2}}$ as shown below.

a) Identify, explain, and correct any errors.

b) Verify your corrected solution.

Malcolm's solution:

$$\begin{aligned}\frac{4}{3 - 2\sqrt{2}} &= \left(\frac{4}{3 - 2\sqrt{2}}\right)\left(\frac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}}\right)\\ &= \frac{12 + 8\sqrt{4(2)}}{9 - 8}\\ &= 12 + 16\sqrt{2}\end{aligned}$$

14. In a golden rectangle, the ratio of the side dimensions is $\frac{2}{\sqrt{5} - 1}$. Determine an equivalent expression with a rational denominator.

15. The period, T, in seconds, of a pendulum is related to its length, L, in metres. The period is the time to complete one full cycle and can be approximated with the formula $T = 2\pi\sqrt{\frac{L}{10}}$.

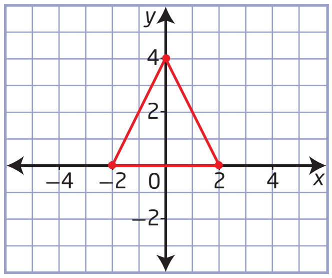
   a) Write an equivalent formula with a rational denominator.

   b) The length of the pendulum in the HSBC building in downtown Vancouver is 27 m. How long would the pendulum take to complete 3 cycles?

Did You Know?

The pendulum in the HSBC building in downtown Vancouver has a mass of approximately 1600 kg. It is made from buffed aluminum and is assisted at the top by a hydraulic mechanical system.

16. Jonasie and Iblauk are planning a skidoo race for their community of Uqsuqtuuq or Gjoa Haven, Nunavut. They sketch the triangular course on a Cartesian plane. The area of 1 grid square represents 9245 m². What is the exact length of the red track?



17. Simplify $\left(\frac{1 + \sqrt{5}}{2 - \sqrt{3}}\right)\left(\frac{1 - \sqrt{5}}{2 - \sqrt{3}}\right)$.

18. In a scale model of a cube, the ratio of the volume of the model to the volume of the cube is 1:4. Express your answers to each of the following questions as mixed radicals in simplest form.

   a) What is the edge length of the actual cube if its volume is 192 mm³?

   b) What is the edge length of the model cube?

   c) What is the ratio of the edge length of the actual cube to the edge length of the model cube?

19. Lev simplifies the expression, $\frac{2x\sqrt{14}}{\sqrt{3 - 5x}}$. He determines the restrictions on the values for x as follows:

$$\begin{aligned} 3 - 5x &> 0 \\ -5x &> -3 \\ x &> \frac{3}{5} \end{aligned}$$

   a) Identify, explain, and correct any errors.

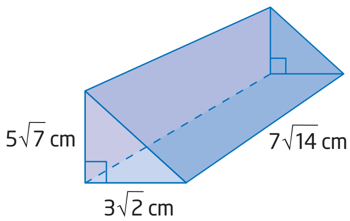
   b) Why do variables involved in radical expressions sometimes have restrictions on their values?

   c) Create an expression involving radicals that does not have any restrictions. Justify your response.

20. Olivia simplifies the following expression. Identify, explain, and correct any errors in her work.

$$\begin{aligned}\frac{2c - c\sqrt{25}}{\sqrt{3}} &= \frac{(2c - c\sqrt{25})}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)\\ &= \frac{\sqrt{3}(2c - c\sqrt{25})}{3}\\ &= \frac{\sqrt{3}(2c \pm 5c)}{3}\\ &= \frac{\sqrt{3}(7c)}{3} \quad \text{or} \quad \frac{\sqrt{3}(-3c)}{3}\\ &= \frac{7c\sqrt{3}}{3} \quad \text{or} \quad -c\sqrt{3}\end{aligned}$$

21. What is the volume of the right triangular prism?



Extend

22. A cube is inscribed in a sphere with radius 1 m. What is the surface area of the cube?

23. Line segment AB has endpoints $A(\sqrt{27}, -\sqrt{50})$ and $B(3\sqrt{48}, 2\sqrt{98})$. What is the midpoint of AB?

24. Rationalize the denominator of $(3(\sqrt{x})^{-1} - 5)^{-2}$. Simplify the expression.

25. a) What are the exact roots of the quadratic equation $x^2 + 6x + 3 = 0$?
    b) What is the sum of the two roots from part a)?
    c) What is the product of the two roots?
    d) How are your answers from parts b) and c) related to the original equation?

26. Rationalize the denominator of $\frac{\sqrt[c]{a}}{\sqrt[n]{r}}$.

27. What is the exact surface area of the right triangular prism in #21?

Create Connections

28. Describe the similarities and differences between multiplying and dividing radical expressions and multiplying and dividing polynomial expressions.

29. How is rationalizing a square-root binomial denominator related to the factors of a difference of squares? Explain, using an example.

30. A snowboarder departs from a jump. The quadratic function that approximately relates height above landing area, h, in metres, and time in air, t, in seconds, is $h(t) = -5t^2 + 10t + 3$.
    a) What is the snowboarder's height above the landing area at the beginning of the jump?
    b) Complete the square of the expression on the right to express the function in vertex form. Isolate the variable t.
    c) Determine the exact height of the snowboarder halfway through the jump.

Did You Know?

Maelle Ricker from North Vancouver won a gold medal at the 2010 Vancouver Olympics in snowboard cross. She is the first Canadian woman to win a gold medal at a Canadian Olympics.

31. Are $m = \frac{-5 + \sqrt{13}}{6}$ and $m = \frac{-5 - \sqrt{13}}{6}$ solutions of the quadratic equation, $3m^2 + 5m + 1 = 0$? Explain your reasoning.

32. Two stacking bowls are in the shape of hemispheres. They have radii that can be represented by $\sqrt[3]{\frac{3V}{2\pi}}$ and $\sqrt[3]{\frac{V-1}{4\pi}}$, where V represents the volume of the bowl.

a) What is the ratio of the larger radius to the smaller radius in simplest form?

b) For which volumes is the ratio a real number?

33. MINI LAB

Step 1 Copy and complete the table of values for each equation using technology.

$y = \sqrt{x}$			$y = x^2$	
x	y		x	y
	0		0	
	1		1	
	2		2	
	3		3	
	4		4	

Step 2 Describe any similarities and differences in the patterns of numbers. Compare your answers with those of a classmate.

Step 3 Plot the points for both functions. Compare the shapes of the two graphs. How are the restrictions on the variable for the radical function related to the quadratic function?

Project Corner — Space Exploration

- Earth has a diameter of about 12 800 km and a mass of about 6.0×10^{24} kg. It is about 150 000 000 km from the Sun.
- Artificial gravity is the emulation in outer space of the effects of gravity felt on a planetary surface.
- When travelling into space, it is necessary to overcome the force of gravity. A spacecraft leaving Earth must reach a gravitational escape velocity greater than 11.2 km/s. Research the formula for calculating the escape velocity. Use the formula to determine the escape velocities for the Moon and the Sun.

Key Ideas

- You can model some real-world relationships with radical equations.
- When solving radical equations, begin by isolating one of the radical terms.
- To eliminate a square root, raise both sides of the equation to the exponent two. For example, in $3 = \sqrt{c + 5}$, square both sides.

$$3^2 = \left(\sqrt{c+5}\right)^2$$
$$9 = c + 5$$
$$4 = c$$

- To identify whether a root is extraneous, substitute the value into the original equation. Raising both sides of an equation to an even exponent may introduce an extraneous root.
- When determining restrictions on the values for variables, consider the following:
  - Denominators cannot be equal to zero.
  - For radicals to be real numbers, radicands must be non-negative if the index is an even number.

Check Your Understanding

Practise

Determine any restrictions on the values for the variable in each radical equation, unless given.

1. Square each expression.

a) $\sqrt{3z}$, $z \geq 0$

b) $\sqrt{x - 4}$, $x \geq 4$

c) $2\sqrt{x + 7}$, $x \geq -7$

d) $-4\sqrt{9 - 2y}$, $\frac{9}{2} \geq y$

2. Describe the steps to solve the equation $\sqrt{x} + 5 = 11$, where $x \geq 0$.

3. Solve each radical equation. Verify your solutions and identify any extraneous roots.

a) $\sqrt{2x} = 3$

b) $\sqrt{-8x} = 4$

c) $7 = \sqrt{5 - 2x}$

4. Solve each radical equation. Verify your solutions.

a) $\sqrt{z} + 8 = 13$

b) $2 - \sqrt{y} = -4$

c) $\sqrt{3x} - 8 = -6$

d) $-5 = 2 - \sqrt{-6m}$

5. In the solution to $k + 4 = \sqrt{-2k}$, identify whether either of the values, $k = -8$ or $k = -2$, is extraneous. Explain your reasoning.

6. Isolate each radical term. Then, solve the equation.

a) $-3\sqrt{n - 1} + 7 = -14$, $n \geq 1$

b) $-7 - 4\sqrt{2x - 1} = 17$, $x \geq \frac{1}{2}$

c) $12 = -3 + 5\sqrt{8 - x}$, $x \leq 8$

7. Solve each radical equation.

a) $\sqrt{m^2 - 3} = 5$

b) $\sqrt{x^2 + 12x} = 8$

c) $\sqrt{\frac{q^2}{2} + 11} = q - 1$

d) $2n + 2\sqrt{n^2 - 7} = 14$

8. Solve each radical equation.

a) $5 + \sqrt{3x - 5} = x$

b) $\sqrt{x^2 + 30x} = 8$

c) $\sqrt{d + 5} = d - 1$

d) $\sqrt{\frac{j + 1}{3}} + 5j = 3j - 1$

9. Solve each radical equation.

a) $\sqrt{2k} = \sqrt{8}$

b) $\sqrt{-3m} = \sqrt{-7m}$

c) $5\sqrt{\frac{j}{2}} = \sqrt{200}$

d) $5 + \sqrt{n} = \sqrt{3n}$

10. Solve.

a) $\sqrt{z + 5} = \sqrt{2z - 1}$

b) $\sqrt{6y - 1} = \sqrt{-17 + y^2}$

c) $\sqrt{5r - 9} - 3 = \sqrt{r + 4} - 2$

d) $\sqrt{x + 19} + \sqrt{x - 2} = 7$

Apply

11. By inspection, determine which one of the following equations will have an extraneous root. Explain your reasoning.

$\sqrt{3y - 1} - 2 = 5$

$4 - \sqrt{m + 6} = -9$

$\sqrt{x + 8} + 9 = 2$

12. The following steps show how Jerry solved the equation $3 + \sqrt{x + 17} = x$. Is his work correct? Explain your reasoning and provide a correct solution if necessary.

Jerry's Solution

$$\begin{aligned} 3 + \sqrt{x + 17} &= x \\ \sqrt{x + 17} &= x - 3 \\ \left(\sqrt{x + 17}\right)^2 &= x^2 - 3^2 \\ x + 17 &= x^2 - 9 \\ 0 &= x^2 - x - 26 \\ x &= \frac{1 \pm \sqrt{1 + 104}}{2} \\ x &= \frac{1 \pm \sqrt{105}}{2} \end{aligned}$$

13. Collision investigators can approximate the initial velocity, v, in kilometres per hour, of a car based on the length, l, in metres, of the skid mark. The formula $v = 12.6\sqrt{l} + 8$, $l \geq 0$, models the relationship. What length of skid is expected if a car is travelling 50 km/h when the brakes are applied? Express your answer to the nearest tenth of a metre.

14. In 1805, Rear-Admiral Beaufort created a numerical scale to help sailors quickly assess the strength of the wind. The integer scale ranges from 0 to 12. The wind scale, B, is related to the wind velocity, v, in kilometres per hour, by the formula $B = 1.33\sqrt{v + 10.0} - 3.49$, $v \geq -10$.

a) Determine the wind scale for a wind velocity of 40 km/h.

b) What wind velocity results in a wind scale of 3?

Web Link

To learn more about the Beaufort scale, go to www.mhrprecalc11.ca and follow the links.

15. The mass, m, in kilograms, that a beam with a fixed width and length can support is related to its thickness, t, in centimetres. The formula is $t = \frac{1}{5}\sqrt{\frac{m}{3}}$, $m \geq 0$. If a beam is 4 cm thick, what mass can it support?

16. Two more than the square root of a number, n, is equal to the number. Model this situation using a radical equation. Determine the value(s) of n algebraically.

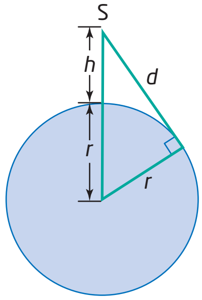
17. The speed, v, in metres per second, of water being pumped into the air to fight a fire is the square root of twice the product of the maximum height, h, in metres, and the acceleration due to gravity. At sea level, the acceleration due to gravity is 9.8 m/s^2.

a) Write the formula that models the relationship between the speed and the height of the water.

b) Suppose the speed of the water being pumped is 30 m/s. What expected height will the spray reach?

c) A local fire department needs to buy a pump that reaches a height of 60 m. An advertisement for a pump claims that it can project water at a speed of 35 m/s. Will this pump meet the department's requirements? Justify your answer.

18. The distance d, in kilometres, to the horizon from a height, h, in kilometres, can be modelled by the formula $d = \sqrt{2rh + h^2}$, where r represents Earth's radius, in kilometres. A spacecraft is 200 km above Earth, at point S. If the distance to the horizon from the spacecraft is 1609 km, what is the radius of Earth?



Extend

19. Solve for a in the equation $\sqrt{3x} = \sqrt{ax} + 2$, $a \geq 0$, $x > 0$.

20. Create a radical equation that results in the following types of solution. Explain how you arrived at your equation.

a) one extraneous solution and no valid solutions

b) one extraneous solution and one valid solution

21. The time, t, in seconds, for an object to fall to the ground is related to its height, h, in metres, above the ground. The formulas for determining this time are $t_m = \sqrt{\frac{h}{1.8}}$ for the moon and $t_E = \sqrt{\frac{h}{4.9}}$ for Earth. The same object is dropped from the same height on both the moon and Earth. If the difference in times for the object to reach the ground is 0.5 s, determine the height from which the object was dropped. Express your answer to the nearest tenth of a metre.

22. Refer to #18. Use the formula $d = \sqrt{2rh + h^2}$ to determine the height of a spacecraft above the moon, where the radius is 1740 km and the distance to the horizon is 610 km. Express your answer to the nearest kilometre.

23. The profit, P, in dollars, of a business can be expressed as $P = -n^2 + 200n$, where n represents the number of employees.

a) What is the maximum profit? How many employees are required for this value?

b) Rewrite the equation by isolating n.

c) What are the restrictions on the radical portion of your answer to part b)?

d) What are the domain and range for the original function? How does your answer relate to part c)?

Create Connections

24. Describe the similarities and differences between solving a quadratic equation and solving a radical equation.

25. Why are extraneous roots sometimes produced when solving radical equations? Include an example and show how the root was produced.

26. An equation to determine the annual growth rate, r, of a population of moose in Wells Gray Provincial Park, British Columbia, over a 3-year period is

$r = -1 + \sqrt[3]{\dfrac{P_f}{P_i}}$, $P_f \geq 0$, $P_i > 0$. In the equation, P_i represents the initial population 5 years ago and P_f represents the final population after 3 years.

a) If $P_i = 320$ and $P_f = 390$, what is the annual growth rate? Express your answer as a percent to the nearest tenth.

b) Rewrite the equation by isolating P_f.

c) Determine the four populations of moose over this 3-year period.

d) What kind of sequence does the set of populations in part c) represent?

27. **MINI LAB** A continued radical is a series of nested radicals that may be infinite but has a finite rational result. Consider the following continued radical:

$$\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}}}$$

Step 1 Using a calculator or spreadsheet software, determine a decimal approximation for the expressions in the table.

Number of Nested Radicals	Expression	Decimal Approximation
1	$\sqrt{6 + \sqrt{6}}$	
2	$\sqrt{6 + \sqrt{6 + \sqrt{6}}}$	
3	$\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6}}}}$	
4		
5		
6		
7		
8		
9		

Step 2 From your table, predict the value of the expression

$$\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}}}.$$

Step 3 Let $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}}}$. Solve the equation algebraically.

Step 4 Check your result with a classmate. Why does one of the roots need to be rejected?

Step 5 Generate another continued radical expression that will result in a finite real-number solution. Does your answer have a rational or an irrational root?

Step 6 Exchange your radical expression in step 5 with a classmate and solve their problem.

Chapter 5 Review

5.1 Working With Radicals, pages 272-281

1. Convert each mixed radical to an entire radical.

a) $8\sqrt{5}$

b) $-2\sqrt[5]{3}$

c) $3y^3\sqrt{7}$

d) $-3z(\sqrt[3]{4z})$

2. Convert each entire radical to a mixed radical in simplest form.

a) $\sqrt{72}$

b) $3\sqrt{40}$

c) $\sqrt{27m^2}$, $m \geq 0$

d) $\sqrt[3]{80x^5y^6}$

3. Simplify.

a) $-\sqrt{13} + 2\sqrt{13}$

b) $4\sqrt{7} - 2\sqrt{112}$

c) $-\sqrt[3]{3} + \sqrt[3]{24}$

4. Simplify radicals and collect like terms. State any restrictions on the values for the variables.

a) $4\sqrt{45x^3} - \sqrt{27x} + 17\sqrt{3x} - 9\sqrt{125x^3}$

b) $\frac{2}{5}\sqrt{44a} + \sqrt{144a^3} - \frac{\sqrt{11a}}{2}$

5. Which of the following expressions is not equivalent to $8\sqrt{7}$?

$2\sqrt{112}$, $\sqrt{448}$, $3\sqrt{42}$, $4\sqrt{28}$

Explain how you know without using technology.

6. Order the following numbers from least to greatest: $3\sqrt{7}$, $\sqrt{65}$, $2\sqrt{17}$, 8

7. The speed, v, in kilometres per hour, of a car before a collision can be approximated from the length, d, in metres, of the skid mark left by the tire. On a dry day, one formula that approximates this speed is $v = \sqrt{169d}$, $d \geq 0$.

a) Rewrite the formula as a mixed radical.

b) What is the approximate speed of a car if the skid mark measures 13.4 m? Express your answer to the nearest kilometre per hour.

8. The city of Yorkton, Saskatchewan, has an area of 24.0 km^2. If this city were a perfect square, what would its exact perimeter be? Express your answer as a mixed radical in simplest form.

9. State whether each equation is true or false. Justify your reasoning.

a) $-3^2 = \pm 9$

b) $(-3)^2 = 9$

c) $\sqrt{9} = \pm 3$

5.2 Multiplying and Dividing Radical Expressions, pages 282-293

10. Multiply. Express each product as a radical in simplest form.

a) $\sqrt{2}(\sqrt{6})$

b) $(-3f\sqrt{15})(2f^3\sqrt{5})$

c) $(\sqrt[4]{8})(3\sqrt[4]{18})$

11. Multiply and simplify. Identify any restrictions on the values for the variable in part c).

a) $(2 - \sqrt{5})(2 + \sqrt{5})$

b) $(5\sqrt{3} - \sqrt{8})^2$

c) $(a + 3\sqrt{a})(a + 7\sqrt{4a})$

12. Are $x = \frac{5 + \sqrt{17}}{2}$ and $x = \frac{5 - \sqrt{17}}{2}$ a conjugate pair? Justify your answer.

Are they solutions of the quadratic equation $x^2 - 5x + 2 = 0$? Explain.

13. Rationalize each denominator.

a) $\dfrac{\sqrt{6}}{\sqrt{12}}$

b) $\dfrac{-1}{\sqrt[3]{25}}$

c) $-4\sqrt{\dfrac{2a^2}{9}}$, $a \geq 0$

14. Rationalize each denominator. State any restrictions on the values for the variables.

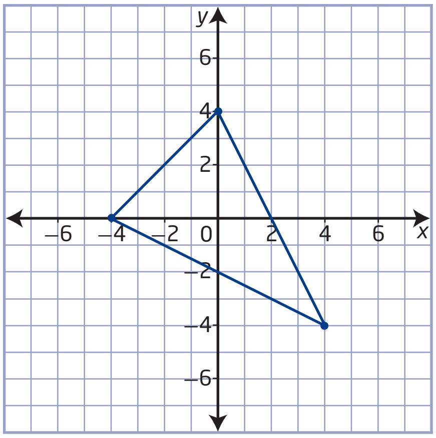
a) $\dfrac{-2}{4 - \sqrt{3}}$

b) $\dfrac{\sqrt{7}}{2\sqrt{5} - \sqrt{7}}$

c) $\dfrac{18}{6 + \sqrt{27m}}$

d) $\dfrac{a + \sqrt{b}}{a - \sqrt{b}}$

15. What is the exact perimeter of the triangle?



16. Simplify.

a) $\left(\dfrac{-5\sqrt{3}}{\sqrt{6}}\right)\left(\dfrac{-\sqrt{7}}{3\sqrt{21}}\right)$

b) $\left(\dfrac{2a\sqrt{a^3}}{9}\right)\left(\dfrac{12}{-\sqrt{8a}}\right)$

17. The area of a rectangle is 12 square units. The width is $(4 - \sqrt{2})$ units. Determine an expression for the length of the rectangle in simplest radical form.

5.3 Radical Equations, pages 294–303

18. Identify the values of x for which the radicals are defined. Solve for x and verify your answers.

a) $-\sqrt{x} = -7$

b) $\sqrt{4 - x} = -2$

c) $5 - \sqrt{2x} = -1$

d) $1 + \sqrt{\dfrac{7x}{3}} = 8$

19. Solve each radical equation. Determine any restrictions on the values for the variables.

a) $\sqrt{5x - 3} = \sqrt{7x - 12}$

b) $\sqrt{y - 3} = y - 3$

c) $\sqrt{7n + 25} - n = 1$

d) $\sqrt{8 - \dfrac{m}{3}} = \sqrt{3m} - 4$

e) $\sqrt[3]{3x - 1} + 7 = 3$

20. Describe the steps in your solution to #19c). Explain why one of the roots was extraneous.

21. On a calm day, the distance, d, in kilometres, that the coast guard crew on the Coast Guard cutter Vakta can see to the horizon depends on their height, h, in metres, above the water. The formula $d = \sqrt{\dfrac{3h}{2}}$, $h \geq 0$ models this relationship. What is the height of the crew above the water if the distance to the horizon is 7.1 km?

Did You Know?

The Vakta is a 16.8-m cutter used by Search and Rescue to assist in water emergencies on Lake Winnipeg. It is based in Gimli, Manitoba.

Chapter 5 Practice Test

Multiple Choice

For #1 to #6, choose the best answer.

1. What is the entire radical form of $-3(\sqrt[3]{2})$?

A $\sqrt[3]{54}$ **B** $\sqrt[3]{-54}$

C $\sqrt[3]{-18}$ **D** $\sqrt[3]{18}$

2. What is the condition on the variable in $2\sqrt{-7n}$ for the radicand to be a real number?

A $n \geq 7$ **B** $n \leq -7$

C $n \geq 0$ **D** $n \leq 0$

3. What is the simplest form of the sum $-2x\sqrt{6x} + 5x\sqrt{6x}$, $x \geq 0$?

A $3\sqrt{6x}$ **B** $6\sqrt{12x}$

C $3x\sqrt{6x}$ **D** $6x\sqrt{12}$

4. What is the product of $\sqrt{540}$ and $\sqrt{6y}$, $y \geq 0$, in simplest form?

A $3y\sqrt{360}$ **B** $6y\sqrt{90}$

C $10\sqrt{32y}$ **D** $18\sqrt{10y}$

5. Determine any root of the equation, $x + 7 = \sqrt{23 - x}$, where $x \leq 23$.

A $x = 13$

B $x = -2$

C $x = 2$ and 13

D $x = -2$ and -13

6. Suppose $\frac{5}{7}\sqrt{\frac{3}{2}}$ is written in simplest form as $a\sqrt{b}$, where a is a real number and b is an integer. What is the value of b?

A 2 **B** 3

C 6 **D** 14

Short Answer

7. Order the following numbers from least to greatest:

$3\sqrt{11}$, $5\sqrt{6}$, $9\sqrt{2}$, $\sqrt{160}$

8. Express as a radical in simplest form.

$\dfrac{(2\sqrt{5n})(3\sqrt{8n})}{1 - 12\sqrt{2}}$, $n \geq 0$.

9. Solve $3 - x = \sqrt{x^2 - 5}$. State any extraneous roots that you found. Identify the values of x for which the radical is defined.

10. Solve $\sqrt{9y + 1} = 3 + \sqrt{4y - 2}$, $y \geq \frac{1}{2}$. Verify your solution. Justify your method. Identify any extraneous roots that you found.

11. Masoud started to simplify $\sqrt{450}$ by rewriting 450 as a product of prime factors: $\sqrt{2(3)(3)(5)(5)}$. Explain how he can convert his expression to a mixed radical.

12. For sailboats to travel into the wind, it is sometimes necessary to tack, or move in a zigzag pattern. A sailboat in Lake Winnipeg travels 4 km due north and then 4 km due west. From there the boat travels 5 km due north and then 5 km due west. How far is the boat from its starting point? Express your answer as a mixed radical.

13. You wish to rationalize the denominator in each expression. By what number will you multiply each expression? Justify your answer.

a) $\dfrac{4}{\sqrt{6}}$

b) $\dfrac{22}{\sqrt{y - 3}}$

c) $\dfrac{2}{\sqrt[3]{7}}$

14. For diamonds of comparable quality, the cost, C, in dollars, is related to the mass, m, in carats, by the formula $m = \sqrt{\dfrac{C}{700}}$, $C \geq 0$. What is the cost of a 3-carat diamond?

Did You Know?

Snap Lake Mine is 220 km northeast of Yellowknife, Northwest Territories. It is the first fully underground diamond mine in Canada.

15. Teya tries to rationalize the denominator in the expression $\dfrac{5\sqrt{2} + \sqrt{3}}{4\sqrt{2} - \sqrt{3}}$. Is Teya correct? If not, identify and explain any errors she made.

Teya's Solution

$$\begin{aligned}\frac{5\sqrt{2} + \sqrt{3}}{4\sqrt{2} - \sqrt{3}} &= \left(\frac{5\sqrt{2} + \sqrt{3}}{4\sqrt{2} - \sqrt{3}}\right)\left(\frac{4\sqrt{2} + \sqrt{3}}{4\sqrt{2} + \sqrt{3}}\right) \\ &= \frac{20\sqrt{4} + 5\sqrt{6} + 4\sqrt{6} + \sqrt{9}}{32 - 3} \\ &= \frac{40 + 9\sqrt{6} + 3}{32 - 3} \\ &= \frac{43 + 9\sqrt{6}}{29}\end{aligned}$$

16. A right triangle has one leg that measures 1 unit.

a) Model the length of the hypotenuse using a radical equation.

b) The length of the hypotenuse is 11 units. What is the length of the unknown leg? Express your answer as a mixed radical in simplest form.

Extended Response

17. A 100-W light bulb operates with a current of 0.5 A. The formula relating current, I, in amperes (A); power, P, in watts (W); and resistance, R, in ohms (Ω), is $I = \sqrt{\dfrac{P}{R}}$.

a) Isolate R in the formula.

b) What is the resistance in the light bulb?

18. Sylvie built a model of a cube-shaped house.

a) Express the edge length of a cube in terms of the surface area using a radical equation.

b) Suppose the surface area of the cube in Sylvie's model is 33 cm^2. Determine the exact edge length in simplest form.

c) If the surface area of a cube doubles, by what scale factor will the edge length change?

19. Beverley invested \$3500 two years ago. The investment earned compound interest annually according to the formula $A = P(1 + i)^n$. In the formula, A represents the final amount of the investment, P represents the principal or initial amount, i represents the interest rate per compounding period, and n represents the number of compounding periods. The current amount of her investment is \$3713.15.

a) Model Beverley's investment using the formula.

b) What is the interest rate? Express your answer as a percent.

Key Ideas

- A rational expression is an algebraic fraction of the form $\frac{p}{q}$, where p and q are polynomials and $q \neq 0$.
- A non-permissible value is a value of the variable that causes an expression to be undefined. For a rational expression, this occurs when the denominator is zero.
- Rational expressions can be simplified by:
  - factoring the numerator and the denominator
  - determining non-permissible values
  - dividing both the numerator and denominator by all common factors

Check Your Understanding

Practise

1. What should replace $\blacksquare$ to make the expressions in each pair equivalent?

a) $\frac{3}{5}, \frac{\blacksquare}{30}$

b) $\frac{2}{5}, \frac{\blacksquare}{35x}, x \neq 0$

c) $\frac{4}{\blacksquare}, \frac{44}{77}$

d) $\frac{x + 2}{x - 3}, \frac{4x + 8}{\blacksquare}, x \neq 3$

e) $\frac{3(6)}{\blacksquare(6)}, \frac{3}{8}$

f) $\frac{1}{y - 2}, \frac{\blacksquare}{y^2 - 4}, y \neq \pm 2$

2. State the operation and quantity that must be applied to both the numerator and the denominator of the first expression to obtain the second expression.

a) $\frac{3p^2q}{pq^2}, \frac{3p}{q}$

b) $\frac{2}{x + 4}, \frac{2x - 8}{x^2 - 16}$

c) $\frac{-4(m - 3)}{m^2 - 9}, \frac{-4}{m + 3}$

d) $\frac{1}{y - 1}, \frac{y^2 + y}{y^3 - y}$

3. What value(s) of the variable, if any, make the denominator of each expression equal zero?

a) $\frac{-4}{x}$

b) $\frac{3c - 1}{c - 1}$

c) $\frac{y}{y + 5}$

d) $\frac{m + 3}{5}$

e) $\frac{1}{d^2 - 1}$

f) $\frac{x - 1}{x^2 + 1}$

4. Determine the non-permissible value(s) for each rational expression. Why are these values not permitted?

a) $\frac{3a}{4 - a}$

b) $\frac{2e + 8}{e}$

c) $\frac{3(y + 7)}{(y - 4)(y + 2)}$

d) $\frac{-7(r - 1)}{(r - 1)(r + 3)}$

e) $\frac{2k + 8}{k^2}$

f) $\frac{6x - 8}{(3x - 4)(2x + 5)}$

5. What value(s) for the variables must be excluded when working with each rational expression?

a) $\frac{4\pi r^2}{8\pi r^3}$

b) $\frac{2t + t^2}{t^2 - 1}$

c) $\frac{x - 2}{10 - 5x}$

d) $\frac{3g}{g^3 - 9g}$

6. Simplify each rational expression. State any non-permissible values for the variables.

a) $\frac{2c(c - 5)}{3c(c - 5)}$

b) $\frac{3w(2w + 3)}{2w(3w + 2)}$

c) $\frac{(x - 7)(x + 7)}{(2x - 1)(x - 7)}$

d) $\frac{5(a - 3)(a + 2)}{10(3 - a)(a + 2)}$

7. Consider the rational expression $\frac{x^2 - 1}{x^2 + 2x - 3}$.

a) Explain why you cannot divide out the x^2 in the numerator with the x^2 in the denominator.

b) Explain how to determine the non-permissible values. State the non-permissible values.

c) Explain how to simplify a rational expression. Simplify the rational expression.

8. Write each rational expression in simplest form. State any non-permissible values for the variables.

a) $\frac{6r^2p^3}{4rp^4}$

b) $\frac{3x - 6}{10 - 5x}$

c) $\frac{b^2 + 2b - 24}{2b^2 - 72}$

d) $\frac{10k^2 + 55k + 75}{20k^2 - 10k - 150}$

e) $\frac{x - 4}{4 - x}$

f) $\frac{5(x^2 - y^2)}{x^2 - 2xy + y^2}$

Apply

9. Since $\frac{x^2 + 2x - 15}{x - 3}$ can be written as $\frac{(x - 3)(x + 5)}{x - 3}$, you can say that $\frac{x^2 + 2x - 15}{x - 3}$ and $x + 5$ are equivalent expressions. Is this statement always, sometimes, or never true? Explain.

10. Explain why 6 may not be the only non-permissible value for a rational expression that is written in simplest form as $\frac{y}{y - 6}$. Give examples to support your answer.

11. Mike always looks for shortcuts. He claims, "It is easy to simplify expressions such as $\frac{5 - x}{x - 5}$ because the top and bottom are opposites of each other and any time you divide opposites the result is −1." Is Mike correct? Explain why or why not.

12. Suppose you are tutoring a friend in simplifying rational expressions. Create three sample expressions written in the form $\frac{ax^2 + bx + c}{dx^2 + ex + f}$ where the numerators and denominators factor and the expressions can be simplified. Describe the process you used to create one of your expressions.

13. Shali incorrectly simplifies a rational expression as shown below.

$$\begin{aligned}\frac{g^2 - 4}{2g - 4} &= \frac{(g - 2)(g + 2)}{2(g - 2)} \\ &= \frac{g + 2}{2} \\ &= g + 1\end{aligned}$$

What is Shali's error? Explain why the step is incorrect. Show the correct solution.

14. Create a rational expression with variable p that has non-permissible values of 1 and −2.

15. The distance, d, can be determined using the formula $d = rt$, where r is the rate of speed and t is the time.

a) If the distance is represented by $2n^2 + 11n + 12$ and the rate of speed is represented by $2n^2 - 32$, what is an expression for the time?

b) Write your expression from part a) in simplest form. Identify any non-permissible values.

16. You have been asked to draw the largest possible circle on a square piece of paper. The side length of the piece of paper is represented by $2x$.

a) Draw a diagram showing your circle on the piece of paper. Label your diagram.

b) Create a rational expression comparing the area of your circle to the area of the piece of paper.

c) Identify any non-permissible values for your rational expression.

d) What is your rational expression in simplest form?

e) What percent of the paper is included in your circle? Give your answer to the nearest percent.

17. A chemical company is researching the effect of a new pesticide on crop yields. Preliminary results show that the extra yield per hectare is given by the expression $\frac{900p}{2 + p}$, where p is the mass of pesticide, in kilograms. The extra yield is also measured in kilograms.

a) Explain whether the non-permissible value needs to be considered in this situation.

b) What integral value for p gives the least extra yield?

c) Substitute several values for p and determine what seems to be the greatest extra yield possible.

18. Write an expression in simplest form for the time required to travel 100 km at each rate. Identify any non-permissible values.

a) $2q$ kilometres per hour

b) $(p - 4)$ kilometres per hour

19. A school art class is planning a day trip to the Glenbow Museum in Calgary. The cost of the bus is \$350 and admission is \$9 per student.

a) What is the total cost for a class of 30 students?

b) Write a rational expression that could be used to determine the cost per student if n students go on the trip.

c) Use your expression to determine the cost per student if 30 students go.

Did You Know?

The Glenbow Museum in Calgary is one of western Canada's largest museums. It documents life in western Canada from the 1800s to the present day. Exhibits trace the traditions of the First Nations peoples as well as the hardships of ranching and farming in southern Alberta.

First Nations exhibit at Glenbow Museum

20. Terri believes that $\frac{5}{m + 5}$ can be expressed in simplest form as $\frac{1}{m + 1}$.

a) Do you agree with Terri? Explain in words.

b) Use substitution to show whether $\frac{5}{m + 5}$ and $\frac{1}{m + 1}$ are equivalent or not.

21. Sometimes it is useful to write more complicated equivalent rational expressions. For example, $\frac{3x}{4}$ is equivalent to $\frac{15x}{20}$ and to $\frac{3x^2 - 6x}{4x - 8}$, $x \neq 2$.

a) How can you change $\frac{3x}{4}$ into its equivalent form, $\frac{15x}{20}$?

b) What do you need to do to $\frac{3x}{4}$ to get $\frac{3x^2 - 6x}{4x - 8}$?

22. Write a rational expression equivalent to $\frac{x - 2}{3}$ that has

a) a denominator of 12

b) a numerator of $3x - 6$

c) a denominator of $6x + 15$

23. Write a rational expression that satisfies each set of conditions.

a) equivalent to 5, with $5b$ as the denominator

b) equivalent to $\frac{x + 1}{3}$, with a denominator of $12a^2b$

c) equivalent to $\frac{a - b}{7x}$, with a numerator of $2b - 2a$

24. The area of right △PQR is $(x^2 - x - 6)$ square units, and the length of side PQ is $(x - 3)$ units. Side PR is the hypotenuse.

a) Draw a diagram of △PQR.

b) Write an expression for the length of side QR. Express your answer in simplest form.

c) What are the non-permissible values?

25. The work shown to simplify each rational expression contains at least one error. Rewrite each solution, correcting the errors.

a)
$$\begin{aligned}\frac{6x^2 - x - 1}{9x^2 - 1} &= \frac{(2x + 1)(3x - 1)}{(3x + 1)(3x - 1)}\\ &= \frac{2x + 1}{3x + 1}, x \neq -\frac{1}{3}\end{aligned}$$

b)
$$\begin{aligned}\frac{2n^2 + n - 15}{5n - 2n^2} &= \frac{(n + 3)(2n - 5)}{n(5 - 2n)}\\ &= \frac{(n + 3)(2n - 5)}{-n(2n - 5)}\\ &= \frac{n + 3}{-n}\\ &= \frac{n - 3}{n}, n \neq 0, \frac{5}{2}\end{aligned}$$

Extend

26. Write in simplest form. Identify any non-permissible values.

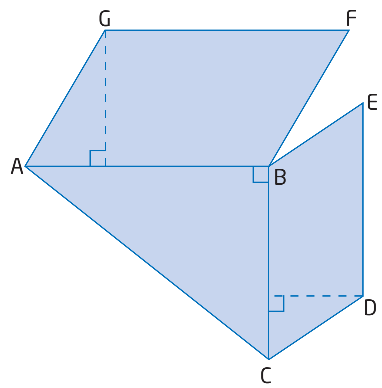
a) $\dfrac{(x + 2)^2 - (x + 2) - 20}{x^2 - 9}$

b) $\dfrac{4(x^2 - 9)^2 - (x + 3)^2}{x^2 + 6x + 9}$

c) $\dfrac{(x^2 - x)^2 - 8(x^2 - x) + 12}{(x^2 - 4)^2 - (x - 2)^2}$

d) $\dfrac{(x^2 + 4x + 4)^2 - 10(x^2 + 4x + 4) + 9}{(2x + 1)^2 - (x + 2)^2}$

27. Parallelogram ABFG has an area of $(16x^2 - 1)$ square units and a height of $(4x - 1)$ units. Parallelogram BCDE has an area of $(6x^2 - x - 12)$ square units and a height of $(2x - 3)$ units. What is an expression for the area of △ABC? Leave your answer in the form $ax^2 + bx + c$. What are the non-permissible values?



28. Carpet sellers need to know if a partial roll contains enough carpet to complete an order. Your task is to create an expression that gives the approximate length of carpet on a roll using measurements from the end of the roll.

a) Let t represent the thickness of the carpet and L the length of carpet on a roll. What is an expression for the area of the rolled edge of the carpet?

b) Draw a diagram showing the carpet rolled on a centre tube. Label the radius of the centre tube as r and the radius of the entire carpet roll as R. What is an expression for the approximate area of the edge of the carpet on the roll? Write your answer in factored form.

c) Write an expression for the length of carpet on a roll of thickness t. What conditions apply to t, L, R, and r?

Create Connections

29. Write a rational expression using one variable that satisfies the following conditions.

a) The non-permissible values are -2 and 5.

b) The non-permissible values are 1 and -3 and the expression is $\frac{x}{x - 1}$ in simplest form. Explain how you found your expression.

30. Consider the rational expressions $\frac{y - 3}{4}$ and $\frac{2y^2 - 5y - 3}{8y + 4}$, $y \neq -\frac{1}{2}$.

a) Substitute a value for y to show that the two expressions are equivalent.

b) Use algebra to show that the expressions are equivalent.

c) Which approach proves the two expressions are equivalent? Why?

31. Two points on a coordinate grid are represented by A(p, 3) and B($2p + 1$, $p - 5$).

a) Write a rational expression for the slope of the line passing through A and B. Write your answer in simplest form.

b) Determine a value for p such that the line passing through A and B has a negative slope.

c) Describe the line through A and B for any non-permissible value of p.

32. Use examples to show how writing a fraction in lowest terms and simplifying a rational expression involve the same mathematical processes.

Check Your Understanding

Practise

1. Simplify each product. Identify all non-permissible values.

   a) $\frac{12m^2f}{5cf} \times \frac{15c}{4m}$

   b) $\frac{3(a-b)}{(a-1)(a+5)} \times \frac{(a-5)(a+5)}{15(a-b)}$

   c) $\frac{(y-7)(y+3)}{(2y-3)(2y+3)} \times \frac{4(2y+3)}{(y+3)(y-1)}$

2. Write each product in simplest form. Determine all non-permissible values.

   a) $\frac{d^2-100}{144} \times \frac{36}{d+10}$

   b) $\frac{a+3}{a+1} \times \frac{a^2-1}{a^2-9}$

   c) $\frac{4z^2-25}{2z^2-13z+20} \times \frac{z-4}{4z+10}$

   d) $\frac{2p^2+5p-3}{2p-3} \times \frac{p^2-1}{6p-3} \times \frac{2p-3}{p^2+2p-3}$

3. What is the reciprocal of each rational expression?

   a) $\frac{2}{t}$

   b) $\frac{2x-1}{3}$

   c) $\frac{-8}{3-y}$

   d) $\frac{2p-3}{p-3}$

4. What are the non-permissible values in each quotient?

   a) $\frac{4t^2}{3s} \div \frac{2t}{s^2}$

   b) $\frac{r^2-7r}{r^2-49} \div \frac{3r^2}{r+7}$

   c) $\frac{5}{n+1} \div \frac{10}{n^2-1} \div (n-1)$

5. What is the simplified product of $\frac{2x-6}{x+3}$ and $\frac{x+3}{2}$? Identify any non-permissible values.

6. What is the simplified quotient of $\frac{y^2}{y^2-9}$ and $\frac{y}{y-3}$? Identify any non-permissible values.

7. Show how to simplify each rational expression or product.

   a) $\frac{3-p}{p-3}$

   b) $\frac{7k-1}{3k} \times \frac{1}{1-7k}$

8. Express each quotient in simplest form. Identify all non-permissible values.

   a) $\frac{2w^2-w-6}{3w+6} \div \frac{2w+3}{w+2}$

   b) $\frac{v-5}{v} \div \frac{v^2-2v-15}{v^3}$

   c) $\frac{9x^2-1}{x+5} \div \frac{3x^2-5x-2}{2-x}$

   d) $\frac{8y^2-2y-3}{y^2-1} \div \frac{2y^2-3y-2}{2y-2} \div \frac{3-4y}{y+1}$

9. Explain why the non-permissible values in the quotient $\frac{x-5}{x+3} \div \frac{x+1}{x-2}$ are -3, -1 and 2.

Apply

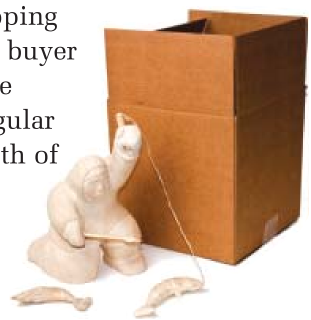
10. The height of a stack of plywood is represented by $\frac{n^2-4}{n+1}$. If the number of sheets is defined by $n-2$, what expression could be used to represent the thickness of one sheet? Express your answer in simplest form.

11. Write an expression involving a product or a quotient of rational expressions for each situation. Simplify each expression.

a) The Mennonite Heritage Village in Steinbach, Manitoba, has a working windmill. If the outer end of a windmill blade turns at a rate of $\frac{x-3}{5}$ metres per minute, how far does it travel in 1 h?

b) A plane travels from Victoria to Edmonton, a distance of 900 km, in $\frac{600}{n+1}$ hours. What is the average speed of the plane?

c) Simione is shipping his carving to a buyer in Winnipeg. He makes a rectangular box with a length of $(2x - 3)$ metres and a width of $(x + 1)$ metres. The volume of the box is $(x^2 + 2x + 1)$ cubic metres. What is an expression for the height of the box?



12. How does the quotient of $\frac{3m+1}{m-1}$ and $\frac{3m+1}{m^2-1}$ compare to the quotient of $\frac{3m+1}{m^2-1}$ and $\frac{3m+1}{m-1}$? Is this always true or sometimes true? Explain your thinking.

13. Simplifying a rational expression is similar to using unit analysis to convert from one unit to another. For example, to convert 68 cm to kilometres, you can use the following steps.

$$(68 \text{ cm})\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)$$
$$= (68 \cancel{\text{cm}}) \times \left(\frac{1 \cancel{\text{m}}}{100 \cancel{\text{cm}}}\right) \times \left(\frac{1 \text{ km}}{1000 \cancel{\text{m}}}\right)$$
$$= \frac{68 \text{ km}}{(100)(1000)}$$
$$= 0.000\ 68 \text{ km}$$

Therefore, 68 cm is equivalent to 0.000 68 km. Create similar ratios that you can use to convert a measurement in yards to its equivalent in centimetres. Use 1 in. = 2.54 cm. Provide a specific example.

14. Tessa is practising for a quiz. Her work on one question is shown below.

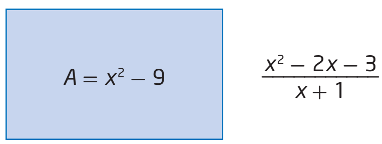
$$\frac{c^2-36}{2c} \div \frac{c+6}{8c^2}$$
$$= \frac{2c}{(c-6)(c+6)} \times \frac{c+6}{(2c)(4c)}$$
$$= \frac{\overset{1}{\cancel{2c}}}{(c-6)\underset{1}{\cancel{(c+6)}}} \times \frac{\overset{1}{\cancel{c+6}}}{\underset{1}{\cancel{(2c)}}(4c)}$$
$$= \frac{1}{4c(c-6)}$$

a) Identify any errors that Tessa made.

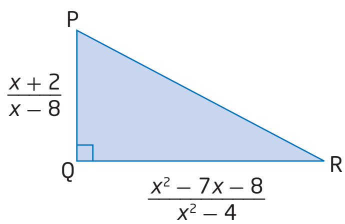
b) Complete the question correctly.

c) How does the correct answer compare with Tessa's answer? Explain.

15. Write an expression to represent the length of the rectangle. Simplify your answer.



16. What is an expression for the area of △PQR? Give your answer in simplest form.



17. You can manipulate variables in a formula by substituting from one formula into another. Try this on the following. Give all answers in simplest form.

a) If $K = \frac{P}{2m}$ and $m = \frac{h}{w}$, express K in terms of P, h, and w.

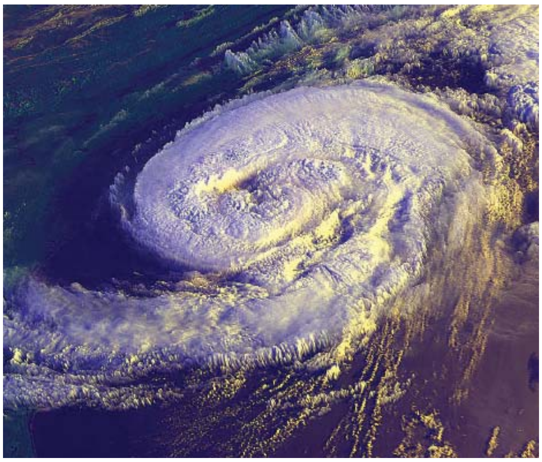
b) If $y = \frac{2\pi}{d}$ and $x = dr$, express y in terms of π, r, and x.

c) Use the formulas $v = wr$ and $a = w^2r$ to determine a formula for a in terms of v and w.

18. The volume, V, of a gas increases or decreases with its temperature, T, according to Charles's law by the formula $\frac{V_1}{V_2} = \frac{T_1}{T_2}$. Determine V_1 if $V_2 = \frac{n^2 - 16}{n - 1}$, $T_1 = \frac{n - 1}{3}$, and $T_2 = \frac{n + 4}{6}$.
Express your answer in simplest form.

Did You Know?

Geostrophic winds are driven by pressure differences that result from temperature differences. Geostrophic winds occur at altitudes above 1000 m.



Hurricane Bonnie

Extend

19. Normally, expressions such as $x^2 - 5$ are not factored. However, you could express $x^2 - 5$ as $(x - \sqrt{5})(x + \sqrt{5})$.

a) Do you agree that $x^2 - 5$ and $(x - \sqrt{5})(x + \sqrt{5})$ are equivalent? Explain why or why not.

b) Show how factoring could be used to simplify the product $\left(\frac{x + \sqrt{3}}{x^2 - 3}\right)\left(\frac{x^2 - 7}{x - \sqrt{7}}\right)$.

c) What is the simplest form of $\frac{x^2 - 7}{x - \sqrt{7}}$ if you rationalize the denominator? How does this answer compare to the value of $\frac{x^2 - 7}{x - \sqrt{7}}$ that you obtained by factoring in part b)?

20. Fog can be cleared from airports, highways, and harbours using fog-dissipating materials. One device for fog dissipation launches canisters of dry ice to a height defined by $\frac{V^2 \sin x}{2g}$, where V is the exit velocity of the canister, x is the angle of elevation, and g is the acceleration due to gravity.

a) What approximate height is achieved by a canister with $V = 85$ m/s, $x = 52°$, and $g = 9.8$ m/s^2?

b) What height can be achieved if $V = \frac{x + 3}{x - 5}$ metres per second and $x = 30°$?

Fog over Vancouver

Create Connections

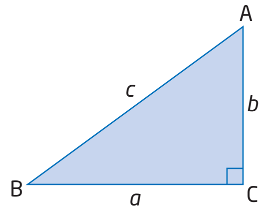
21. Multiplying and dividing rational expressions is very much like multiplying and dividing rational numbers. Do you agree or disagree with this statement? Support your answer with examples.

22. Two points on a coordinate grid are represented by $M(p - 1, 2p + 3)$ and $N(2p - 5, p + 1)$.

a) What is a simplified rational expression for the slope of the line passing through M and N?

b) Write a rational expression for the slope of any line that is perpendicular to MN.

23. Consider △ABC as shown.



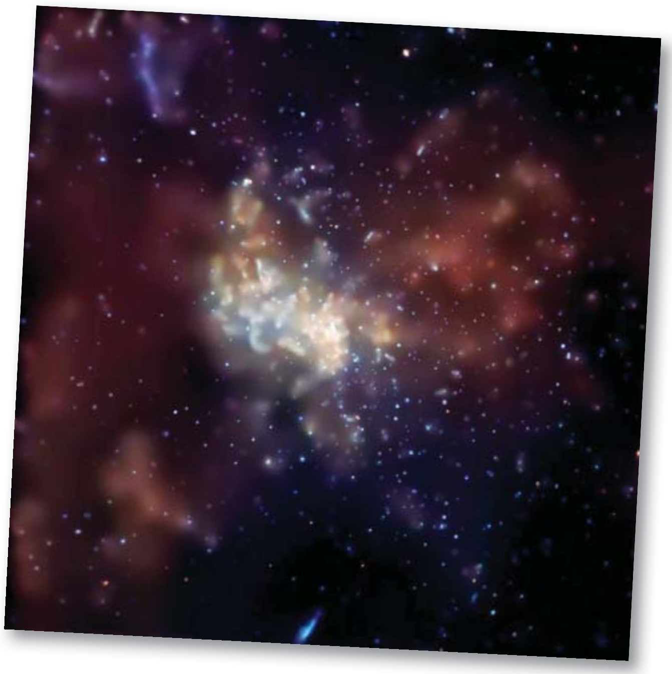
a) What is an expression for tan B?

b) What is an expression for $\frac{\sin B}{\cos B}$? Use your knowledge of rational expressions to help you write the answer in simplest form.

c) How do your expressions for tan B and $\frac{\sin B}{\cos B}$ compare? What can you conclude from this exercise?

Project Corner — Space Anomalies

- Time dilation is a phenomenon described by the theory of general relativity. It is the difference in the rate of the passage of time, and it can arise from the relative velocity of motion between observers and the difference in their distance from a gravitational mass.
- A planetary year is the length of time it takes a planet to revolve around the Sun. An Earth year is about 365 days long.
- There is compelling evidence that a super-massive black hole of more than 4 million solar masses is located near the Sagittarius A* region in the centre of the Milky Way galaxy.
- To predict space weather and the effects of solar activity on Earth, an understanding of both solar flares and coronal mass ejections is needed.
- What other types of information about the universe would be useful when predicting the future of space travel?



Black hole near Sagittarius A*

Check Your Understanding

Practise

1. Add or subtract. Express answers in simplest form. Identify any non-permissible values.
   a) $\frac{11x}{6} - \frac{4x}{6}$
   b) $\frac{7}{x} + \frac{3}{x}$
   c) $\frac{5t + 3}{10} + \frac{3t + 5}{10}$
   d) $\frac{m^2}{m + 1} + \frac{m}{m + 1}$
   e) $\frac{a^2}{a - 4} - \frac{a}{a - 4} - \frac{12}{a - 4}$

2. Show that x and $\frac{3x - 7}{9} + \frac{6x + 7}{9}$ are equivalent expressions.

3. Simplify. Identify all non-permissible values.
   a) $\frac{1}{(x - 3)(x + 1)} - \frac{4}{(x + 1)}$
   b) $\frac{x - 5}{x^2 + 8x - 20} + \frac{2x + 1}{x^2 - 4}$

4. Identify two common denominators for each question. What is the LCD in each case?
   a) $\frac{x - 3}{6} - \frac{x - 2}{4}$
   b) $\frac{2}{5ay^2} + \frac{3}{10a^2y}$
   c) $\frac{4}{9 - x^2} - \frac{7}{3 + x}$

5. Add or subtract. Give answers in simplest form. Identify all non-permissible values.
   a) $\frac{1}{3a} + \frac{2}{5a}$
   b) $\frac{3}{2x} + \frac{1}{6}$
   c) $4 - \frac{6}{5x}$
   d) $\frac{4z}{xy} - \frac{9x}{yz}$
   e) $\frac{2s}{5t^2} + \frac{1}{10t} - \frac{6}{15t^3}$
   f) $\frac{6xy}{a^2b} - \frac{2x}{ab^2y} + 1$

6. Add or subtract. Give answers in simplest form. Identify all non-permissible values.
   a) $\frac{8}{x^2 - 4} - \frac{5}{x + 2}$
   b) $\frac{1}{x^2 - x - 12} + \frac{3}{x + 3}$
   c) $\frac{3x}{x + 2} - \frac{x}{x - 2}$
   d) $\frac{5}{y + 1} - \frac{1}{y} - \frac{y - 4}{y^2 + y}$
   e) $\frac{2h}{h^2 - 9} + \frac{h}{h^2 + 6h + 9} - \frac{3}{h - 3}$
   f) $\frac{2}{x^2 + x - 6} + \frac{3}{x^3 + 2x^2 - 3x}$

7. Simplify each rational expression, and then add or subtract. Express answers in simplest form. Identify all non-permissible values.
   a) $\frac{3x + 15}{x^2 - 25} + \frac{4x^2 - 1}{2x^2 + 9x - 5}$
   b) $\frac{2x}{x^3 + x^2 - 6x} - \frac{x - 8}{x^2 - 5x - 24}$
   c) $\frac{n + 3}{n^2 - 5n + 6} + \frac{6}{n^2 - 7n + 12}$
   d) $\frac{2w}{w^2 + 5w + 6} - \frac{w - 6}{w^2 + 6w + 8}$

Apply

8. Linda has made an error in simplifying the following. Identify the error and correct the answer.

$$\frac{6}{x - 2} + \frac{4}{x^2 - 4} - \frac{7}{x + 2}$$
$$= \frac{6(x + 2) + 4 - 7(x - 2)}{(x - 2)(x + 2)}$$
$$= \frac{6x + 12 + 4 - 7x - 14}{(x - 2)(x + 2)}$$
$$= \frac{-x + 2}{(x - 2)(x + 2)}$$

9. Can the rational expression $\frac{-x + 5}{(x - 5)(x + 5)}$ be simplified further? Explain.

10. Simplify. State any non-permissible values.

a) $\dfrac{2 - \dfrac{6}{x}}{1 - \dfrac{9}{x^2}}$

b) $\dfrac{\dfrac{3}{2} + \dfrac{3}{t}}{\dfrac{t}{t + 6} - \dfrac{1}{t}}$

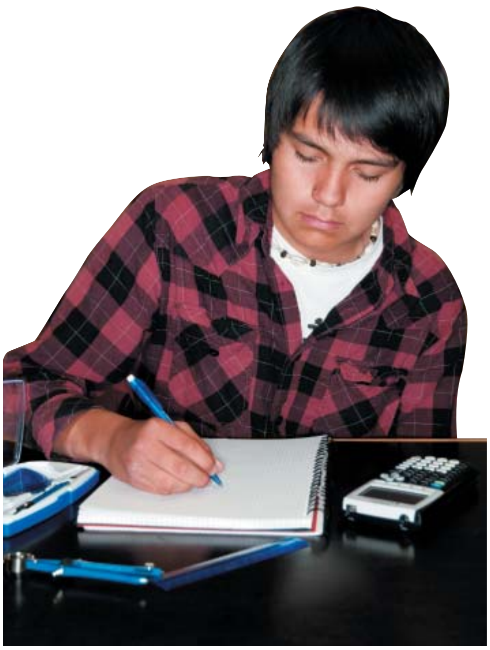
c) $\dfrac{\dfrac{3}{m} - \dfrac{3}{2m + 3}}{\dfrac{3}{m^2} + \dfrac{1}{2m + 3}}$

d) $\dfrac{\dfrac{1}{x + 4} + \dfrac{1}{x - 4}}{\dfrac{x}{x^2 - 16} + \dfrac{1}{x + 4}}$

11. Calculators often perform calculations in a different way to accommodate the machine's logic. For each pair of rational expressions, show that the second expression is equivalent to the first one.

a) $\dfrac{A}{B} + \dfrac{C}{D}$; $\dfrac{\dfrac{AD}{B} + C}{D}$

b) $AB + CD + EF$; $\left[\dfrac{\left(\dfrac{AB}{D} + C\right)D}{F} + E\right]F$



12. A right triangle has legs of length $\dfrac{x}{2}$ and $\dfrac{x - 1}{4}$. If all measurements are in the same units, what is a simplified expression for the length of the hypotenuse?

13. Ivan is concerned about an underweight calf. He decides to put the calf on a healthy growth program. He expects the calf to gain m kilograms per week and 200 kg in total. However, after some time on the program, Ivan finds that the calf has been gaining $(m + 4)$ kilograms per week.

a) Explain what each of the following rational expressions tells about the situation: $\dfrac{200}{m}$ and $\dfrac{200}{m + 4}$.

b) Write an expression that shows the difference between the number of weeks Ivan expected to have the calf on the program and the number of weeks the calf actually took to gain 200 kg.

c) Simplify your rational expression from part b). Does your simplified expression still represent the difference between the expected and actual times the calf took to gain 200 kg? Explain how you know.

14. Suppose you can type an average of n words per minute.

a) What is an expression for the number of minutes it would take to type an assignment with 200 words?

b) Write a sum of rational expressions to represent the time it would take you to type three assignments of 200, 500, and 1000 words, respectively.

c) Simplify the sum in part b). What does the simplified rational expression tell you?

d) Suppose your typing speed decreases by 5 words per minute for each new assignment. Write a rational expression to represent how much longer it would take to type the three assignments. Express your answer in simplest form.

15. Simplify. Identify all non-permissible values.

a) $\dfrac{x-2}{x+5}+\dfrac{x^2-2x-3}{x^2-x-6}\times\dfrac{x^2+2x}{x^2-4x}$

b) $\dfrac{2x^2-x}{x^2+3x}\times\dfrac{x^2-x-12}{2x^2-3x+1}-\dfrac{x-1}{x+2}$

c) $\dfrac{x-2}{x+5}-\dfrac{x^2-2x-3}{x^2-x-6}\times\dfrac{x^2+2x}{x^2-4x}$

d) $\dfrac{x+1}{x+6}-\dfrac{x^2-4}{x^2+2x}\div\dfrac{2x^2+7x+3}{2x^2+x}$

16. A cyclist rode the first 20-kilometre portion of her workout at a constant speed. For the remaining 16-kilometre portion of her workout, she reduced her speed by 2 km/h. Write an algebraic expression for the total time of her bike ride.

17. Create a scenario involving two or more rational expressions. Use your expressions to create a sum or difference. Simplify. Explain what the sum or difference represents in your scenario. Exchange your work with another student in your class. Check whether your classmate's work is correct.

18. Math teachers can identify common errors that students make when adding or subtracting rational expressions. Decide whether each of the following statements is correct or incorrect. Fix each incorrect statement. Indicate how you could avoid making each error.

a) $\dfrac{a}{b}-\dfrac{b}{a}=\dfrac{a-b}{ab}$

b) $\dfrac{ca+cb}{c+cd}=\dfrac{a+b}{d}$

c) $\dfrac{a}{4}-\dfrac{6-b}{4}=\dfrac{a-6-b}{4}$

d) $\dfrac{1}{1-\dfrac{a}{b}}=\dfrac{b}{1-a}$

e) $\dfrac{1}{a-b}=\dfrac{-1}{a+b}$

19. Keander thinks that you can split up a rational expression by reversing the process for adding or subtracting fractions with common denominators. One example is shown below.

$$\begin{aligned}\frac{3x-7}{x}&=\frac{3x}{x}-\frac{7}{x}\\&=3-\frac{7}{x}\end{aligned}$$

a) Do you agree with Keander? Explain.

b) Keander also claims that by using this method, you can arrive at the rational expressions that were originally added or subtracted. Do you agree or disagree with Keander? Support your decision with several examples.

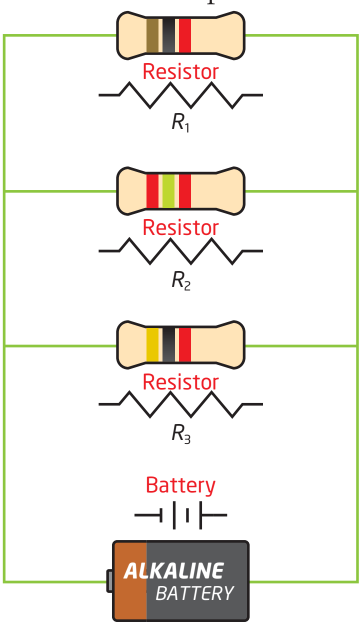
20. A formula for the total resistance, R, in ohms (Ω), of an electric circuit with three resistors in parallel is $R = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$, where R_1 is the resistance of the first resistor, R_2 is the resistance of the second resistor, and R_3 is the resistance of the third resistor, all in ohms.

a) What is the total resistance if the resistances of the three resistors are 2 Ω, 3 Ω, and 4 Ω, respectively?

b) Express the right side of the formula in simplest form.

c) Find the total resistance for part a) using your new expression from part b).

d) Which expression for R did you find easier to use? Explain.



Did You Know?

The English physicist James Joule (1818–1889) showed experimentally that the resistance, R, of a resistor can be calculated as $R = \frac{P}{I^2}$, where P is the power, in watts (W), dissipated by the resistor and I is the current, in amperes (A), flowing through the resistor. This is known as Joule's law. A resistor has a resistance of 1 Ω if it dissipates energy at the rate of 1 W when the current is 1 A.

Extend

21. Suppose that $\frac{a}{c} = \frac{b}{d}$, where a, b, c, and d are real numbers. Use both arithmetic and algebra to show that $\frac{a}{c} = \frac{a - b}{c - d}$ is true.

22. Two points on a coordinate grid are represented by $\mathrm{A}\left(\frac{p - 1}{2}, \frac{p}{3}\right)$ and $\mathrm{B}\left(\frac{p}{3}, \frac{2p - 3}{4}\right)$.

a) What is a simplified rational expression for the slope of the line passing through A and B?

b) What can you say about the slope of AB when $p = 3$? What does this tell you about the line through A and B?

c) Determine whether the slope of AB is positive or negative when $p < 3$ and p is an integer.

d) Predict whether the slope of AB is positive or negative when $p > 3$ and p is an integer. Check your prediction using $p = 4, 5, 6, \ldots, 10$. What did you find?

23. What is the simplified value of the following expression?

$$\left(\frac{p}{p - x} + \frac{q}{q - x} + \frac{r}{r - x}\right) - \left(\frac{x}{p - x} + \frac{x}{q - x} + \frac{x}{r - x}\right)$$

Create Connections

24. Adding or subtracting rational expressions follows procedures that are similar to those for adding or subtracting rational numbers. Show that this statement is true for expressions with and without common denominators.

25. Two students are asked to find a fraction halfway between two given fractions. After thinking for a short time, one of the students says, "That's easy. Just find the average."

a) Show whether the student's suggestion is correct using arithmetic fractions.

b) Determine a rational expression halfway between $\frac{3}{a}$ and $\frac{7}{2a}$. Simplify your answer. Identify any non-permissible values.

26. Mila claims you can add fractions with the same numerator using a different process.

$$\frac{1}{4} + \frac{1}{3} = \frac{1}{\frac{12}{7}}$$
$$= \frac{7}{12}$$

where the denominator in the first step is calculated as $\frac{4 \times 3}{4 + 3}$.

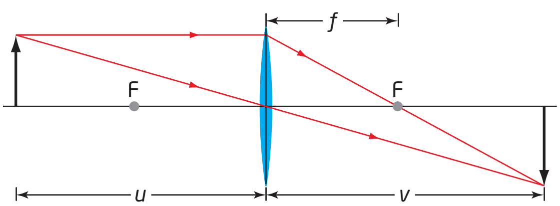
Is Mila's method correct? Explain using arithmetic and algebraic examples.

27. An image found by a convex lens is described by the equation $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, where f is the focal length (distance from the lens to the focus), u is the distance from the object to the lens, and v is the distance from the image to the lens. All distances are measured in centimetres.

a) Use the mathematical ideas from this section to show that $\frac{1}{f} = \frac{u + v}{uv}$.

b) What is the value of f when $u = 80$ and $v = 6.4$?

c) If you know that $\frac{1}{f} = \frac{u + v}{uv}$, what is a rational expression for f?



28. **MINI LAB** In this section, you have added and subtracted rational expressions to get a single expression. For example, $\frac{3}{x - 4} - \frac{2}{x - 1} = \frac{x + 5}{(x - 4)(x - 1)}$. What if you are given a rational expression and you want to find two expressions that can be added to get it? In other words, you are reversing the situation.

$$\frac{x + 5}{(x - 4)(x - 1)} = \frac{A}{x - 4} + \frac{B}{x - 1}$$

Step 1 To determine A, cover its denominator in the expression on the left, leaving you with $\frac{x + 5}{x - 1}$. Determine the value of $\frac{x + 5}{x - 1}$ when $x = 4$, the non-permissible value for the factor of $x - 4$.

$$\frac{x + 5}{x - 1} = \frac{4 + 5}{4 - 1}$$
$$= 3$$

You can often do this step mentally.

This means $A = 3$.

Step 2 Next, cover $(x - 1)$ in the expression on the left and substitute $x = 1$ into $\frac{x + 5}{x - 4}$ to get -2. This means $B = -2$.

Step 3 Check that it works. Does $\frac{3}{x - 4} - \frac{2}{x - 1} = \frac{x + 5}{(x - 4)(x - 1)}$?

Step 4 What are the values for A and B in the following?

a) $\frac{3x - 1}{(x - 3)(x + 1)} = \frac{A}{x - 3} + \frac{B}{x + 1}$

b) $\frac{6x + 15}{(x + 7)(x - 2)} = \frac{A}{x + 7} + \frac{B}{x - 2}$

Step 5 Do you believe that the method in steps 1 to 3 always works, or sometimes works? Use algebraic reasoning to show that if $\frac{x + 5}{(x - 4)(x - 1)} = \frac{A}{x - 4} + \frac{B}{x - 1}$, then $A = 3$ and $B = -2$.

Key Ideas

- You can solve a rational equation by multiplying both sides by a common denominator. This eliminates the fractions from the equation. Then, solve the resulting equation.
- When solving a word problem involving rates, it is helpful to use a table.
- Check that the potential roots satisfy the original equation, are not non-permissible values, and, in the case of a word problem, are realistic in the context.

Check Your Understanding

Practise

1. Use the LCD to eliminate the fractions from each equation. Do not solve.

a) $\frac{x - 1}{3} - \frac{2x - 5}{4} = \frac{5}{12} + \frac{x}{6}$

b) $\frac{2x + 3}{x + 5} + \frac{1}{2} = \frac{7}{2x + 10}$

c) $\frac{4x}{x^2 - 9} - \frac{5}{x + 3} = 2$

2. Solve and check each equation. Identify all non-permissible values.

a) $\frac{f + 3}{2} - \frac{f - 2}{3} = 2$

b) $\frac{3 - y}{3y} + \frac{1}{4} = \frac{1}{2y}$

c) $\frac{9}{w - 3} - \frac{4}{w - 6} = \frac{18}{w^2 - 9w + 18}$

3. Solve each rational equation. Identify all non-permissible values.

a) $\frac{6}{t} + \frac{t}{2} = 4$

b) $\frac{6}{c - 3} = \frac{c + 3}{c^2 - 9} - 5$

c) $\frac{d}{d + 4} = \frac{2 - d}{d^2 + 3d - 4} + \frac{1}{d - 1}$

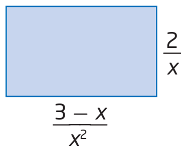
d) $\frac{x^2 + x + 2}{x + 1} - x = \frac{x^2 - 5}{x^2 - 1}$

4. Joline solved the following rational equation. She claims that the solution is $y = 1$. Do you agree? Explain.

$\frac{-3y}{y - 1} + 6 = \frac{6y - 9}{y - 1}$

Apply

5. A rectangle has the dimensions shown.



a) What is an expression for the difference between the length and the width of the rectangle? Simplify your answer.

b) What is an expression for the area of the rectangle? Express the answer in simplest form.

c) If the perimeter of the rectangle is 28 cm, find the value(s) for x.

6. Solve. Round answers to the nearest hundredth.

a) $\frac{26}{b + 5} = 1 + \frac{3}{b - 2}$

b) $\frac{c}{c + 2} - 3 = \frac{-6}{c^2 - 4}$

7. Experts claim that the golden rectangle is most pleasing to the eye. It has dimensions that satisfy the equation $\frac{l}{w} = \frac{l + w}{l}$, where w is the width and l is the length.

According to this relationship, how long should a rectangular picture frame be if its width is 30 cm? Give the exact answer and an approximate answer, rounded to the nearest tenth of a centimetre.

8. The sum of two numbers is 25. The sum of their reciprocals is $\frac{1}{4}$. Determine the two numbers.

9. Two consecutive numbers are represented by x and $x + 1$. If 6 is added to the first number and two is subtracted from the second number, the quotient of the new numbers is $\frac{9}{2}$. Determine the numbers algebraically.

10. A French club collected the same amount from each student going on a trip to Le Cercle Molière in Winnipeg. When six students could not go, each of the remaining students was charged an extra \$3. If the total cost was \$540, how many students went on the trip?

Did You Know?

Le Cercle Molière is the oldest continuously running theatre company in Canada, founded in 1925. It is located in St. Boniface, Manitoba, and moved into its new building in 2009.

11. The sum of the reciprocals of two consecutive integers is $\frac{11}{30}$. What are the integers?

12. Suppose you are running water into a tub. The tub can be filled in 2 min if only the cold tap is used. It fills in 3 min if only the hot tap is turned on. How long will it take to fill the tub if both taps are on simultaneously?

a) Will the answer be less than or greater than 2 min? Why?

b) Complete a table in your notebook similar to the one shown.

	Time to Fill Tub (min)	Fraction Filled in 1 min	Fraction Filled in x minutes
Cold Tap			
Hot Tap			
Both Taps	x	$\frac{1}{x}$	$\frac{x}{x}$ or 1

c) What is one equation that represents both taps filling the tub?

d) Solve your equation to determine the time with both taps running.

13. Two hoses together fill a pool in 2 h. If only hose A is used, the pool fills in 3 h. How long would it take to fill the pool if only hose B were used?

14. Two kayakers paddle 18 km downstream with the current in the same time it takes them to go 8 km upstream against the current. The rate of the current is 3 km/h.

a) Complete a table like the following in your notebook. Use the formula distance = rate × time.

	Distance (km)	Rate (km/h)	Time (h)
Downstream			
Upstream			

b) What equation could you use to find the rate of the kayakers in still water?

c) Solve your equation.

d) Which values are non-permissible?

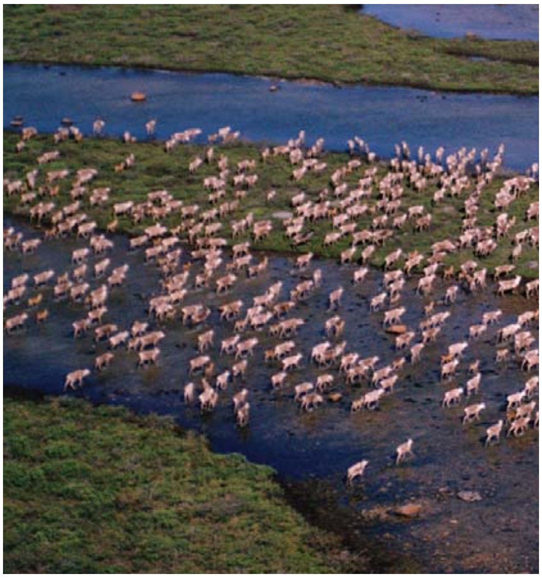
Did You Know?

When you are travelling with the current, add the speed of the current to your rate of speed. When you are travelling against the current, subtract the speed of the current.

Kyuquot Sound, British Columbia

15. Nikita lives in Kindersley, Saskatchewan. With her old combine, she can harvest her entire wheat crop in 72 h. Her neighbour offers to help. His new combine can do the same job in 48 h. How long would it take to harvest the wheat crop with both combines working together?

16. Several cows from the Qamanirjuaq Caribou Herd took 4 days longer to travel 70 km to Forde Lake, Nunavut, than it took them to travel 60 km north beyond Forde Lake. They averaged 5 km/h less before Forde Lake because foraging was better. What was their average speed for the part beyond Forde Lake? Round your answer to the nearest tenth of a kilometre per hour.



Caribou migrating across the tundra, Hudson Bay

Did You Know?

The Qamanirjuaq (ka ma nir you ak) and Beverly barren ground caribou herds winter in the same areas of northern Manitoba, Saskatchewan, Northeast Alberta, and the Northwest Territories and migrate north in the spring into different parts of Nunavut for calving.

Web Link

This is the caribou herd that Inuit depended on in Farley Mowat's book *People of the Deer*. For more information, go to www.mhrprecalc11.ca and follow the links.

17. Ted is a long-distance driver. It took him 30 min longer to drive 275 km on the Trans-Canada Highway west of Swift Current, Saskatchewan, than it took him to drive 300 km east of Swift Current. He averaged 10 km/h less while travelling west of Swift Current due to more severe snow conditions. What was Ted's average speed for each part of the trip?

18. Two friends can paddle a canoe at a rate of 6 km/h in still water. It takes them 1 h to paddle 2 km up a river and back again. Find the speed of the current.

19. Suppose you have 21 days to read a 518-page novel. After finishing half the book, you realize that you must read 12 pages more per day to finish on time. What is your reading rate for the first half of the book? Use a table like the following to help you solve the problem.

	Reading Rate in Pages per Day	Number of Pages Read	Number of Days
First Half			
Second Half			

20. The concentration, C, of salt in a solution is determined by the formula $C = \frac{A}{s + w}$, where A is the constant amount of salt, s is the initial amount of solution, and w is the amount of water added.

a) How much water must be added to a 1-L bottle of 30% salt solution to get a 10% solution?

b) How much water must be added to a half-litre bottle of 10% salt solution to get a 2% solution?

Extend

21. If $b = \frac{1}{a}$ and $\frac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{a} + \frac{1}{b}} = \frac{4}{5}$, solve for a.

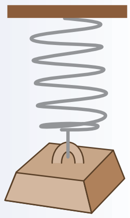
22. A number x is the harmonic mean of a and b if $\frac{1}{x}$ is the average of $\frac{1}{a}$ and $\frac{1}{b}$.

a) Write a rational equation for the statement above. Solve for x.

b) Find two numbers that differ by 8 and have a harmonic mean of 6.

Did You Know?

The distance that a spring stretches is represented by the formula $d = km$, where d is the distance, in centimetres, m is the mass, in grams, and k is a constant.



When two springs with constants k and j are attached to one another, the new spring constant, c, can be found using the formula $\frac{1}{c} = \frac{1}{k} + \frac{1}{j}$. The new spring constant is the harmonic mean of the spring constants for the separate springs.

23. Sometimes is it helpful to solve for a specific variable in a formula. For example, if you solve for x in the equation $\frac{1}{x} - \frac{1}{y} = a$, the answer is $x = \frac{y}{ay + 1}$.

a) Show algebraically how you could solve for x if $\frac{1}{x} - \frac{1}{y} = a$. Show two different ways to get the answer.

b) In the formula $d = v_0t + \frac{1}{2}gt^2$, solve for v_0. Simplify your answer.

c) Solve for n in the formula $I = \frac{E}{R + \frac{r}{n}}$.

Create Connections

24. **a)** Explain the difference between rational expressions and rational equations. Use examples.

b) Explain the process you would use to solve a rational equation. As a model, describe each step you would use to solve the following equation.

$$\frac{5}{x} - \frac{1}{x - 1} = \frac{1}{x - 1}$$

c) There are at least two different ways to begin solving the equation in part b). Identify a different first step from how you began your process in part b).

25. **a)** A laser printer prints 24 more sheets per minute than an ink-jet printer. If it takes the two printers a total of 14 min to print 490 pages, what is the printing rate of the ink-jet printer?

Did You Know?

Is a paperless office possible? According to Statistics Canada, our consumption of paper for printing and writing more than doubled between 1983 and 2003 to 91 kg, or about 20 000 pages per person per year. This is about 55 pages per person per day.

b) Examine the data in Did You Know? Is your paper use close to the national average? Explain.

c) We can adopt practices to conserve paper use. Work with a partner to identify eco-responsible ways you can conserve paper and ink.

26. Suppose there is a quiz in your mathematics class every week. The value of each quiz is 50 points. After the first 6 weeks, your average mark on these quizzes is 36.

a) What average mark must you receive on the next 4 quizzes so that your average is 40 on the first 10 quizzes? Use a rational equation to solve this problem.

b) There are 15 quizzes in your mathematics course. Show if it is possible to have an average of 90% on your quizzes at the end of the course if your average is 40 out of 50 on the first 10 quizzes.

27. Tyler has begun to solve a rational equation. His work is shown below.

$$\begin{aligned} \frac{2}{x - 1} - 3 &= \frac{5x}{x + 1} \\ 2(x + 1) - 3(x + 1)(x - 1) &= 5x(x - 1) \\ 2x + 2 - 3x^2 + 1 &= 5x^2 - 5x \\ 0 &= 8x^2 - 7x - 3 \end{aligned}$$

a) Re-work the solution to correct any errors that Tyler made.

b) Solve for x. Give your answers as exact values.

c) What are the approximate values of x, to the nearest hundredth?

Chapter 6 Review

6.1 Rational Expressions, pages 310–321

1. A rational number is of the form $\frac{a}{b}$, where a and b are integers.

a) What integer cannot be used for b? Why?

b) How does your answer to part a) relate to rational expressions? Explain using examples.

2. You can write an unlimited number of equivalent expressions for any given rational expression. Do you agree or disagree with this statement? Explain.

3. What are the non-permissible values, if any, for each rational expression?

a) $\frac{5x^2}{2y}$ **b)** $\frac{x^2 - 1}{x + 1}$

c) $\frac{27x^2 - 27}{3}$ **d)** $\frac{7}{(a - 3)(a + 2)}$

e) $\frac{-3m + 1}{2m^2 - m - 3}$ **f)** $\frac{t + 2}{2t^2 - 8}$

4. What is the numerical value for each rational expression? Test your result using some permissible values for the variable. Identify any non-permissible values.

a) $\frac{2s - 8s}{s}$

b) $\frac{5x - 3}{3 - 5x}$

c) $\frac{2 - b}{4b - 8}$

5. Write an expression that satisfies the given conditions in each case.

a) equivalent to $\frac{x - 3}{5}$, with a denominator of $10x$

b) equivalent to $\frac{x - 3}{x^2 - 9}$, with a numerator of 1

c) equivalent to $\frac{c - 2d}{3f}$, with a numerator of $3c - 6d$

d) equivalent to $\frac{m + 1}{m + 4}$, with non-permissible values of ± 4

6. a) Explain how to determine non-permissible values for a rational expression. Use an example in your explanation.

b) Simplify. Determine all non-permissible values for the variables.

i) $\frac{3x^2 - 13x - 10}{3x + 2}$

ii) $\frac{a^2 - 3a}{a^2 - 9}$

iii) $\frac{3y - 3x}{4x - 4y}$

iv) $\frac{81x^2 - 36x + 4}{18x - 4}$

7. A rectangle has area $x^2 - 1$ and width $x - 1$.

a) What is a simplified expression for the length?

b) Identify any non-permissible values. What do they mean in this context?

6.2 Multiplying and Dividing Rational Expressions, pages 322–330

8. Explain how multiplying and dividing rational expressions is similar to multiplying and dividing fractions. Describe how they differ. Use examples to support your response.

9. Simplify each product. Determine all non-permissible values.

a) $\frac{2p}{r} \times \frac{10q}{8p}$

b) $4m^3t \times \frac{1}{16mt^4}$

c) $\frac{3a + 3b}{8} \times \frac{4}{a + b}$

d) $\frac{x^2 - 4}{x^2 + 25} \times \frac{2x^2 + 10x}{x^2 + 2x}$

e) $\frac{d^2 + 3d + 2}{2d + 2} \times \frac{2d + 6}{d^2 + 5d + 6}$

f) $\frac{y^2 - 8y - 9}{y^2 - 10y + 9} \times \frac{y^2 - 9y + 8}{y^2 - 1} \times \frac{y^2 - 25}{5 - y}$

10. Divide. Express answers in simplest form. Identify any non-permissible values.

a) $2t \div \frac{1}{4}$

b) $\frac{a^3}{b^4} \div \frac{a^3}{b^3}$

c) $\frac{7}{x^2 - y^2} \div \frac{-35}{x - y}$

d) $\frac{3a + 9}{a - 3} \div \frac{a^2 + 6a + 9}{a - 3}$

e) $\frac{3x - 2}{x^3 + 3x^2 + 2x} \div \frac{9x^2 - 4}{3x^2 + 8x + 4} \div \frac{1}{x}$

f) $\dfrac{\frac{4 - x^2}{6}}{\frac{x - 2}{2}}$

11. Multiply or divide as indicated. Express answers in simplest form. Determine all non-permissible values.

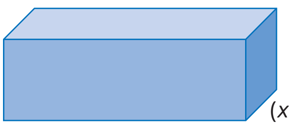
a) $\frac{9}{2m} \div \frac{3}{m} \times \frac{m}{3}$

b) $\frac{x^2 - 3x + 2}{x^2 - 4} \times \frac{x + 3}{x^2 + 3x} \div \frac{1}{x + 2}$

c) $\frac{a - 3}{a - 4} \div \frac{30}{a + 3} \times \frac{5a - 20}{a^2 - 9}$

d) $\frac{3x + 12}{3x^2 - 5x - 12} \times \frac{x - 3}{x + 4} \div \frac{15}{3x + 4}$

12. The volume of a rectangular prism is $(2x^3 + 5x^2 - 12x)$ cubic centimetres. If the length of the prism is $(2x - 3)$ centimetres and its width is $(x + 4)$ centimetres, what is an expression for the height of the prism?



6.3 Adding and Subtracting Rational Expressions, pages 331–340

13. Determine a common denominator for each sum or difference. What is the lowest common denominator (LCD) in each case? What is the advantage of using the LCD?

a) $\frac{4}{5x} + \frac{3}{10x}$

b) $\frac{5}{x - 2} + \frac{2}{x + 1} - \frac{1}{x - 2}$

14. Perform the indicated operations. Express answers in simplest form. Identify any non-permissible values.

a) $\frac{m}{5} + \frac{3}{5}$

b) $\frac{2m}{x} - \frac{m}{x}$

c) $\frac{x}{x + y} + \frac{y}{x + y}$

d) $\frac{x - 2}{3} - \frac{x + 1}{3}$

e) $\frac{x}{x^2 - y^2} - \frac{y}{y^2 - x^2}$

15. Add or subtract. Express answers in simplest form. Identify any non-permissible values.

a) $\frac{4x - 3}{6} - \frac{x - 2}{4}$

b) $\frac{2y - 1}{3y} + \frac{y - 2}{2y} - \frac{y - 8}{6y}$

c) $\frac{9}{x - 3} + \frac{7}{x^2 - 9}$

d) $\frac{a}{a + 3} - \frac{a^2 - 3a}{a^2 + a - 6}$

e) $\frac{a}{a - b} - \frac{2ab}{a^2 - b^2} + \frac{b}{a + b}$

f) $\frac{2x}{4x^2 - 9} + \frac{x}{2x^2 + 5x + 3} - \frac{1}{2x - 3}$

16. The sum of the reciprocals of two numbers will always be the same as the sum of the numbers divided by their product.

a) If the numbers are represented by a and b, translate the sentence above into an equation.

b) Use your knowledge of adding rational expressions to prove that the statement is correct by showing that the left side is equivalent to the right side.

17. Three tests and one exam are given in a course. Let a, b, and c represent the marks of the tests and d be the mark from the final exam. Each is a mark out of 100. In the final mark, the average of the three tests is worth the same amount as the exam. Write a rational expression for the final mark. Show that your expression is equivalent to $\frac{a + b + c + 3d}{6}$. Choose a sample of four marks and show that the simplified expression works.

18. Two sisters go to an auction sale to buy some antique chairs. They intend to pay no more than c dollars for a chair. Beth is worried she will not get the chairs. She bids $10 more per chair than she intended and spends $250. Helen is more patient and buys chairs for $10 less per chair than she intended. She spends $200 in total.

a) Explain what each of the following expressions represents from the information given about the auction sale.

i) $c + 10$ **ii)** $c - 10$

iii) $\frac{200}{c - 10}$ **iv)** $\frac{250}{c + 10}$

v) $\frac{200}{c - 10} + \frac{250}{c + 10}$

b) Determine the sum of the rational expressions in part v) and simplify the result.

6.4 Rational Equations, pages 341–351

19. What is different about the processes used to solve a rational equation from those used to add or subtract rational expressions? Explain using examples.

20. Solve each rational equation. Identify all non-permissible values.

a) $\frac{s - 3}{s + 3} = 2$

b) $\frac{x + 2}{3x + 2} = \frac{x + 3}{x - 1}$

c) $\frac{z - 2}{z} + \frac{1}{5} = \frac{-4}{5z}$

d) $\frac{3m}{m - 3} + 2 = \frac{3m - 1}{m + 3}$

e) $\frac{x}{x - 3} = \frac{3}{x - 3} - 3$

f) $\frac{x - 2}{2x + 1} = \frac{1}{2} + \frac{x - 3}{2x}$

g) $\frac{3}{x + 2} + \frac{5}{x - 3} = \frac{3x}{x^2 - x - 6} - 1$

21. The sum of two numbers is 12. The sum of their reciprocals is $\frac{3}{8}$. What are the numbers?

22. Matt and Elaine, working together, can paint a room in 3 h. It would take Matt 5 h to paint the room by himself. How long would it take Elaine to paint the room by herself?

23. An elevator goes directly from the ground up to the observation deck of the Calgary Tower, which is at 160 m above the ground. The elevator stops at the top for 36 s before it travels directly back down to the ground. The time for the round trip is 2.5 min. The elevator descends at 0.7 m/s faster than it goes up.

a) Determine an equation that could be used to find the rate of ascent of the elevator.

b) Simplify your equation to the form $ax^2 + bx + c = 0$, where a, b, and c are integers, and then solve.

c) What is the rate of ascent in kilometres per hour, to the nearest tenth?

Chapter 6 Practice Test

Multiple Choice

For #1 to #5, choose the best answer.

1. What are the non-permissible values for the rational expression $\dfrac{x(x+2)}{(x-3)(x+1)}$?

A 0 and -2 **B** -3 and 1

C 0 and 2 **D** 3 and -1

2. Simplify the rational expression $\dfrac{x^2 - 7x + 6}{x^2 - 2x - 24}$ for all permissible values of x.

A $\dfrac{x+1}{x-4}$ **B** $\dfrac{x-1}{x+4}$

C $\dfrac{x+1}{x+4}$ **D** $\dfrac{x-1}{x-4}$

3. Simplify $\dfrac{8}{3y} + \dfrac{5y}{4} - \dfrac{5}{8}$ for all permissible values of y.

A $\dfrac{30y^2 - 15y + 64}{24y}$ **B** $\dfrac{30y^2 + 79}{24y}$

C $\dfrac{15y^2 + 64}{24y}$ **D** $\dfrac{5y + 3}{24y}$

4. Simplify $\dfrac{3x - 12}{9x^2} \div \dfrac{x - 4}{3x}$, $x \neq 0$ and $x \neq 4$.

A $\dfrac{1}{x}$ **B** $\dfrac{16}{3x}$

C x **D** $\dfrac{-12}{x-4}$

5. Solve $\dfrac{6}{t-3} = \dfrac{4}{t+4}$, $t \neq 3$ and $t \neq -4$.

A $-\dfrac{1}{2}$ **B** -1

C -6 **D** -18

Short Answer

6. Identify all non-permissible values.

$$\frac{3x - 5}{x^2 - 9} \times \frac{2x - 6}{3x^2 - 2x - 5} \div \frac{x - 3}{x + 3}$$

7. If both rational expressions are defined and equivalent, what is the value of k?

$$\frac{2x^2 + kx - 10}{2x^2 + 7x + 6} = \frac{2x - 5}{2x + 3}$$

8. Add or subtract as indicated. Give your answer in simplest form.

$$\frac{5y}{6} + \frac{1}{y - 2} - \frac{y + 1}{3y - 6}$$

9. Create an equation you could use to solve the following problem. Indicate what your variable represents. Do not solve your equation.

A large auger can fill a grain bin in 5 h less time than a smaller auger. Together they fill the bin in 6 h. How long would it take the larger auger, by itself, to fill the bin?

10. List similarities and differences between the processes of adding and subtracting rational expressions and solving rational equations. Use examples.

Extended Response

11. Solve $2 - \dfrac{5}{x^2 - x - 6} = \dfrac{x + 3}{x + 2}$. Identify all non-permissible values.

12. The following rational expressions form an arithmetic sequence: $\dfrac{3 - x}{x}$, $\dfrac{2x - 1}{2x}$, $\dfrac{5x + 3}{5x}$. Use common differences to create a rational equation. Solve for x.

13. A plane is flying from Winnipeg to Calgary against a strong headwind of 50 km/h. The plane takes $\frac{1}{2}$ h longer for this flight than it would take in calm air. If the distance from Winnipeg to Calgary is 1200 km, what is the speed of the plane in calm air, to the nearest kilometre per hour?

Key Ideas

- The absolute value of a real number a is defined as

$$|a| = \begin{cases} a, \text{ if } a \geq 0 \\ -a, \text{ if } a < 0 \end{cases}$$

- Geometrically, the absolute value of a real number a, written as $|a|$, is its distance from zero on the number line, regardless of direction.
- Determine the absolute value of a numerical expression by
  - evaluating the numerical expression inside the absolute value symbol using the order of operations
  - taking the absolute value of the resulting expression

Check Your Understanding

Practise

1. Evaluate.

a) $|9|$ **b)** $|0|$

c) $|-7|$ **d)** $|-4.728|$

e) $|6.25|$ **f)** $\left|-5\frac{1}{2}\right|$

2. Order the numbers from least to greatest.

$|0.8|$, 1.1, $|-2|$, $\left|\frac{3}{5}\right|$, -0.4, $\left|-1\frac{1}{4}\right|$, -0.8

3. Order the numbers from greatest to least.

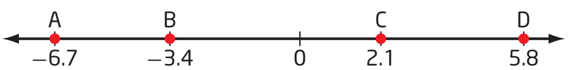
-2.4, $|1.3|$, $\left|-\frac{7}{5}\right|$, -1.9, $|-0.6|$, $\left|1\frac{1}{10}\right|$, 2.2

4. Evaluate each expression.

a) $|8 - 15|$ **b)** $|3| - |-8|$

c) $|7 - (-3)|$ **d)** $|2 - 5(3)|$

5. Use absolute value symbols to write an expression for the distance between each pair of specified points on the number line. Determine the distance.



a) A and C **b)** B and D

c) C and B **d)** D and A

6. Determine the value of each absolute value expression.

a) $2|-6 - (-11)|$

b) $|-9.5| - |12.3|$

c) $3\left|\frac{1}{2}\right| + 5\left|-\frac{3}{4}\right|$

d) $|3(-2)^2 + 5(-2) + 7|$

e) $|-4 + 13| + |6 - (-9)| - |8 - 17| + |-2|$

Apply

7. Use absolute value symbols to write an expression for the length of each horizontal or vertical line segment. Determine each length.

a) A(8, 1) and B(3, 1)

b) A(12, 9) and B(−8, 9)

c) A(6, 2) and B(6, 9)

d) A(−1, −7) and B(−1, 15)

e) A(a, y) and B(b, y)

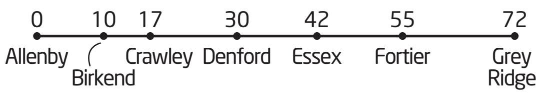
f) A(x, m) and B(x, n)

8. Southern Alberta often experiences dry chinook winds in winter and spring that can change temperatures by a large amount in a short time. On a particular day in Warner, Alberta, the temperature was -11 °C in the morning. A chinook wind raised the temperature to $+7$ °C by afternoon. The temperature dropped to -9 °C during the night. Use absolute value symbols to write an expression for the total change in temperature that day. What is the total change in temperature for the day?

Did You Know?

A First Nations legend of the St'at'imc Nation of British Columbia says that a girl named Chinook-Wind married Glacier and moved to his country. In this foreign land, she longed for her home and sent a message to her people. They came to her first in a vision of snowflakes, then rain, and finally as a melting glacier that took her home.

9. Suppose a straight stretch of highway running west to east begins at the town of Allenby (0 km). The diagram shows the distances from Allenby east to various towns. A new grain storage facility is to be built along the highway 24 km east of Allenby. Write an expression using absolute value symbols to determine the total distance of the grain storage facility from all seven towns on the highway for this proposed location. What is the distance?



10. The Alaska Highway runs from Dawson Creek, British Columbia, to Delta Junction, Alaska. Travel guides along the highway mark historic mileposts, from mile 0 in Dawson Creek to mile 1422 in Delta Junction. The table shows the Ramsay family's trip along this highway.

	Destination	Mile Number
Starting Point	Charlie Lake campground	51
Tuesday	Liard River, British Columbia	496
Wednesday	Whitehorse, Yukon Territory	918
Thursday	Beaver Creek, Yukon Territory	1202
	Haines Junction, Yukon Territory	1016
Friday	Delta Junction, Alaska	1422

Use an expression involving absolute value symbols to determine the total distance, in miles, that the Ramsay family travelled in these four days.

Did You Know?

In 1978, the mileposts along the Canadian section were replaced with kilometre posts. Some mileposts at locations of historic significance remain, although reconstruction and rerouting mean that these markers no longer represent accurate driving distances.

11. When Vanessa checks her bank account on-line, it shows the following balances:

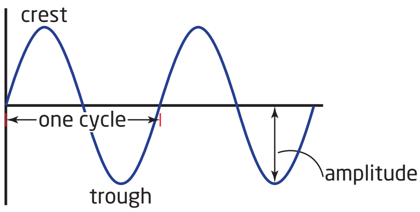
Date	Balance
Oct. 4	$359.22
Oct. 12	$310.45
Oct. 17	$295.78
Oct. 30	$513.65
Nov. 5	$425.59

a) Use an absolute value expression to determine the total change in Vanessa's bank balance during this period.

b) How is this different from the net change in her bank balance?

12. In physics, the amplitude of a wave is measured as the absolute value of the difference between the crest height and the trough height of the wave, divided by 2.

$$\text{Amplitude} = \frac{|\text{crest height} - \text{trough height}|}{2}$$



Determine the amplitude of waves with the following characteristics.

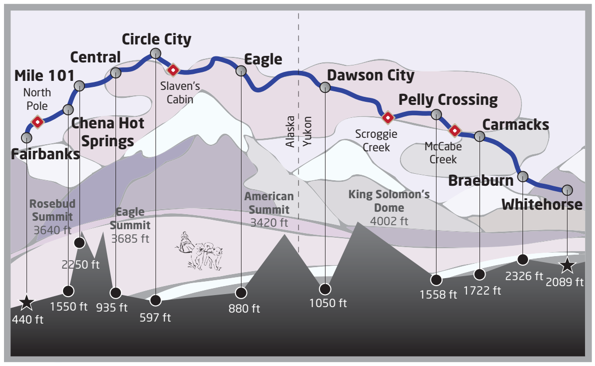
a) crest at height 17 and trough at height 2

b) crest at height 90 and trough at height -90

c) crest at height 1.25 and trough at height -0.5

13. The Festival du Voyageur is an annual Francophone winter festival held in Manitoba. One of the outdoor events at the festival is a snowshoe race. A possible trail for the race is 2 km long, with the start at 0 km; checkpoints at 500 m, 900 m, and 1600 m; and the finish at 2 km. Suppose a race organizer travels by snowmobile from the start to the 1600-m checkpoint, back to the 900-m checkpoint, then out to the finish line, and finally back to the 500-m checkpoint. Use an absolute value expression to determine the total distance travelled by the race organizer, in metres and in kilometres.

14. The Yukon Quest dog sled race runs between Fairbanks, Alaska, and Whitehorse, Yukon Territory, a distance of more than 1000 mi. It lasts for 2 weeks. The elevation at Fairbanks is 440 ft, and the elevation at Whitehorse is 2089 ft.



a) Determine the net change in elevation from Fairbanks to Whitehorse.

b) The race passes through Central, at an elevation of 935 ft; Circle City, whose elevation is 597 ft; and Dawson City, at an elevation of 1050 ft. What is the total change in elevation from Fairbanks to Whitehorse, passing through these cities?

Did You Know?

The Yukon Quest has run every year since 1984. The race follows the historic Gold Rush and mail delivery dog sled routes from the turn of the 20th century. In even-numbered years, the race starts in Fairbanks and ends in Whitehorse. In odd-numbered years, the race starts in Whitehorse and ends in Fairbanks.

Extend

15. A trading stock opens the day at a value of $7.65 per share, drops to $7.28 by noon, then rises to $8.10, and finally falls an unknown amount to close the trading day. If the absolute value of the total change for this stock is $1.55, determine the amount that the stock dropped in the afternoon before closing.

16. As part of a scavenger hunt, Toby collects items along a specified trail. Starting at the 2-km marker, he bicycles east to the 7-km marker, and then turns around and bicycles west back to the 3-km marker. Finally, Toby turns back east and bicycles until the total distance he has travelled is 15 km.

a) How many kilometres does Toby travel in the last interval?

b) At what kilometre marker is Toby at the end of the scavenger hunt?

17. Mikhala and Jocelyn are examining some effects of absolute value on the sum of the squares of the set of values $\{-2.5, 3, -5, 7.1\}$. Mikhala takes the absolute value of each number and then squares it. Jocelyn squares each value and then takes the absolute value.

a) What result does each student get?

b) Explain the results.

c) Is this always true? Explain.

18. a) Michel writes the expression $|x - 5|$, where x is a real number, without the absolute value symbol. His answer is shown below.

$$|x - 5| = \begin{cases} x - 5, \text{ if } x - 5 \geq 0 \\ -(x - 5), \text{ if } x - 5 < 0 \end{cases}$$

$$|x - 5| = \begin{cases} x - 5, \text{ if } x \geq 5 \\ 5 - x, \text{ if } x < 5 \end{cases}$$

Explain the steps in Michel's solution.

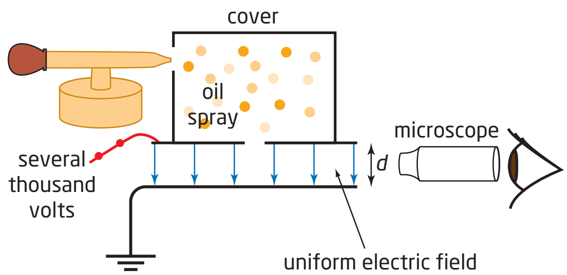
b) If x is a real number, then write each of the following without the absolute value symbol.

i) $|x - 7|$ **ii)** $|2x - 1|$

iii) $|3 - x|$ **iv)** $|x^2 + 4|$

19. Julia states, "To determine the absolute value of any number, change the sign of the number." Use an example to show that Julia is incorrect. In your own words, correctly complete the statement, "To determine the absolute value of a number, …."

20. In the Millikan oil drop experiment, oil drops are electrified with either a positive or a negative charge and then sprayed between two oppositely charged metal plates. Since an individual oil drop will be attracted by one plate and repelled by the other plate, the drop will move either upward or downward toward the plate of opposite charge. If the charge on the plates is reversed, the oil drop is forced to reverse its direction and thus stay suspended indefinitely.



Suppose an oil drop starts out at 45 mm below the upper plate, moves to a point 67 mm below the upper plate, then to a point 32 mm from the upper plate, and finally to a point 58 mm from the upper plate. What total distance does the oil drop travel during the experiment?

Create Connections

21. Describe a situation in which the absolute value of a measurement is preferable to the actual signed value.

22. When an object is thrown into the air, it moves upward, and then changes direction as it returns to Earth. Would it be more appropriate to use signed values (+ or −) or the absolute value for the velocity of the object at any point in its flight? Why? What does the velocity of the object at the top of its flight have to be?

23. A school volleyball team has nine players. The heights of the players are 172 cm, 181 cm, 178 cm, 175 cm, 180 cm, 168 cm, 177 cm, 175 cm, and 178 cm.

a) What is the mean height of the players?

b) Determine the absolute value of the difference between each individual's height and the mean. Determine the sum of the values.

c) Divide the sum by the number of players.

d) Interpret the result in part c) in terms of the height of players on this team.

24. When writing a quadratic function in vertex form, $y = a(x - p)^2 + q$, the vertex of the graph is located at (p, q). If the function has zeros, or its graph has x-intercepts, you can find them using the equation $x = p \pm \sqrt{\left|\frac{q}{a}\right|}$.

a) Use this equation to find the zeros of each quadratic function.

i) $y = 2(x + 1)^2 - 8$

ii) $y = -(x + 2)^2 + 9$

How could you verify that the zeros are correct?

b) What are the zeros of the function $y = 4(x - 3)^2 + 16$? Explain whether or not you could use this method to determine the zeros for all quadratic functions written in vertex form.

25. Explain, using examples, why $\sqrt{x^2} = |x|$.

Project Corner — Space Tourism

Assume the following for the future of space tourism.

- The comfort and quality of a cruise ship are available for travel in outer space.
- The space vehicle in which tourists travel is built within specified tolerances for aerodynamics, weight, payload capacity, and life-support systems, to name a few criteria.
- Each space vehicle leaves Earth within a given launch window.
- The distance of Earth from other celestial destinations changes at different times of the year.
- The quantity of fuel on board the space vehicle determines its range of travel.
- How might absolute value be involved in these design and preparation issues?

Key Ideas

- You can analyse absolute value functions in several ways:
  - graphically, by sketching and identifying the characteristics of the graph, including the x-intercepts and the y-intercept, the minimum values, the domain, and the range
  - algebraically, by rewriting the function as a piecewise function
  - In general, you can express the absolute value function $y = |f(x)|$ as the piecewise function

    $$y = \begin{cases} f(x), \text{ if } f(x) \geq 0 \\ -f(x), \text{ if } f(x) < 0 \end{cases}$$

- The domain of an absolute value function $y = |f(x)|$ is the same as the domain of the function $y = f(x)$.
- The range of an absolute value function $y = |f(x)|$ depends on the range of the function $y = f(x)$. For the absolute value of a linear or quadratic function, the range will generally, but not always, be $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

Check Your Understanding

Practise

1. Given the table of values for $y = f(x)$, create a table of values for $y = |f(x)|$.

a)

x	$y = f(x)$
−2	−3
−1	−1
0	1
1	3
2	5

b)

x	$y = f(x)$
−2	0
−1	−2
0	−2
1	0
2	4

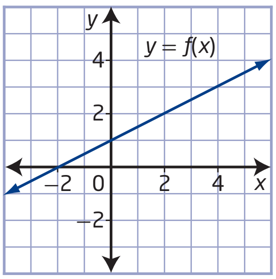
2. The point $(-5, -8)$ is on the graph of $y = f(x)$. Identify the corresponding point on the graph of $y = |f(x)|$.

3. The graph of $y = f(x)$ has an x-intercept of 3 and a y-intercept of −4. What are the x-intercept and the y-intercept of the graph of $y = |f(x)|$?

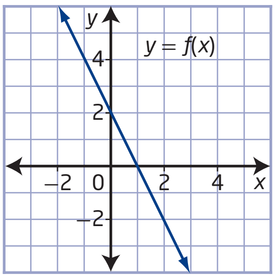
4. The graph of $y = f(x)$ has x-intercepts of −2 and 7, and a y-intercept of $-\frac{3}{2}$. State the x-intercepts and the y-intercept of the graph of $y = |f(x)|$.

5. Copy the graph of $y = f(x)$. On the same set of axes, sketch the graph of $y = |f(x)|$.

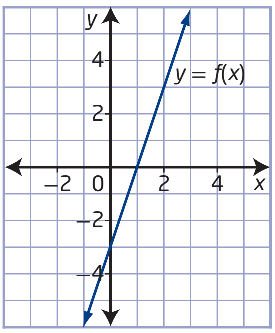
a)



b)



c)



6. Sketch the graph of each absolute value function. State the intercepts and the domain and range.

a) $y = |2x - 6|$

b) $y = |x + 5|$

c) $f(x) = |-3x - 6|$

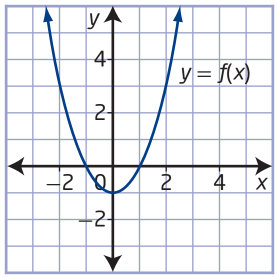
d) $g(x) = |-x - 3|$

e) $y = \left|\frac{1}{2}x - 2\right|$

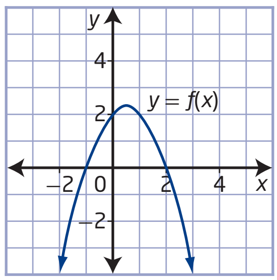
f) $h(x) = \left|\frac{1}{3}x + 3\right|$

7. Copy the graph of $y = f(x)$. On the same set of axes, sketch the graph of $y = |f(x)|$.

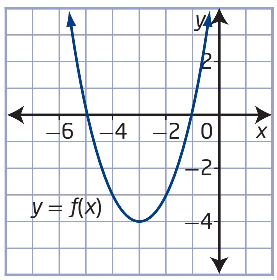
a)



b)



c)



8. Sketch the graph of each function. State the intercepts and the domain and range.

a) $y = |x^2 - 4|$

b) $y = |x^2 + 5x + 6|$

c) $f(x) = |-2x^2 - 3x + 2|$

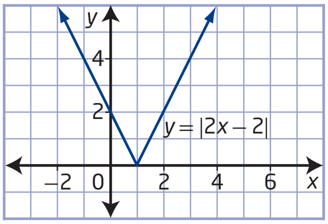
d) $y = \left|\frac{1}{4}x^2 - 9\right|$

e) $g(x) = |(x - 3)^2 + 1|$

f) $h(x) = |-3(x + 2)^2 - 4|$

9. Write the piecewise function that represents each graph.

a)



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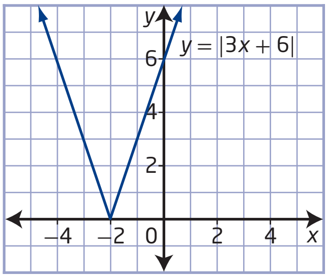

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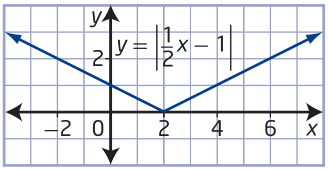
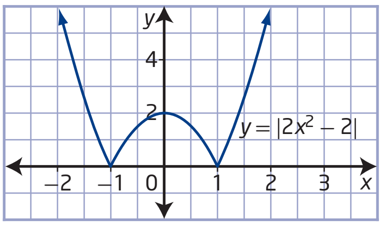


10. What piecewise function could you use to represent each graph of an absolute value function?

a)



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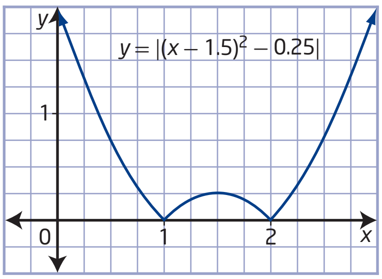

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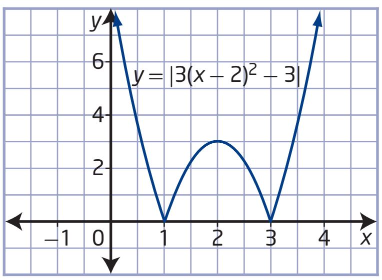


11. Express each function as a piecewise function.

a) $y = |x - 4|$

b) $y = |3x + 5|$

c) $y = |-x^2 + 1|$

d) $y = |x^2 - x - 6|$

Apply

12. Consider the function $g(x) = |6 - 2x|$.

a) Create a table of values for the function using values of -1, 0, 2, 3, and 5 for x.

b) Sketch the graph.

c) Determine the domain and range for $g(x)$.

d) Write the function in piecewise notation.

13. Consider the function $g(x) = |x^2 - 2x - 8|$.

a) What are the y-intercept and x-intercepts of the graph of the function?

b) Graph the function.

c) What are the domain and range of $g(x)$?

d) Express the function as a piecewise function.

14. Consider the function $g(x) = |3x^2 - 4x - 4|$.

a) What are the intercepts of the graph?

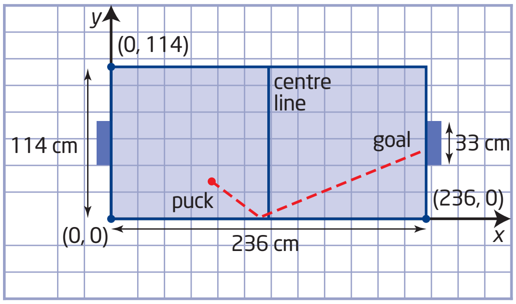
b) Graph the function.

c) What are the domain and range of $g(x)$?

d) What is the piecewise notation form of the function?

15. Raza and Michael are discussing the functions $p(x) = 2x^2 - 9x + 10$ and $q(x) = |2x^2 - 9x + 10|$. Raza says that the two functions have identical graphs. Michael says that the absolute value changes the graph so that $q(x)$ has a different range and a different graph from $p(x)$. Who is correct? Explain your answer.

16. Air hockey is a table game where two players try to score points by hitting a puck into the other player's goal. The diameter of the puck is 8.26 cm. Suppose a Cartesian plane is superimposed over the playing surface of an air hockey table so that opposite corners have the coordinates (0, 114) and (236, 0), as shown. The path of the puck hit by a player is given by $y = |0.475x - 55.1|$.



a) Graph the function.

b) At what point does the puck ricochet off the side of the table?

c) If the other player does not touch the puck, verify whether or not the puck goes into the goal.

17. The velocity, v, in metres per second, of a go-cart at a given time, t, in seconds, is modelled by the function $v(t) = -2t + 4$. The distance travelled, in metres, can be determined by calculating the area between the graph of $v(t) = |-2t + 4|$ and the x-axis. What is the distance travelled in the first 5 s?

18. a) Graph $f(x) = |3x - 2|$ and $g(x) = |-3x + 2|$. What do you notice about the two graphs? Explain why.

b) Graph $f(x) = |4x + 3|$. Write a different absolute value function of the form $g(x) = |ax + b|$ that has the same graph.

19. Graph $f(x) = |x^2 - 6x + 5|$. Write a different absolute value function of the form $g(x) = |ax^2 + bx + c|$ that has the same graph as $f(x) = |x^2 - 6x + 5|$.

20. An absolute value function has the form $f(x) = |ax + b|$, where $a \neq 0$, $b \neq 0$, and $a, b \in \mathrm{R}$. If the function $f(x)$ has a domain of $\{x \mid x \in \mathrm{R}\}$, a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$, an x-intercept occurring at $\left(\frac{3}{2}, 0\right)$, and a y-intercept occurring at (0, 6), what are the values of a and b?

21. An absolute value function has the form $f(x) = |x^2 + bx + c|$, where $b \neq 0$, $c \neq 0$, and $b, c \in \mathrm{R}$. If the function $f(x)$ has a domain of $\{x \mid x \in \mathrm{R}\}$, a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$, x-intercepts occurring at (−6, 0) and (2, 0), and a y-intercept occurring at (0, 12), determine the values of b and c.

22. Explain why the graphs of $y = |x^2|$ and $y = x^2$ are identical.

Extend

23. Is the following statement true for all $x, y \in \mathrm{R}$? Justify your answer.

$|x| + |y| = |x + y|$

24. Draw the graph of $|x| + |y| = 5$.

25. Use the piecewise definition of $y = |x|$ to prove that for all $x, y \in \mathrm{R}$, $|x|(|y|) = |xy|$.

26. Compare the graphs of $f(x) = |3x - 6|$ and $g(x) = |3x| - 6$. Discuss the similarities and differences.

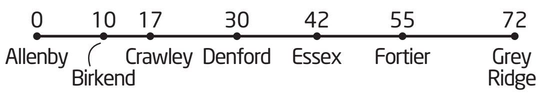
Create Connections

27. Explain how to use a piecewise function to graph an absolute value function.

28. Consider the quadratic function $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. Describe the nature of the discriminant, $b^2 - 4ac$, for the graphs of $y = ax^2 + bx + c$ and $y = |ax^2 + bx + c|$ to be identical.

29. MINI LAB In Section 7.1, you solved the following problem:

Suppose a straight stretch of highway running west to east begins at the town of Allenby (0 km). The diagram shows the distances from Allenby east along the highway to various towns. A new grain storage facility is to be built along the highway 24 km from Allenby. Find the total distance of the grain storage facility from all of the seven towns on the highway for this proposed location.



Step 1 Rather than building the facility at a point 24 km east of Allenby, as was originally planned, there may be a more suitable location along the highway that would minimize the total distance of the grain storage facility from all of the towns. Do you think that point exists? If so, predict its location.

Step 2 Let the location of the grain storage facility be at point x (x kilometres east of Allenby). Then, the absolute value of the distance of the facility from Allenby is $|x|$ and from Birkend is $|x - 10|$. Why do you need to use absolute value?

Continue this process to write absolute value expressions for the distance of the storage facility from each of the seven towns. Then, combine them to create a function for the total of the absolute value distances from the different towns to point x.

Step 3 Graph the combined function using a graphing calculator. Set an appropriate window to view the graph. What are the window settings?

Step 4 **a)** What does the graph indicate about placing the point x at different locations along the highway?

b) What are the coordinates of the minimum point on the graph?

c) Interpret this point with respect to the location of the grain storage facility.

30. Each set of transformations is applied to the graph of $f(x) = x^2$ in the order listed. Write the function of each transformed graph.

a) a horizontal translation of 3 units to the right, a vertical translation of 7 units up, and then take its absolute value

b) a change in the width by a factor of $\frac{4}{5}$, a horizontal translation of 3 units to the left, and then take its absolute value

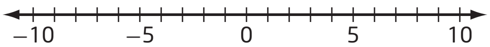
c) a reflection in the x-axis, a vertical translation of 6 units down, and then take its absolute value

d) a change in the width by a factor of 5, a horizontal translation of 3 units to the left, a vertical translation of 3 units up, and then take its absolute value

Check Your Understanding

Practise

1. Use the number line to geometrically solve each equation.



a) $|x| = 7$ **b)** $|x| + 8 = 12$

c) $|x| + 4 = 4$ **d)** $|x| = -6$

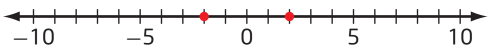
2. Solve each absolute value equation by graphing.

a) $|x - 4| = 10$ **b)** $|x + 3| = 2$

c) $6 = |x + 8|$ **d)** $|x + 9| = -3$

3. Determine an absolute value equation in the form $|ax + b| = c$ given its solutions on the number line.

a)



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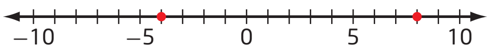
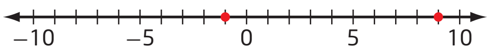


c)



4. Solve each absolute value equation algebraically. Verify your solutions.

a) $|x + 7| = 12$

b) $|3x - 4| + 5 = 7$

c) $2|x + 6| + 12 = -4$

d) $-6|2x - 14| = -42$

5. Solve each equation.

a) $|2a + 7| = a - 4$

b) $|7 + 3x| = 11 - x$

c) $|1 - 2m| = m + 2$

d) $|3x + 3| = 2x - 5$

e) $3|2a + 7| = 3a + 12$

6. Solve each equation and verify your solutions graphically.

a) $|x| = x^2 + x - 3$

b) $|x^2 - 2x + 2| = 3x - 4$

c) $|x^2 - 9| = x^2 - 9$

d) $|x^2 - 1| = x$

e) $|x^2 - 2x - 16| = 8$

Apply

7. Bolts are manufactured at a certain factory to have a diameter of 18 mm and are rejected if they differ from this by more than 0.5 mm.

a) Write an absolute value equation in the form $|d - a| = b$ to describe the acceptance limits for the diameter, d, in millimetres, of these bolts, where a and b are real numbers.

b) Solve the resulting absolute value equation to find the maximum and minimum diameters of the bolts.

8. One experiment measured the speed of light as 299 792 456.2 m/s with a measurement uncertainty of 1.1 m/s.

a) Write an absolute value equation in the form $|c - a| = b$ to describe the measured speed of light, c, metres per second, where a and b are real numbers.

b) Solve the absolute value equation to find the maximum and minimum values for the speed of light for this experiment.

9. In communities in Nunavut, aviation fuel is stored in huge tanks at the airport. Fuel is re-supplied by ship yearly. The fuel tank in Kugaaruk holds 50 000 L. The fuel re-supply brings a volume, V, in litres, of fuel plus or minus 2000 L.

a) Write an absolute value equation in the form $|V - a| = b$ to describe the limits for the volume of fuel delivered, where a and b are real numbers.

b) Solve your absolute value equation to find the maximum and minimum volumes of fuel.

10. Consider the statement $x = 7 \pm 4.8$.

a) Describe the values of x.

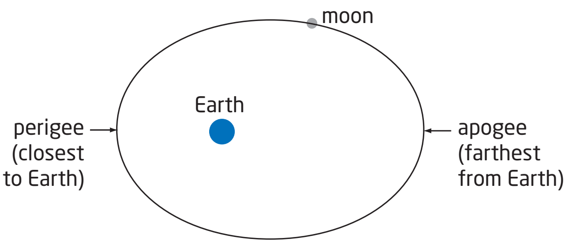
b) Translate the statement into an equation involving absolute value.

11. When measurements are made in science, there is always a degree of error possible. Absolute error is the uncertainty of a measurement. For example, if the mass of an object is known to be 125 g, but the absolute error is said to be ±4 g, then the measurement could be as high as 129 g and as low as 121 g.

a) If the mass of a substance is measured once as 64 g and once as 69 g, and the absolute error is ±2.5 g, what is the actual mass of the substance?

b) If the volume of a liquid is measured to be 258 mL with an absolute error of ±7 mL, what are the least and greatest possible measures of the volume?

12. The moon travels in a elliptical orbit around Earth. The distance between Earth and the moon changes as the moon travels in this orbit. The point where the moon's orbit is closest to Earth is called *perigee*, and the point when it is farthest from Earth is called *apogee*. You can use the equation $|d - 381\ 550| = 25\ 150$ to find these distances, d, in kilometres.



a) Solve the equation to find the perigee and apogee of the moon's orbit of Earth.

b) Interpret the given values 381 550 and 25 150 with respect to the distance between Earth and the moon.

Did You Know?

When a full moon is at perigee, it can appear as much as 14% larger to us than a full moon at apogee.

13. Determine whether $n \geq 0$ or $n \leq 0$ makes each equation true.

a) $n + |-n| = 2n$ **b)** $n + |-n| = 0$

14. Solve each equation for x, where $a, b, c \in \mathrm{R}$.

a) $|ax| - b = c$ **b)** $|x - b| = c$

15. Erin and Andrea each solve $|x - 4| + 8 = 12$. Who is correct? Explain your reasoning.

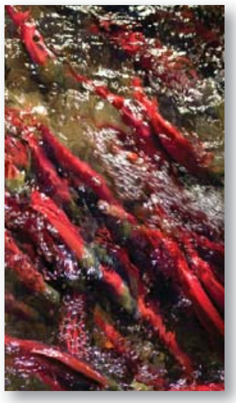
Erin's solution:

$$|x - 4| + 8 = 12$$
$$|x - 4| = 4$$
$$x + 4 = 4 \quad \text{or} \quad -x + 4 = 4$$
$$x = 0 \qquad\qquad x = 0$$

Andrea's solution:

$$|x - 4| + 8 = 12$$
$$|x - 4| = 4$$
$$x - 4 = 4 \quad \text{or} \quad -x + 4 = 4$$
$$x = 8 \qquad\qquad x = 0$$

16. Mission Creek in the Okanagan Valley of British Columbia is the site of the spawning of Kokanee salmon every September. Kokanee salmon are sensitive to water temperature. If the water is too cold, egg hatching is delayed, and if the water is too warm, the eggs die. Biologists have found that the spawning rate of the salmon is greatest when the water is at an average temperature of 11.5 °C with an absolute value difference of 2.5 °C. Write and solve an absolute value equation that determines the limits of the ideal temperature range for the Kokanee salmon to spawn.



Did You Know?

In recent years, the September temperature of Mission Creek has been rising. Scientists are considering reducing the temperature of the water by planting more vegetation along the creek banks. This would create shade, cooling the water.

17. Low-dose aspirin contains 81 mg of the active ingredient acetylsalicylic acid (ASA) per tablet. It is used to regulate and reduce heart attack risk associated with high blood pressure by thinning the blood.

a) Given a tolerance of 20% for generic brands, solve an absolute value equation for the maximum and minimum amount of ASA per tablet.

b) Which limit might the drug company tend to lean toward? Why?

Extend

18. For the launch of the Ares I-X rocket from the Kennedy Space Center in Florida in 2009, scientists at NASA indicated they had a launch window of 08:00 to 12:00 eastern time. If a launch at any time in this window is acceptable, write an absolute value equation to express the earliest and latest acceptable times for launch.

19. Determine whether each statement is sometimes true, always true, or never true, where a is a natural number. Explain your reasoning.

a) The value of $|x + 1|$ is greater than zero.

b) The solution to $|x + a| = 0$ is greater than zero.

c) The value of $|x + a| + a$ is greater than zero.

20. Write an absolute value equation with the indicated solutions or type of solution.

a) -2 and 8

b) no solution

c) one integral solution

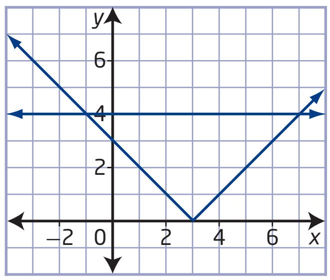
d) two integral solutions

21. Does the absolute value equation $|ax + b| = 0$, where $a, b \in R$, always have a solution? Explain.

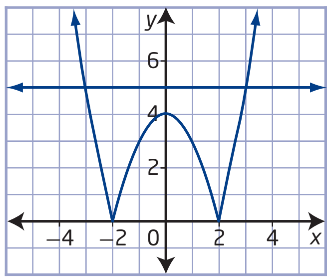
Create Connections

22. For each graph, an absolute value function and a linear function intersect to produce solutions to an equation composed of the two functions. Determine the equation that is being solved in each graph.

a)



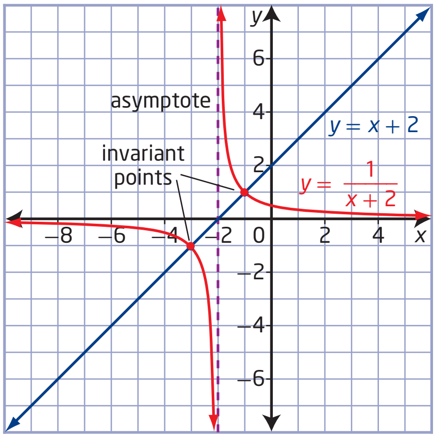
b)



23. Explain, without solving, why the equation $|3x + 1| = -2$ has no solutions, while the equation $|3x + 1| - 4 = -2$ has solutions.

24. Why do some absolute value equations produce extraneous roots when solved algebraically? How are these roots created in the algebraic process if they are not actual solutions of the equation?

- The domain of the reciprocal function is the same as the domain of the original function, excluding the non-permissible values.



Check Your Understanding

Practise

1. Given the function $y = f(x)$, write the corresponding reciprocal function.

a) $y = -x + 2$

b) $y = 3x - 5$

c) $y = x^2 - 9$

d) $y = x^2 - 7x + 10$

2. For each function,

i) state the zeros

ii) write the reciprocal function

iii) state the non-permissible values of the corresponding rational expression

iv) explain how the zeros of the original function are related to the non-permissible values of the reciprocal function

v) state the equation(s) of the vertical asymptote(s)

a) $f(x) = x + 5$

b) $g(x) = 2x + 1$

c) $h(x) = x^2 - 16$

d) $t(x) = x^2 + x - 12$

3. State the equation(s) of the vertical asymptote(s) for each function.

a) $f(x) = \dfrac{1}{5x - 10}$

b) $f(x) = \dfrac{1}{3x + 7}$

c) $f(x) = \dfrac{1}{(x - 2)(x + 4)}$

d) $f(x) = \dfrac{1}{x^2 - 9x + 20}$

4. The calculator screen gives a function table for $f(x) = \dfrac{1}{x - 3}$. Explain why there is an undefined statement.

x	f1(x):= 1/(x−3)
0.	−0.3333...
1.	−0.5
2.	−1.
3.	#UNDEF
4.	1.

5. What are the x-intercept(s) and the y-intercept of each function?

a) $f(x) = \dfrac{1}{x + 5}$

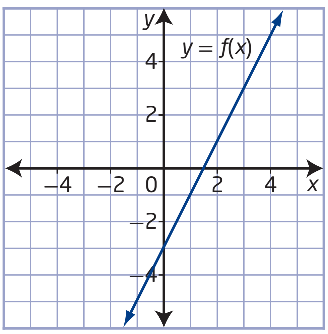
b) $f(x) = \dfrac{1}{3x - 4}$

c) $f(x) = \dfrac{1}{x^2 - 9}$

d) $f(x) = \dfrac{1}{x^2 + 7x + 12}$

6. Copy each graph of $y = f(x)$, and sketch the graph of the reciprocal function $y = \dfrac{1}{f(x)}$. Describe your method.

a)



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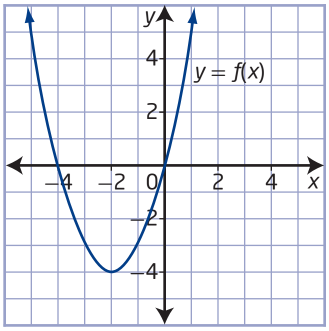

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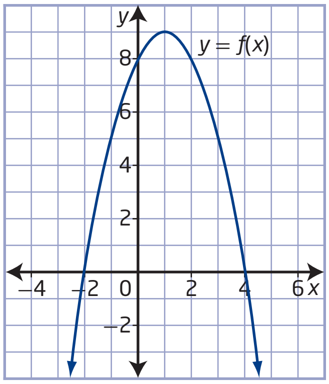


7. Sketch the graphs of $y = f(x)$ and $y = \dfrac{1}{f(x)}$ on the same set of axes. Label the asymptotes, the invariant points, and the intercepts.

a) $f(x) = x - 16$

b) $f(x) = 2x + 4$

c) $f(x) = 2x - 6$

d) $f(x) = x - 1$

8. Sketch the graphs of $y = f(x)$ and $y = \dfrac{1}{f(x)}$ on the same set of axes. Label the asymptotes, the invariant points, and the intercepts.

a) $f(x) = x^2 - 16$

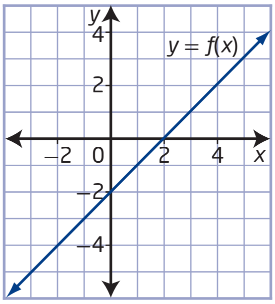
b) $f(x) = x^2 - 2x - 8$

c) $f(x) = x^2 - x - 2$

d) $f(x) = x^2 + 2$

9. Match the graph of the function with the graph of its reciprocal.

a)



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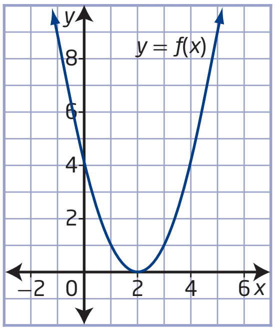

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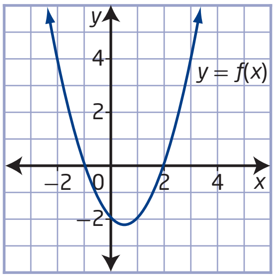

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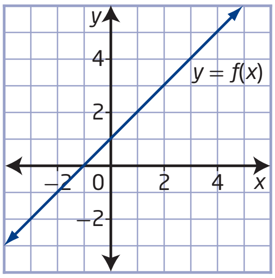
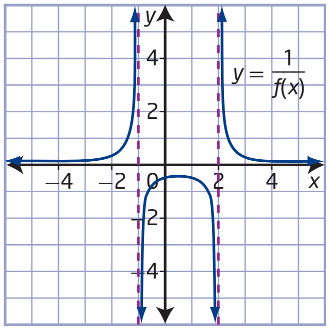


A



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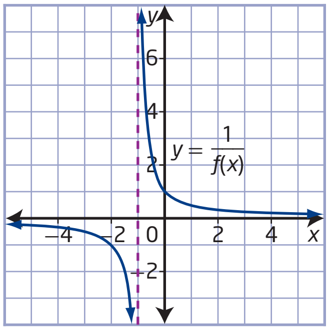
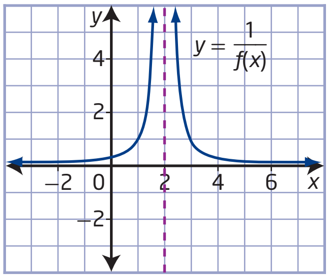


C



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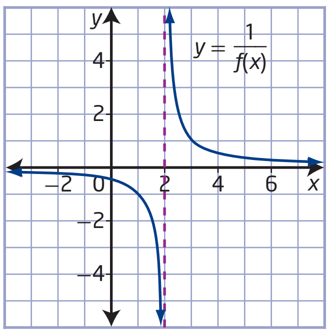


Apply

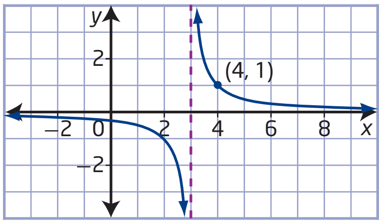
10. Each of the following is the graph of a reciprocal function, $y = \frac{1}{f(x)}$.

i) Sketch the graph of the original function, $y = f(x)$.

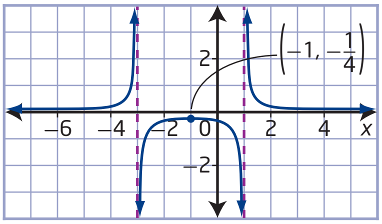
ii) Explain the strategies you used.

iii) What is the original function, $y = f(x)$?

a)



b)



11. You can model the swinging motion of a pendulum using many mathematical rules. For example, the frequency, f, or number of vibrations per second of one swing, in hertz (Hz), equals the reciprocal of the period, T, in seconds, of the swing. The formula is $f = \frac{1}{T}$.

a) Sketch the graph of the function $f = \frac{1}{T}$.

b) What is the reciprocal function?

c) Determine the frequency of a pendulum with a period of 2.5 s.

d) What is the period of a pendulum with a frequency of 1.6 Hz?

Did You Know?

Much of the mathematics of pendulum motion was described by Galileo, based on his curiosity about a swinging lamp in the Cathedral of Pisa, Italy. His work led to much more accurate measurement of time on clocks.

12. The greatest amount of time, t, in minutes, that a scuba diver can take to rise toward the water surface without stopping for decompression is defined by the function $t = \frac{525}{d - 10}$, where d is the depth, in metres, of the diver.

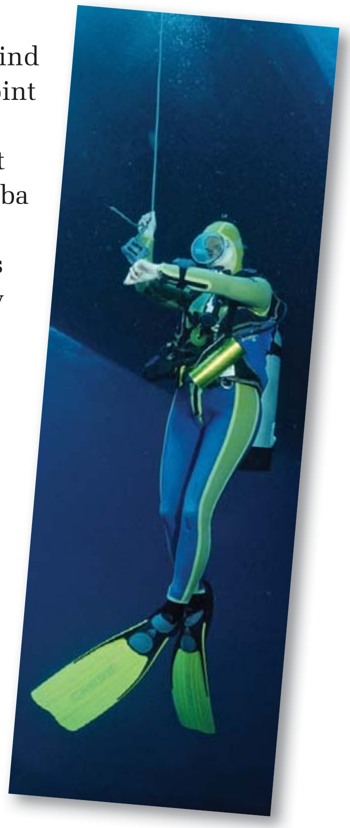
a) Graph the function using graphing technology.

b) Determine a suitable domain which represents this application.

c) Determine the maximum time without stopping for a scuba diver who is 40 m deep.

d) Graph a second function, $t = 40$. Find the intersection point of the two graphs. Interpret this point in terms of the scuba diver rising to the surface. Check this result algebraically with the original function.

e) Does this graph have a horizontal asymptote? What does this mean with respect to the scuba diver?



Did You Know?

If scuba divers rise to the water surface too quickly, they may experience decompression sickness or *the bends*, which is caused by breathing nitrogen or other gases under pressure. The nitrogen bubbles are released into the bloodstream and obstruct blood flow, causing joint pain.

13. The pitch, p, in hertz (Hz), of a musical note is the reciprocal of the period, P, in seconds, of the sound wave for that note created by the air vibrations.

a) Write a function for pitch, p, in terms of period, P.

b) Sketch the graph of the function.

c) What is the pitch, to the nearest 0.1 Hz, for a musical note with period 0.048 s?

14. The intensity, I, in watts per square metre (W/m^2), of a sound equals 0.004 multiplied by the reciprocal of the square of the distance, d, in metres, from the source of the sound.

a) Write a function for I in terms of d to represent this relationship.

b) Graph this function for a domain of $d > 0$.

c) What is the intensity of a car horn for a person standing 5 m from the car?

15. a) Describe how to find the vertex of the parabola defined by $f(x) = x^2 - 6x - 7$.

b) Explain how knowing the vertex in part a) would help you to graph the function $g(x) = \frac{1}{x^2 - 6x - 7}$.

c) Sketch the graph of $g(x) = \frac{1}{x^2 - 6x - 7}$.

16. The amount of time, t, to complete a large job is proportional to the reciprocal of the number of workers, n, on the job. This can be expressed as $t = k\left(\frac{1}{n}\right)$ or $t = \frac{k}{n}$, where k is a constant. For example, the Spiral Tunnels built by the Canadian Pacific Railroad in Kicking Horse Pass, British Columbia, were a major engineering feat when they opened in 1909. Building two spiral tracks each about 1 km long required 1000 workers to work about 720 days. Suppose that each worker performed a similar type of work.

a) Substitute the given values of t and n into the formula to find the constant k.

b) Use technology to graph the function $t = \frac{k}{n}$.

c) How much time would have been required to complete the Spiral Tunnels if only 400 workers were on the job?

d) Determine the number of workers needed if the job was to be completed in 500 days.

Did You Know?

Kicking Horse Pass is in Yoho National Park. *Yoho* is a Cree word meaning great awe or astonishment. This may be a reference to the soaring peaks, the rock walls, and the spectacular Takakkaw Falls nearby.

Extend

17. Use the summary of information to produce the graphs of both $y = f(x)$ and $y = \frac{1}{f(x)}$, given that $f(x)$ is a linear function.

Interval of x	$x < 3$	$x > 3$
Sign of $f(x)$	+	−
Direction of $f(x)$	decreasing	decreasing
Sign of $\frac{1}{f(x)}$	+	−
Direction of $\frac{1}{f(x)}$	increasing	increasing

18. Determine whether each statement is true or false, and explain your reasoning.

a) The graph of $y = \frac{1}{f(x)}$ always has a vertical asymptote.

b) A function in the form of $y = \frac{1}{f(x)}$ always has at least one value for which it is not defined.

c) The domain of $y = \frac{1}{f(x)}$ is always the same as the domain of $y = f(x)$.

Create Connections

19. Rita and Jerry are discussing how to determine the asymptotes of the reciprocal of a given function. Rita concludes that you can determine the roots of the corresponding equation, and those values will lead to the equations of the asymptotes. Jerry assumes that when the function is written in rational form, you can determine the non-permissible values. The non-permissible values will lead to the equations of the asymptotes.

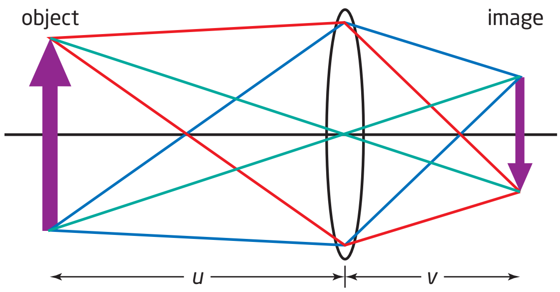
a) Which student has made a correct assumption? Explain your choice.

b) Is this true for both a linear and a quadratic function?

20. The diagram shows how an object forms an inverted image on the opposite side of a convex lens, as in many cameras. Scientists discovered the relationship

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

where u is the distance from the object to the lens, v is the distance from the lens to the image, and f is the focal length of the lens being used.



a) Determine the distance, v, between the lens and the image if the distance, u, to the object is 300 mm and the lens has a focal length, f, of 50 mm.

b) Determine the focal length of a zoom lens if an object 10 000 mm away produces an inverted image 210 mm behind the lens.

21. MINI LAB Use technology to explore the behaviour of a graph near the vertical asymptote and the end behaviour of the graph.

Consider the function

$f(x) = \frac{1}{4x - 2}, x \neq \frac{1}{2}$.

Step 1 Sketch the graph of the function $f(x) = \frac{1}{4x - 2}, x \neq \frac{1}{2}$, drawing in the vertical asymptote.

Step 2 **a)** Copy and complete the tables to show the behaviour of the function as $x \to \left(\frac{1}{2}\right)^-$ and as $x \to \left(\frac{1}{2}\right)^+$, meaning when x approaches $\frac{1}{2}$ from the left (−) and from the right (+).

As $x \to \left(\frac{1}{2}\right)^-$:

x	$f(x)$
0	−0.5
0.4	
0.45	
0.47	
0.49	
0.495	
0.499	

As $x \to \left(\frac{1}{2}\right)^+$:

x	$f(x)$
1	0.5
0.6	
0.55	
0.53	
0.51	
0.505	
0.501	

b) Describe the behaviour of the function as the value of x approaches the asymptote. Will this always happen?

Step 3 **a)** To explore the end behaviour of the function, the absolute value of x is made larger and larger. Copy and complete the tables for values of x that are farther and farther from zero.

As x becomes smaller:

x	$f(x)$
−10	$-\frac{1}{42}$
−100	
−1000	
−10 000	
−100 000	

As x becomes larger:

x	$f(x)$
10	$\frac{1}{38}$
100	
1 000	
10 000	
100 000	

b) Describe what happens to the graph of the reciprocal function as $|x|$ becomes very large.

22. Copy and complete the flowchart to describe the relationship between a function and its corresponding reciprocal function.

Functions

$y = f(x)$	$y = \frac{1}{f(x)}$
The absolute value of the function gets very large.	
	Reciprocal values are positive.
Function values are negative.	
The zeros of the function are the x-intercepts of the graph.	
	The value of the reciprocal function is 1.
The absolute value of the function approaches zero.	
The value of the function is −1.	

Chapter 7 Review

7.1 Absolute Value, pages 358–367

1. Evaluate.

a) $|-5|$ **b)** $\left|2\frac{3}{4}\right|$ **c)** $|-6.7|$

2. Rearrange these numbers in order from least to greatest.

-4, $\sqrt{9}$, $|-3.5|$, -2.7, $\left|-\frac{9}{2}\right|$, $|-1.6|$, $\left|1\frac{1}{2}\right|$

3. Evaluate each expression.

a) $|-7 - 2|$

b) $|-3 + 11 - 6|$

c) $5|-3.75|$

d) $|5^2 - 7| + |-10 + 2^3|$

4. A school group travels to Mt. Robson Provincial Park in British Columbia to hike the Berg Lake Trail. From the Robson River bridge, kilometre 0.0, they hike to Kinney Lake, kilometre 4.2, where they stop for lunch. They then trek across the suspension bridge to the campground, kilometre 10.5. The next day they hike to the shore of Berg Lake and camp, kilometre 19.6. On day three, they hike to the Alberta/British Columbia border, kilometre 21.9, and turn around and return to the campground near Emperor Falls, kilometre 15.0. On the final day, they walk back out to the trailhead, kilometre 0.0. What total distance did the school group hike?

5. Over the course of five weekdays, one mining stock on the Toronto Stock Exchange (TSX) closed at \$4.28 on Monday, closed higher at \$5.17 on Tuesday, finished Wednesday at \$4.79, and shot up to close at \$7.15 on Thursday, only to finish the week at \$6.40.

a) What is the net change in the closing value of this stock for the week?

b) Determine the total change in the closing value of the stock.

7.2 Absolute Value Functions, pages 368–379

6. Consider the functions $f(x) = 5x + 2$ and $g(x) = |5x + 2|$.

a) Create a table of values for each function, using values of -2, -1, 0, 1, and 2 for x.

b) Plot the points and sketch the graphs of the functions on the same coordinate grid.

c) Determine the domain and range for both $f(x)$ and $g(x)$.

d) List the similarities and the differences between the two functions and their corresponding graphs.

7. Consider the functions $f(x) = 8 - x^2$ and $g(x) = |8 - x^2|$.

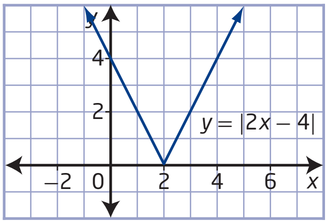
a) Create a table of values for each function, using values of -2, -1, 0, 1, and 2 for x.

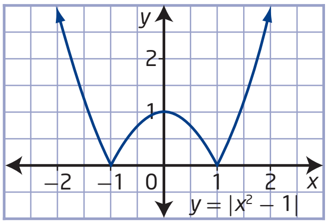
b) Plot the points and sketch the graphs of the functions on the same coordinate grid.

c) Determine the domain and range for both $f(x)$ and $g(x)$.

d) List the similarities and the differences between the two functions and their corresponding graphs.

8. Write the piecewise function that represents each graph.

a) 

b) 

9. a) Explain why the functions $f(x) = 3x^2 + 7x + 2$ and $g(x) = |3x^2 + 7x + 2|$ have different graphs.

b) Explain why the functions $f(x) = 3x^2 + 4x + 2$ and $g(x) = |3x^2 + 4x + 2|$ have identical graphs.

10. An absolute value function has the form $f(x) = |ax + b|$, where $a \neq 0$, $b \neq 0$, and $a, b \in \mathrm{R}$. If the function $f(x)$ has a domain of $\{x \mid x \in \mathrm{R}\}$, a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$, an x-intercept occurring at $\left(-\frac{2}{3}, 0\right)$, and a y-intercept occurring at $(0, 10)$, what are the values of a and b?

7.3 Absolute Value Equations, pages 380–391

11. Solve each absolute value equation graphically. Express answers to the nearest tenth, when necessary.

a) $|2x - 2| = 9$

b) $|7 + 3x| = x - 1$

c) $|x^2 - 6| = 3$

d) $|m^2 - 4m| = 5$

12. Solve each equation algebraically.

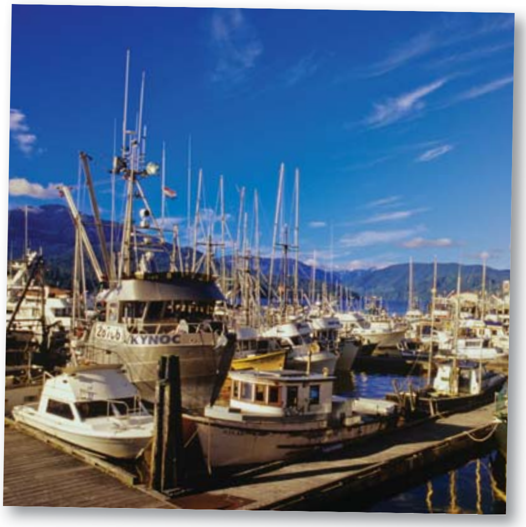
a) $|q + 9| = 2$

b) $|7x - 3| = x + 1$

c) $|x^2 - 6x| = x$

d) $3x - 1 = |4x^2 - x - 4|$

13. In coastal communities, the depth, d, in metres, of water in the harbour varies during the day according to the tides. The maximum depth of the water occurs at high tide and the minimum occurs at low tide. Two low tides and two high tides will generally occur over a 24-h period. On one particular day in Prince Rupert, British Columbia, the depth of the first high tide and the first low tide can be determined using the equation $|d - 4.075| = 1.665$.



a) Find the depth of the water, in metres, at the first high tide and the first low tide in Prince Rupert on this day.

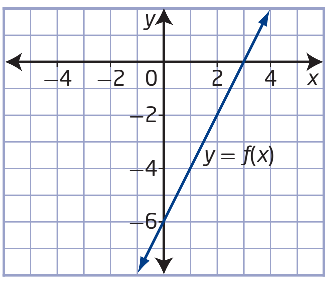
b) Suppose the low tide and high tide depths for Prince Rupert on the next day are 2.94 m, 5.71 m, 2.28 m, and 4.58 m. Determine the total change in water depth that day.

14. The mass, m, in kilograms, of a bushel of wheat depends on its moisture content. Dry wheat has moisture content as low as 5% and wet wheat has moisture content as high as 50%. The equation $|m - 35.932| = 11.152$ can be used to find the extreme masses for both a dry and a wet bushel of wheat. What are these two masses?

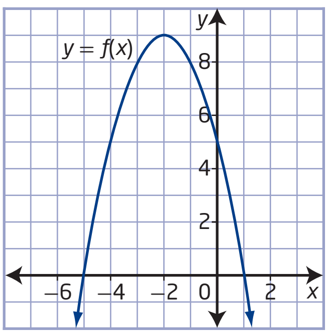
7.4 Reciprocal Functions, pages 392–409

15. Copy each graph of $y = f(x)$ and sketch the graph of the corresponding reciprocal function, $y = \frac{1}{f(x)}$. Label the asymptotes, the invariant points, and the intercepts.

a)



b)



16. Sketch the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ on the same set of axes. Label the asymptotes, the invariant points, and the intercepts.

a) $f(x) = 4x - 9$ **b)** $f(x) = 2x + 5$

17. For each function,

i) determine the corresponding reciprocal function, $y = \frac{1}{f(x)}$

ii) state the non-permissible values of x and the equation(s) of the vertical asymptote(s) of the reciprocal function

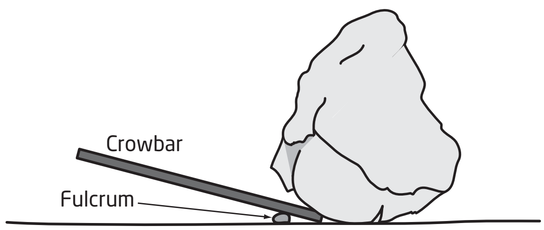
iii) determine the x-intercepts and the y-intercept of the reciprocal function

iv) sketch the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ on the same set of axes

a) $f(x) = x^2 - 25$

b) $f(x) = x^2 - 6x + 5$

18. The force, F, in newtons (N), required to lift an object with a lever is proportional to the reciprocal of the distance, d, in metres, of the force from the fulcrum of a lever. The fulcrum is the point on which a lever pivots. Suppose this relationship can be modelled by the function $F = \frac{600}{d}$.



a) Determine the force required to lift an object if the force is applied 2.5 m from the fulcrum.

b) Determine the distance from the fulcrum of a 450-N force applied to lift an object.

c) How does the force needed to lift an object change if the distance from the fulcrum is doubled? tripled?

Chapter 7 Practice Test

Multiple Choice

For #1 to #5, choose the best answer.

1. The value of the expression $|-9 - 3| - |5 - 2^3| + |-7 + 1 - 4|$ is

A 13

B 19

C 21

D 25

2. The range of the function $f(x) = |x - 3|$ is

A $\{y \mid y > 3, y \in \mathrm{R}\}$

B $\{y \mid y \geq 3, y \in \mathrm{R}\}$

C $\{y \mid y \geq 0, y \in \mathrm{R}\}$

D $\{y \mid y > 0, y \in \mathrm{R}\}$

3. The absolute value equation $|1 - 2x| = 9$ has solution(s)

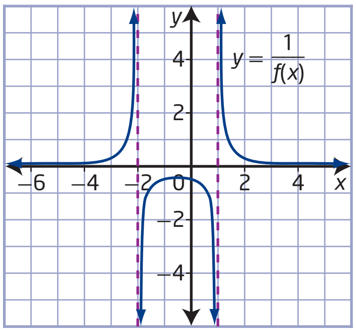
A $x = -4$

B $x = 5$

C $x = -5$ and $x = 4$

D $x = -4$ and $x = 5$

4. The graph represents the reciprocal of which quadratic function?



A $f(x) = x^2 + x - 2$

B $f(x) = x^2 - 3x + 2$

C $f(x) = x^2 - x - 2$

D $f(x) = x^2 + 3x + 2$

5. One of the vertical asymptotes of the graph of the reciprocal function $y = \frac{1}{x^2 - 16}$ has equation

A $x = 0$

B $x = 4$

C $x = 8$

D $x = 16$

Short Answer

6. Consider the function $f(x) = |2x - 7|$.

a) Sketch the graph of the function.

b) Determine the intercepts.

c) State the domain and range.

d) What is the piecewise notation form of the function?

7. Solve the equation $|3x^2 - x| = 4x - 2$ algebraically.

8. Solve the equation $|2w - 3| = w + 1$ graphically.

Extended Response

9. Determine the error(s) in the following solution. Explain how to correct the solution.

Solve $|x - 4| = x^2 + 4x$.

Case 1

$$\begin{aligned} x + 4 &= x^2 + 4x \\ 0 &= x^2 + 3x - 4 \\ 0 &= (x + 4)(x - 1) \end{aligned}$$

$x + 4 = 0$ or $x - 1 = 0$

$x = -4$ or $x = 1$

Case 2

$$\begin{aligned} -x - 4 &= x^2 + 4x \\ 0 &= x^2 + 5x + 4 \\ 0 &= (x + 4)(x + 1) \end{aligned}$$

$x + 4 = 0$ or $x + 1 = 0$

$x = -4$ or $x = -1$

The solutions are $x = -4$, $x = -1$, and $x = 1$.

10. Consider the function $f(x) = 6 - 5x$.

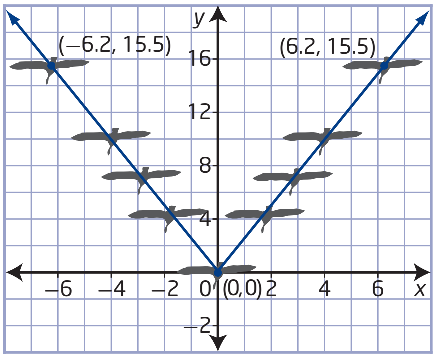
a) Determine its reciprocal function.

b) State the equations of any vertical asymptotes of the reciprocal function.

c) Graph the function $f(x)$ and its reciprocal function. Describe a strategy that could be used to sketch the graph of any reciprocal function.

11. A biologist studying Canada geese migration analysed the vee flight formation of a particular flock using a coordinate system, in metres. The centre of each bird was assigned a coordinate point. The lead bird has the coordinates (0, 0), and the coordinates of two birds at the ends of each leg are (6.2, 15.5) and (−6.2, 15.5).

Bottom View of Flying Geese



a) Write an absolute value function whose graph contains each leg of the vee formation.

b) What is the angle between the legs of the vee formation, to the nearest tenth of a degree?

c) The absolute value function $y = |2.8x|$ describes the flight pattern of a different flock of geese. What is the angle between the legs of this vee formation, to the nearest tenth of a degree?

12. Astronauts in space feel lighter because weight decreases as a person moves away from the gravitational pull of Earth. Weight, W_h, in newtons (N), at a particular height, h, in kilometres, above Earth is related to the reciprocal of that height by the formula $W_h = \dfrac{W_e}{\left(\dfrac{h}{6400} + 1\right)^2}$, where W_e is the person's weight, in newtons (N), at sea level on Earth.

Canadian astronauts Julie Payette and Bob Thirsk

a) Sketch the graph of the function for an astronaut whose weight is 750 N at sea level.

b) Determine this astronaut's weight at a height of

i) 8 km **ii)** 2000 km

c) Determine the range of heights for which this astronaut will have a weight of less than 30 N.

Did You Know?

When people go into space, their mass remains constant but their weight decreases because of the reduced gravity.

Cumulative Review, Chapters 5–7

Chapter 5 Radical Expressions and Equations

1. Express $3xy^3\sqrt{2x}$ as an entire radical.

2. Express $\sqrt{48a^3b^2c^5}$ as a simplified mixed radical.

3. Order the set of numbers from least to greatest.

$3\sqrt{6}$, $\sqrt{36}$, $2\sqrt{3}$, $\sqrt{18}$, $2\sqrt{9}$, $\sqrt[3]{8}$

4. Simplify each expression. Identify any restrictions on the values for the variables.

a) $4\sqrt{2a} + 5\sqrt{2a}$

b) $10\sqrt{20x^2} - 3x\sqrt{45}$

5. Simplify. Identify any restrictions on the values of the variable in part c).

a) $2\sqrt[3]{4}(-4\sqrt[3]{6})$

b) $\sqrt{6}(\sqrt{12} - \sqrt{3})$

c) $(6\sqrt{a} + \sqrt{3})(2\sqrt{a} - \sqrt{4})$

6. Rationalize each denominator.

a) $\dfrac{\sqrt{12}}{\sqrt{4}}$

b) $\dfrac{2}{2 + \sqrt{3}}$

c) $\dfrac{\sqrt{7} + \sqrt{28}}{\sqrt{7} - \sqrt{14}}$

7. Solve the radical equation $\sqrt{x + 6} = x$. Verify your answers.

8. On a children's roller coaster ride, the speed in a loop depends on the height of the hill the car has just come down and the radius of the loop. The velocity, v, in feet per second, of a car at the top of a loop of radius, r, in feet, is given by the formula $v = \sqrt{h - 2r}$, where h is the height of the previous hill, in feet.

a) Find the height of the hill when the velocity at the top of the loop is 20 ft/s and the radius of the loop is 15 ft.

b) Would you expect the velocity of the car to increase or decrease as the radius of the loop increases? Explain your reasoning.

Chapter 6 Rational Expressions and Equations

9. Simplify each expression. Identify any non-permissible values.

a) $\dfrac{12a^3b}{48a^2b^4}$

b) $\dfrac{4 - x}{x^2 - 8x + 16}$

c) $\dfrac{(x - 3)(x + 5)}{x^2 - 1} \div \dfrac{x + 2}{x - 3}$

d) $\dfrac{5x - 10}{6x} \times \dfrac{3x}{15x - 30}$

e) $\left(\dfrac{x + 2}{x - 3}\right)\left(\dfrac{x^2 - 9}{x^2 - 4}\right) \div \left(\dfrac{x + 3}{x - 2}\right)$

10. Determine the sum or difference. Express answers in lowest terms. Identify any non-permissible values.

a) $\dfrac{10}{a + 2} + \dfrac{a - 1}{a - 7}$

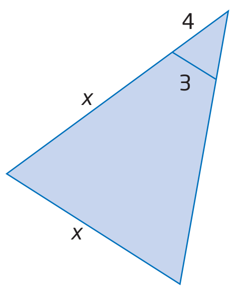
b) $\dfrac{3x + 2}{x + 4} - \dfrac{x - 5}{x^2 - 4}$

c) $\dfrac{2x}{x^2 - 25} - \dfrac{3}{x^2 - 4x - 5}$

11. Sandra simplified the expression $\dfrac{(x + 2)(x + 5)}{x + 5}$ to $x + 2$. She stated that they were equivalent expressions. Do you agree or disagree with Sandra's statement? Provide a reason for your answer.

12. When two triangles are similar, you can use the proportion of corresponding sides to determine an unknown dimension. Solve the rational equation to determine the value of x.

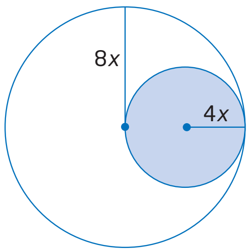
$\dfrac{x + 4}{4} = \dfrac{x}{3}$



13. If a point is selected at random from a figure and is equally likely to be any point inside the figure, then the probability that a point is in the shaded region is given by

$$P = \frac{\text{area of shaded region}}{\text{area of entire figure}}$$

What is the probability that the point is in the shaded region?



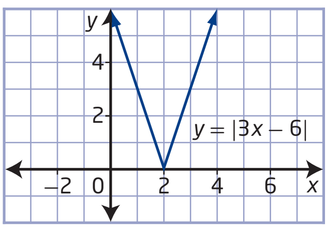
Chapter 7 Absolute Value and Reciprocal Functions

14. Order the values from least to greatest.

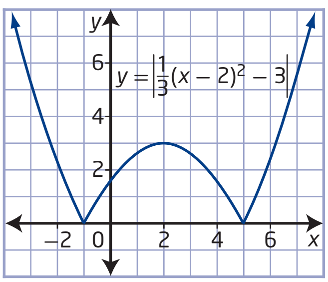
$|-5|$, $|4 - 6|$, $|2(-4) - 5|$, $|8.4|$

15. Write the piecewise function that represents each graph.

a)



b)



16. For each absolute value function,

i) sketch the graph

ii) determine the intercepts

iii) determine the domain and range

a) $y = |3x - 7|$ **b)** $y = |x^2 - 3x - 4|$

17. Solve algebraically. Verify your solutions.

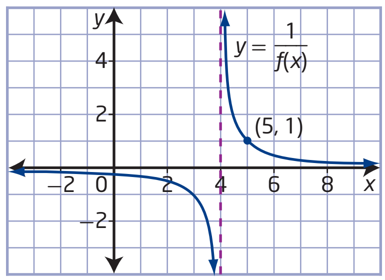
a) $|2x - 1| = 9$ **b)** $|2x^2 - 5| = 13$

18. The area, A, of a triangle on a coordinate grid with vertices at $(0, 0)$, (a, b), and (c, d) can be calculated using the formula $A = \frac{1}{2}|ad - bc|$.

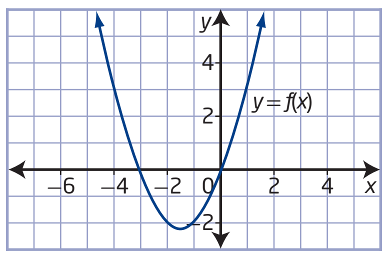
a) Why do you think absolute value must be used in the formula for area?

b) Determine the area of a triangle with vertices at $(0, 0)$, $(-5, 2)$, and $(-3, 4)$.

19. Sketch the graph of $y = f(x)$ given the graph of $y = \frac{1}{f(x)}$. What is the original function, $y = f(x)$?



20. Copy the graph of $y = f(x)$, and sketch the graph of the reciprocal function $y = \frac{1}{f(x)}$. Discuss your method.



21. Sketch the graph of $y = \frac{1}{f(x)}$ given $f(x) = (x + 2)^2$. Label the asymptotes, the invariant points, and the intercepts.

22. Consider the function $f(x) = 3x - 1$.

a) What characteristics of the graph of $y = f(x)$ are different from those of $y = |f(x)|$?

b) Describe how the graph of $y = f(x)$ is different from the graph of $y = \frac{1}{f(x)}$.

Unit 3 Test

Multiple Choice

For #1 to #8, choose the best answer.

1. What is the entire radical form of $2(\sqrt[3]{-27})$?

A $\sqrt[3]{-54}$

B $\sqrt[3]{-108}$

C $\sqrt[3]{-216}$

D $\sqrt[3]{-432}$

2. What is the simplified form of $\dfrac{4\sqrt{72x^5}}{x\sqrt{8}}$, $x > 0$?

A $\dfrac{12\sqrt{2x^3}}{\sqrt{2}}$

B $4x\sqrt{3x}$

C $\dfrac{6\sqrt{2x^3}}{\sqrt{2}}$

D $12x\sqrt{x}$

3. Determine the root(s) of $x + 2 = \sqrt{x^2 + 3}$.

A $x = -\dfrac{1}{4}$ and $x = 3$

B $x = -\dfrac{1}{4}$

C $x = \dfrac{1}{4}$ and $x = -3$

D $x = \dfrac{1}{4}$

4. Simplify the rational expression $\dfrac{9x^4 - 27x^6}{3x^3}$ for all permissible values of x.

A $3x(1 - 3x)$

B $3x(1 - 9x^5)$

C $3x - 9x^3$

D $9x^3 - 9x^4$

5. Which expression could be used to determine the length of the line segment between the points $(4, -3)$ and $(-6, -3)$?

A $-6 - 4$

B $4 - 6$

C $|4 - 6|$

D $|-6 - 4|$

6. Arrange the expressions $|4 - 11|$, $\dfrac{1}{5}|-5|$, $\left|1 - \dfrac{1}{4}\right|$, and $|2| - |4|$ in order from least to greatest.

A $|4 - 11|, \dfrac{1}{5}|-5|, \left|1 - \dfrac{1}{4}\right|, |2| - |4|$

B $|2| - |4|, \left|1 - \dfrac{1}{4}\right|, \dfrac{1}{5}|-5|, |4 - 11|$

C $|2| - |4|, \dfrac{1}{5}|-5|, \left|1 - \dfrac{1}{4}\right|, |4 - 11|$

D $\left|1 - \dfrac{1}{4}\right|, \dfrac{1}{5}|-5|, |4 - 11|, |2| - |4|$

7. Which of the following statements is false?

A $\sqrt{n}\sqrt{m} = \sqrt{mn}$

B $\dfrac{\sqrt{18}}{\sqrt{36}} = \sqrt{\dfrac{1}{2}}$

C $\dfrac{\sqrt{7}}{\sqrt{8n}} = \dfrac{\sqrt{14n}}{4n}$

D $\sqrt{m^2 + n^2} = m + n$

8. The graph of $y = \dfrac{1}{f(x)}$ has vertical asymptotes at $x = -2$ and $x = 5$ and a horizontal asymptote at $y = 0$. Which of the following statements is possible?

A $f(x) = (x + 2)(x + 5)$

B $f(x) = x^2 - 3x - 10$

C The domain of $f(x)$ is $\{x \mid x \neq -2, x \neq -5, x \in \mathrm{R}\}$.

D The range of $y = \dfrac{1}{f(x)}$ is $\{y \mid y \in \mathrm{R}\}$.

Numerical Response

Copy and complete the statements in #9 to #13.

9. The radical $\sqrt{3x - 9}$ results in real numbers when $x \geq$ ■.

10. When the denominator of the expression $\dfrac{\sqrt{5}}{3\sqrt{2}}$ is rationalized, the expression becomes ■.

11. The expression $\frac{3x - 7}{x + 11} - \frac{x - k}{x + 11}$, $x \neq -11$, simplifies to $\frac{2x + 21}{x + 11}$ when the value of k is ■.

12. The lesser solution to the absolute value equation $|1 - 4x| = 9$ is $x =$ ■.

13. The graph of the reciprocal function $f(x) = \frac{1}{x^2 - 4}$ has vertical asymptotes with equations $x =$ ■ and $x =$ ■.

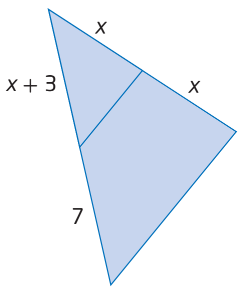
Written Response

14. Order the numbers from least to greatest.
$3\sqrt{7}$, $4\sqrt{5}$, $6\sqrt{2}$, 5

15. Consider the equation $\sqrt{3x + 4} = \sqrt{2x - 5}$.
   a) Describe a possible first step to solve the radical equation.
   b) Determine the restrictions on the values for the variable x.
   c) Algebraically determine all roots of the equation.
   d) Verify the solutions by substitution.

16. Simplify the expression
$$\frac{4x^2 + 4x - 8}{x^2 - 5x + 4} \div \frac{2x^2 + 3x - 2}{4x^2 + 8x - 5}.$$
List all non-permissible values for the variable.

17. The diagram shows two similar triangles.



   a) Write a proportion that relates the sides of the similar triangles.
   b) Determine the non-permissible values for the rational equation.
   c) Algebraically determine the value of x that makes the triangles similar.

18. Consider the function $y = |2x - 5|$.
   a) Sketch the graph of the function.
   b) Determine the intercepts.
   c) State the domain and range.
   d) What is the piecewise notation form of the function?

19. Solve $|x^2 - 3x| = 2$. Verify your solutions graphically.

20. Consider $f(x) = x^2 + 2x - 8$. Sketch the graph of $y = f(x)$ and the graph of $y = \frac{1}{f(x)}$ on the same set of axes. Label the asymptotes, the invariant points, and the intercepts.

21. In the sport of curling, players measure the "weight" of their shots by timing the stone between two marked lines on the ice, usually the hog lines, which are 72 ft apart. The weight, or average speed, s, of the curling stone is proportional to the reciprocal of the time, t, it takes to travel between hog lines.

Canadian women curlers at 2010 Vancouver Olympics

   a) If $d = 72$, rewrite the formula $d = st$ as a function in terms of s.
   b) What is the weight of a stone that takes 14.5 s to travel between hog lines?
   c) How much time is required for a stone to travel between hog lines if its weight is 6.3 ft/s?

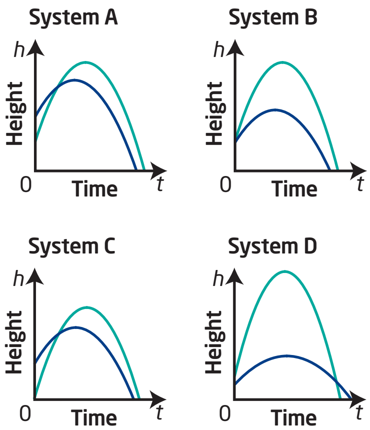
Check Your Understanding

Practise

Where necessary, round answers to the nearest hundredth.

1. The Canadian Arenacross Championship for motocross was held in Penticton, British Columbia, in March 2010. In the competition, riders launch their bikes off jumps and perform stunts. The height above ground level of one rider going off two different jumps at the same speed is plotted. Time is measured from the moment the rider leaves the jump. The launch height and the launch angle of each jump are different.

a) Which system models the situation? Explain your choice. Explain why the other graphs do not model this situation.



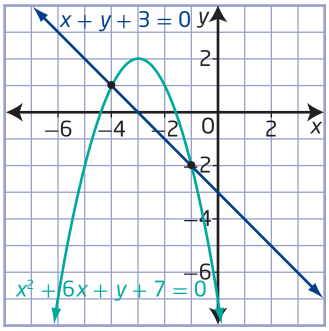
b) Interpret the point(s) of intersection for the graph you selected.

2. Verify that (0, −5) and (3, −2) are solutions to the following system of equations.

$y = -x^2 + 4x - 5$
$y = x - 5$

3. What type of system of equations is represented in each graph? Give the solution(s) to the system.

a)



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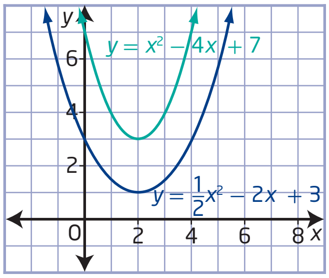
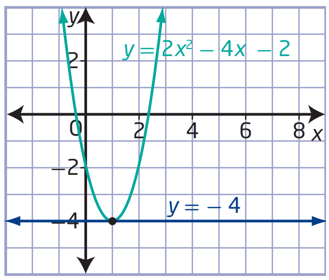


c)



4. Solve each system by graphing. Verify your solutions.

a) $y = x + 7$
$y = (x + 2)^2 + 3$

b) $f(x) = -x + 5$
$g(x) = \frac{1}{2}(x - 4)^2 + 1$

c) $x^2 + 16x + y = -59$
$x - 2y = 60$

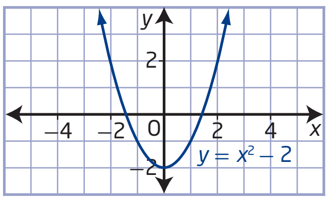
d) $x^2 + y - 3 = 0$
$x^2 - y + 1 = 0$

e) $y = x^2 - 10x + 32$
$y = 2x^2 - 32x + 137$

5. Solve each system by graphing. Verify your solutions.
   a) $h = d^2 - 16d + 60$
   $h = 12d - 55$
   b) $p = 3q^2 - 12q + 17$
   $p = -0.25q^2 + 0.5q + 1.75$
   c) $2v^2 + 20v + t = -40$
   $5v + 2t + 26 = 0$
   d) $n^2 + 2n - 2m - 7 = 0$
   $3n^2 + 12n - m + 6 = 0$
   e) $0 = t^2 + 40t - h + 400$
   $t^2 = h + 30t - 225$

Apply

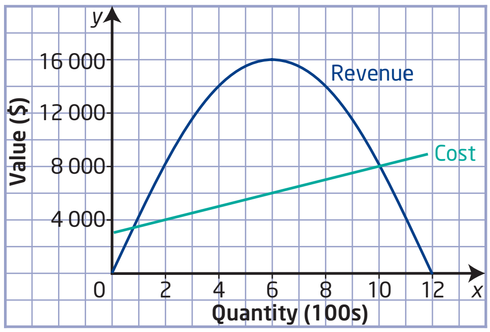
6. Sketch the graph of a system of two quadratic equations with only one real solution. Describe the necessary conditions for this situation to occur.
7. For each situation, sketch a graph to represent a system of quadratic-quadratic equations with two real solutions, so that the two parabolas have
   a) the same axis of symmetry
   b) the same axis of symmetry and the same y-intercept
   c) different axes of symmetry but the same y-intercept
   d) the same x-intercepts
8. Given the graph of a quadratic function as shown, determine the equation of a line such that the quadratic function and the line form a system that has
   a) no real solution
   b) one real solution
   c) two real solutions



9. Every summer, the Folk on the Rocks Music Festival is held at Long Lake in Yellowknife, Northwest Territories.

Dene singer/ songwriter, Leela Gilday from Yellowknife.

Jonas has been selling shirts in the Art on the Rocks area at the festival for the past 25 years. His total costs (production of the shirts plus 15% of all sales to the festival) and the revenue he receives from sales (he has a variable pricing scheme) are shown on the graph below.



   a) What are the solutions to this system? Give answers to the nearest hundred.
   b) Interpret the solution and its importance to Jonas.
   c) You can determine the profit using the equation Profit = Revenue − Cost. Use the graph to estimate the quantity that gives the greatest profit. Explain why this is the quantity that gives him the most profit.

10. Vertical curves are used in the construction of roller coasters. One downward-sloping grade line, modelled by the equation $y = -0.04x + 3.9$, is followed by an upward-sloping grade line modelled by the equation $y = 0.03x + 2.675$. The vertical curve between the two lines is modelled by the equation $y = 0.001x^2 - 0.04x + 3.9$. Determine the coordinates of the beginning and the end of the curve.

11. A car manufacturer does performance tests on its cars. During one test, a car starts from rest and accelerates at a constant rate for 20 s. Another car starts from rest 3 s later and accelerates at a faster constant rate. The equation that models the distance the first car travels is $d = 1.16t^2$, and the equation that models the distance the second car travels is $d = 1.74(t - 3)^2$, where t is the time, in seconds, after the first car starts the test, and d is the distance, in metres.

a) Write a system of equations that could be used to compare the distance travelled by both cars.

b) In the context, what is a suitable domain for the graph? Sketch the graph of the system of equations.

c) Graphically determine the approximate solution to the system.

d) Describe the meaning of the solution in the context.

12. Jubilee Fountain in Lost Lagoon is a popular landmark in Vancouver's Stanley Park. The streams of water shooting out of the fountain follow parabolic paths. Suppose the tallest stream in the middle is modelled by the equation $h = -0.3125d^2 + 5d$, one of the smaller streams is modelled by the equation $h = -0.85d^2 + 5.11d$, and a second smaller stream is modelled by the equation $h = -0.47d^2 + 3.2d$, where h is the height, in metres, of the water stream and d is the distance, in metres, from the central water spout.

a) Solve the system $h = -0.3125d^2 + 5d$ and $h = -0.85d^2 + 5.11d$ graphically. Interpret the solution.

b) Solve the system of equations involving the two smaller streams of water graphically. Interpret the solution.

Did You Know?

Jubilee Fountain was built in 1936 to commemorate the city of Vancouver's golden jubilee (50th birthday).

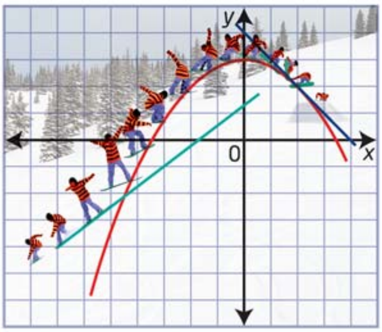
13. The sum of two integers is 21. Fifteen less than double the square of the smaller integer gives the larger integer.

a) Model this information with a system of equations.

b) Solve the system graphically. Interpret the solution.

c) Verify your solution.

14. A Cartesian plane is superimposed over a photograph of a snowboarder completing a 540° Front Indy off a jump. The blue line is the path of the jump and can be modelled by the equation $y = -x + 4$. The red parabola is the path of the snowboarder during the jump and can be modelled by the equation $y = -\frac{1}{4}x^2 + 3$. The green line is the mountainside where the snowboarder lands and can be modelled by the equation $y = \frac{3}{4}x + \frac{5}{4}$.



a) Determine the solutions to the linear-quadratic systems: the blue line and the parabola, and the green line and the parabola.

b) Explain the meaning of the solutions in this context.

15. A frog jumps to catch a grasshopper. The frog reaches a maximum height of 25 cm and travels a horizontal distance of 100 cm. A grasshopper, located 30 cm in front of the frog, starts to jump at the same time as the frog. The grasshopper reaches a maximum height of 36 cm and travels a horizontal distance of 48 cm. The frog and the grasshopper both jump in the same direction.

a) Consider the frog's starting position to be at the origin of a coordinate grid. Draw a diagram to model the given information.

b) Determine a quadratic equation to model the frog's height compared to the horizontal distance it travelled and a quadratic equation to model the grasshopper's height compared to the horizontal distance it travelled.

c) Solve the system of two equations.

d) Interpret your solution in the context of this problem.

Extend

16. The Greek mathematician, Menaechmus (about 380 B.C.E. to 320 B.C.E.) was one of the first to write about parabolas. His goal was to solve the problem of "doubling the cube." He used the intersection of curves to find x and y so that $\frac{a}{x} = \frac{x}{y} = \frac{y}{2a}$, where a is the side of a given cube and x is the side of a cube that has twice the volume. Doubling a cube whose side length is 1 cm is equivalent to solving $\frac{1}{x} = \frac{x}{y} = \frac{y}{2}$.

a) Use a system of equations to graphically solve $\frac{1}{x} = \frac{x}{y} = \frac{y}{2}$.

b) What is the approximate side length of a cube whose volume is twice the volume of a cube with a side length of 1 cm?

c) Verify your answer.

d) Explain how you could use the volume formula to find the side length of a cube whose volume is twice the volume of a cube with a side length of 1 cm. Why was Menaechmus unable to use this method to solve the problem?

Did You Know?

Duplicating the cube is a classic problem of Greek mathematics: *Given the length of an edge of a cube, construct a second cube that has double the volume of the first.* The problem was to find a ruler-and-compasses construction for the cube root of 2. Legend has it that the oracle at Delos requested that this construction be performed to appease the gods and stop a plague. This problem is sometimes referred to as the Delian problem.

17. The solution to a system of two equations is $(-1, 2)$ and $(2, 5)$.

a) Write a linear-quadratic system of equations with these two points as its solutions.

b) Write a quadratic-quadratic system of equations with these two points as its only solutions.

c) Write a quadratic-quadratic system of equations with these two points as two of infinitely many solutions.

18. Determine the possible number of solutions to a system involving two quadratic functions and one linear function. Support your work with a series of sketches. Compare your work with that of a classmate.

Create Connections

19. Explain the similarities and differences between linear systems and the systems you studied in this section.

20. Without graphing, use your knowledge of linear and quadratic equations to determine whether each system has no solution, one solution, two solutions, or an infinite number of solutions. Explain how you know.

a) $y = x^2$
$y = x + 1$

b) $y = 2x^2 + 3$
$y = -2x - 5$

c) $y = (x - 4)^2 + 1$
$y = \frac{1}{3}(x - 4)^2 + 2$

d) $y = 2(x + 8)^2 - 9$
$y = -2(x + 8)^2 - 9$

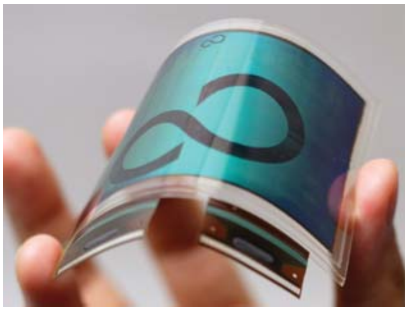
e) $y = 2(x - 3)^2 + 1$
$y = -2(x - 3)^2 - 1$

f) $y = (x + 5)^2 - 1$
$y = x^2 + 10x + 24$

Project Corner Nanotechnology

Nanotechnology has applications in a wide variety of areas.

- **Electronics:** Nanoelectronics will produce new devices that are small, require very little electricity, and produce little (if any) heat.
- **Energy:** There will be advances in solar power, hydrogen fuel cells, thermoelectricity, and insulating materials.
- **Health Care:** New drug-delivery techniques will be developed that will allow medicine to be targeted directly at a disease instead of the entire body.
- **The Environment:** Renewable energy, more efficient use of resources, new filtration systems, water purification processes, and materials that detect and clean up environmental contaminants are some of the potential eco-friendly uses.
- **Everyday Life:** Almost all areas of your life may be improved by nanotechnology: from the construction of your house, to the car you drive, to the clothes you wear.



Which applications of nanotechnology have you used?

Key Ideas

- Solve systems of linear-quadratic or quadratic-quadratic equations algebraically by using either a substitution method or an elimination method.
- To solve a system of equations in two variables using substitution,
  - isolate one variable in one equation
  - substitute the expression into the other equation and solve for the remaining variable
  - substitute the value(s) into one of the original equations to determine the corresponding value(s) of the other variable
  - verify your answer by substituting into both original equations
- To solve a system of equations in two variables using elimination,
  - if necessary, rearrange the equations so that the like terms align
  - if necessary, multiply one or both equations by a constant to create equivalent equations with a pair of variable terms with opposite coefficients
  - add or subtract to eliminate one variable and solve for the remaining variable
  - substitute the value(s) into one of the original equations to determine the corresponding value(s) of the other variable
  - verify your answer(s) by substituting into both original equations

Check Your Understanding

Practise

Where necessary, round your answers to the nearest hundredth.

1. Verify that (5, 7) is a solution to the following system of equations.

$k + p = 12$
$4k^2 - 2p = 86$

2. Verify that $\left(\frac{1}{3}, \frac{3}{4}\right)$ is a solution to the following system of equations.

$18w^2 - 16z^2 = -7$
$144w^2 + 48z^2 = 43$

3. Solve each system of equations by substitution, and verify your solution(s).

a) $x^2 - y + 2 = 0$
$4x = 14 - y$

b) $2x^2 - 4x + y = 3$
$4x - 2y = -7$

c) $7d^2 + 5d - t - 8 = 0$
$10d - 2t = -40$

d) $3x^2 + 4x - y - 8 = 0$
$y + 3 = 2x^2 + 4x$

e) $y + 2x = x^2 - 6$
$x + y - 3 = 2x^2$

4. Solve each system of equations by elimination, and verify your solution(s).

a) $6x^2 - 3x = 2y - 5$
$2x^2 + x = y - 4$

b) $x^2 + y = 8x + 19$
$x^2 - y = 7x - 11$

c) $2p^2 = 4p - 2m + 6$
$5m + 8 = 10p + 5p^2$

d) $9w^2 + 8k = -14$
$w^2 + k = -2$

e) $4h^2 - 8t = 6$
$6h^2 - 9 = 12t$

5. Solve each system algebraically. Explain why you chose the method you used.

a) $y - 1 = -\frac{7}{8}x$
$3x^2 + y = 8x - 1$

b) $8x^2 + 5y = 100$
$6x^2 - x - 3y = 5$

c) $x^2 - \frac{48}{9}x + \frac{1}{3}y + \frac{1}{3} = 0$
$-\frac{5}{4}x^2 - \frac{3}{2}x + \frac{1}{4}y - \frac{1}{2} = 0$

Apply

6. Alex and Kaela are considering the two equations $n - m^2 = 7$ and $2m^2 - 2n = -1$. Without making any calculations, they both claim that the system involving these two equations has no solution.

Alex's reasoning:
If I double every term in the first equation and then use the elimination method, both of the variables will disappear, so the system does not have a solution.

Kaela's reasoning:
If I solve the first equation for n and substitute into the second equation, I will end up with an equation without any variables, so the system does not have a solution.

a) Is each person's reasoning correct?

b) Verify the conclusion graphically.

7. Marie-Soleil solved two systems of equations using elimination. Instead of creating opposite terms and adding, she used a subtraction method. Her work for the elimination step in two different systems of equations is shown below.

First System

$$\begin{array}{rcl} 5x + 2y & = & 12 \\ x^2 - 2x + 2y & = & 7 \\ \hline -x^2 + 7x & = & 5 \end{array}$$

Second System

$$\begin{array}{rcl} 12m^2 - 4m - 8n & = & -3 \\ 9m^2 - m - 8n & = & 2 \\ \hline 3m^2 - 3m & = & -5 \end{array}$$

a) Study Marie-Soleil's method. Do you think this method works? Explain.

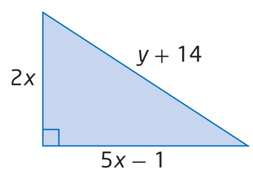
b) Redo the first step in each system by multiplying one of the equations by -1 and adding. Did you get the same results as Marie-Soleil?

c) Do you prefer to add or subtract to eliminate a variable? Explain why.

8. Determine the values of m and n if (2, 8) is a solution to the following system of equations.

$mx^2 - y = 16$
$mx^2 + 2y = n$

9. The perimeter of the right triangle is 60 m. The area of the triangle is $10y$ square metres.



a) Write a simplified expression for the triangle's perimeter in terms of x and y.

b) Write a simplified expression for the triangle's area in terms of x and y.

c) Write a system of equations and explain how it relates to this problem.

d) Solve the system for x and y. What are the dimensions of the triangle?

e) Verify your solution.

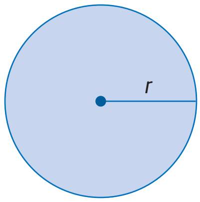
10. Two integers have a difference of −30. When the larger integer is increased by 3 and added to the square of the smaller integer, the result is 189.

a) Model the given information with a system of equations.

b) Determine the value of the integers by solving the system.

c) Verify your solution.

11. The number of centimetres in the circumference of a circle is three times the number of square centimetres in the area of the circle.



a) Write the system of linear-quadratic equations, in two variables, that models the circle with the given property.

b) What are the radius, circumference, and area of the circle with this property?

12. A 250-g ball is thrown into the air with an initial velocity of 22.36 m/s. The kinetic energy, E_k, of the ball is given by the equation $E_k = \frac{5}{32}(d - 20)^2$ and its potential energy, E_p, is given by the equation $E_p = -\frac{5}{32}(d - 20)^2 + 62.5$, where energy is measured in joules (J) and d is the horizontal distance travelled by the ball, in metres.

a) At what distances does the ball have the same amount of kinetic energy as potential energy?

b) How many joules of each type of energy does the ball have at these distances?

c) Verify your solution by graphing.

d) When an object is thrown into the air, the total mechanical energy of the object is the sum of its kinetic energy and its potential energy. On Earth, one of the properties of an object in motion is that the total mechanical energy is a constant. Does the graph of this system show this property? Explain how you could confirm this observation.

13. The 2015-m-tall Mount Asgard in Auyuittuq (ow you eet took) National Park, Baffin Island, Nunavut, was used in the opening scene for the James Bond movie *The Spy Who Loved Me*. A stuntman skis off the edge of the mountain, free-falls for several seconds, and then opens a parachute. The height, h, in metres, of the stuntman above the ground t seconds after leaving the edge of the mountain is given by two functions.

$h(t) = -4.9t^2 + 2015$ represents the height of the stuntman before he opens the parachute.

$h(t) = -10.5t + 980$ represents the height of the stuntman after he opens the parachute.

a) For how many seconds does the stuntman free-fall before he opens his parachute?

b) What height above the ground was the stuntman when he opened the parachute?

c) Verify your solutions.

Did You Know?

Mount Asgard, named after the kingdom of the gods in Norse mythology, is known as Sivanitirutinguak (see va kneek tea goo ting goo ak) to Inuit. This name, in Inuktitut, means "shape of a bell."

14. A table of values is shown for two different quadratic functions.

First Quadratic

x	y
−1	2
0	0
1	2
2	8

Second Quadratic

x	y
−5	4
−4	1
−3	0
−2	1

a) Use paper and pencil to plot each set of ordered pairs on the same grid. Sketch the quadratic functions.

b) Estimate the solution to the system involving the two quadratic functions.

c) Determine a quadratic equation for each function and model a quadratic-quadratic system with these equations.

d) Solve the system of equations algebraically. How does your solution compare to your estimate in part b)?

15. When a volcano erupts, it sends lava fragments flying through the air. From the point where a fragment is blasted into the air, its height, h, in metres, relative to the horizontal distance travelled, x, in metres, can be approximated using the function $h(x) = -\dfrac{4.9}{(v_0 \cos \theta)^2}x^2 + (\tan \theta)x + h_0$, where v_0 is the initial velocity of the fragment, in metres per second; θ is the angle, in degrees, relative to the horizontal at which the fragment starts its path; and h_0 is the initial height, in metres, of the fragment.

a) The height of the summit of a volcano is 2500 m. If a lava fragment blasts out of the middle of the summit at an angle of 45° travelling at 60 m/s, confirm that the function $h(x) = -0.003x^2 + x + 2500$ approximately models the fragment's height relative to the horizontal distance travelled. Confirm that a fragment blasted out at an angle of 60° travelling at 60 m/s can be approximately modelled by the function $h(x) = -0.005x^2 + 1.732x + 2500$.

b) Solve the system

$h(x) = -0.003x^2 + x + 2500$
$h(x) = -0.005x^2 + 1.732x + 2500$

c) Interpret your solution and the conditions required for it to be true.

Did You Know?

Iceland is one of the most active volcanic areas in the world. On April 14, 2010, when Iceland's Eyjafjallajokull (ay yah fyah lah yoh kuul) volcano had a major eruption, ash was sent high into Earth's atmosphere and drifted south and east toward Europe. This large ash cloud wreaked havoc on air traffic. In fear that the airplanes' engines would be clogged by the ash, thousands of flights were cancelled for many days.

16. In western Canada, helicopter "bombing" is used for avalanche control. In high-risk areas, explosives are dropped onto the mountainside to safely start an avalanche. The function $h(x) = -\dfrac{5}{1600}x^2 + 200$ represents the height, h, in metres, of the explosive once it has been thrown from the helicopter, where x is the horizontal distance, in metres, from the base of the mountain. The mountainside is modelled by the function $h(x) = 1.19x$.

a) How can the following system of equations be used for this scenario?

$h = -\dfrac{5}{1600}x^2 + 200$
$h = 1.19x$

b) At what height up the mountain does the explosive charge land?

17. The monthly economic situation of a manufacturing firm is given by the following equations.

$$R = 5000x - 10x^2$$
$$R_M = 5000 - 20x$$
$$C = 300x + \frac{1}{12}x^2$$
$$C_M = 300 + \frac{1}{4}x^2$$

where x represents the quantity sold, R represents the firm's total revenue, R_M represents marginal revenue, C represents total cost, and C_M represents the marginal cost. All costs are in dollars.

a) Maximum profit occurs when marginal revenue is equal to marginal cost. How many items should be sold to maximize profit?

b) Profit is total revenue minus total cost. What is the firm's maximum monthly profit?

Extend

18. Kate is an industrial design engineer. She is creating the program for cutting fabric for a shade sail. The shape of a shade sail is defined by three intersecting parabolas. The equations of the parabolas are

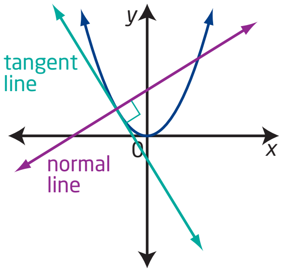
$$y = x^2 + 8x + 16$$
$$y = x^2 - 8x + 16$$
$$y = -\frac{x^2}{8} + 2$$

where x and y are measurements in metres.

a) Use an algebraic method to determine the coordinates of the three vertices of the sail.

b) Estimate the area of material required to make the sail.

19. A normal line is a line that is perpendicular to a tangent line of a curve at the point of tangency.



The line $y = 4x - 2$ is tangent to the curve $y = 2x^2 - 4x + 6$ at a point A.

a) What are the coordinates of point A?

b) What is the equation of the normal line to the curve $y = 2x^2 - 4x + 6$ at the point A?

c) The normal line intersects the curve again at point B, creating chord AB. Determine the length of this chord.

20. Solve the following system of equations using an algebraic method.

$$y = \frac{2x - 1}{x}$$
$$\frac{x}{x + 2} + y - 2 = 0$$

21. Determine the equations for the linear-quadratic system with the following properties. The vertex of the parabola is at $(-1, -4.5)$. The line intersects the parabola at two points. One point of intersection is on the y-axis at $(0, -4)$ and the other point of intersection is on the x-axis.

Create Connections

22. Consider graphing methods and algebraic methods for solving a system of equations. What are the advantages and the disadvantages of each method? Create your own examples to model your answer.

23. A parabola has its vertex at $(-3, -1)$ and passes through the point $(-2, 1)$. A second parabola has its vertex at $(-1, 5)$ and has y-intercept 4. What are the approximate coordinates of the point(s) at which these parabolas intersect?

24. Use algebraic reasoning to show that the graphs of $y = -\frac{1}{2}x - 2$ and $y = x^2 - 4x + 2$ do not intersect.

25. **MINI LAB** In this activity, you will explore the effects that varying the parameters b and m in a linear equation have on a system of linear-quadratic equations.

Step 1 Consider the system of linear-quadratic equations $y = x^2$ and $y = x + b$, where $b \in R$. Graph the system of equations for different values of b. Experiment with changing the value of b so that for some of your values of b the parabola and the line intersect in two points, and not for others. For what value of b do the parabola and the line intersect in exactly one point? Based on your results, predict the values of b for which the system has two real solutions, one real solution, and no real solution.

Step 2 Algebraically determine the values of b for which the system has two real solutions, one real solution, and no real solution.

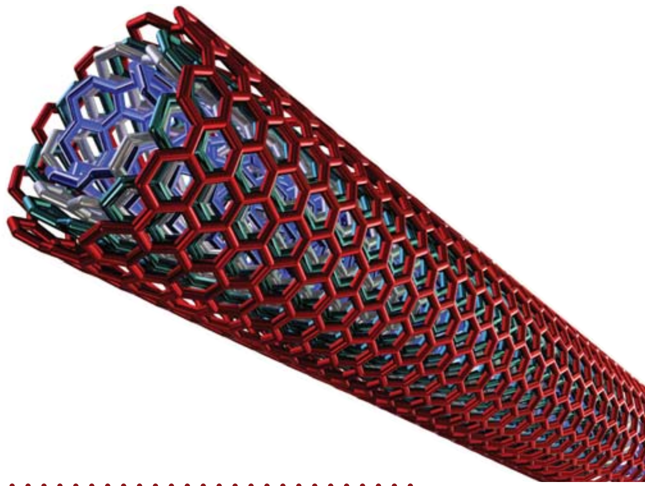
Step 3 Consider the system of linear-quadratic equations $y = x^2$ and $y = mx - 1$, where $m \in R$. Graph the system of equations for different values of m. Experiment with changing the value of m. For what value of m do the parabola and the line intersect in exactly one point? Based on your results, predict the values of m for which the system has two real solutions, one real solution, and no real solution.

Step 4 Algebraically determine the values of m for which the system has two real solutions, one real solution and no real solution.

Step 5 Consider the system of linear-quadratic equations $y = x^2$ and $y = mx + b$, where $m, b \in R$. Determine the conditions on m and b for which the system has two real solutions, one real solution, and no real solution.

Project Corner — Carbon Nanotubes and Engineering

- Carbon nanotubes are cylindrical molecules made of carbon. They have many amazing properties and, as a result, can be used in a number of different applications.
- Carbon nanotubes are up to 100 times stronger than steel and only $\frac{1}{16}$ its mass.
- Researchers mix nanotubes with plastics as reinforcers.



Nanotechnology is already being applied to sports equipment such as bicycles, golf clubs, and tennis rackets. Future uses will include things like aircraft, bridges, and cars. What are some other things that could be enhanced by this stronger and lighter product?

Chapter 8 Review

8.1 Solving Systems of Equations Graphically, pages 424–439

Where necessary, round your answers to the nearest hundredth.

1. Consider the tables of values for $y = -1.5x - 2$ and $y = -2(x - 4)^2 + 3$.

x	y
0.5	−2.75
1	−3.5
1.5	−4.25
2	−5
2.5	−5.75
3	−6.5

x	y
0.5	−21.5
1	−15
1.5	−9.5
2	−5
2.5	−1.5
3	1

a) Use the tables to determine a solution to the system of equations

$y = -1.5x - 2$
$y = -2(x - 4)^2 + 3$

b) Verify this solution by graphing.

c) What is the other solution to the system?

2. State the number of possible solutions to each system. Include sketches to support your answers.

a) a system involving a parabola and a horizontal line

b) a system involving two parabolas that both open upward

c) a system involving a parabola and a line with a positive slope

3. Solve each system of equations by graphing.

a) $y = \frac{2}{3}x + 4$
$y = -3(x + 6)^2$

b) $y = x^2 - 4x + 1$
$y = -\frac{1}{2}(x - 2)^2 + 3$

4. Adam graphed the system of quadratic equations $y = x^2 + 1$ and $y = x^2 + 3$ on a graphing calculator. He speculates that the two graphs will intersect at some large value of y. Is Adam correct? Explain.

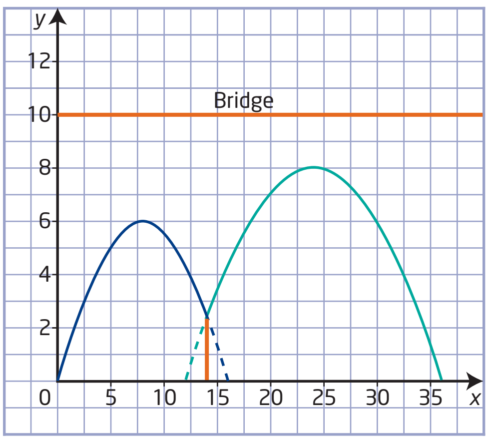
5. Solve each system of equations by graphing.

a) $p = \frac{1}{3}(x + 2)^2 + 2$
$p = \frac{1}{3}(x - 1)^2 + 3$

b) $y = -6x^2 - 4x + 7$
$y = x^2 + 2x - 6$

c) $t = -3d^2 - 2d + 3.25$
$t = \frac{1}{8}d - 5$

6. An engineer constructs side-by-side parabolic arches to support a bridge over a road and a river. The arch over the road has a maximum height of 6 m and a width of 16 m. The river arch has a maximum height of 8 m, but its width is reduced by 4 m because it intersects the arch over the road. Without this intersection, the river arch would have a width of 24 m. A support footing is used at the intersection point of the arches. The engineer sketched the arches on a coordinate system. She placed the origin at the left most point of the road.



a) Determine the equation that models each arch.

b) Solve the system of equations.

c) What information does the solution to the system give the engineer?

7. Caitlin is at the base of a hill with a constant slope. She kicks a ball as hard as she can up the hill.

   a) Explain how the following system models this situation.

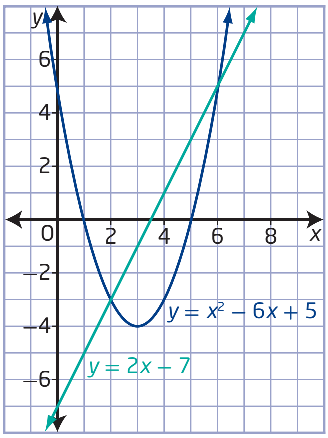
   $h = -0.09d^2 + 1.8d$

   $h = \frac{1}{2}d$

   b) Solve the system.

   c) Interpret the point(s) of intersection in the context.

8.2 Solving Systems of Equations Algebraically, pages 440–456

8. 

   a) Estimate the solutions to the system of equations shown in the graph.

   b) Solve the system algebraically.

9. Without solving the system $4m^2 - 3n = -2$ and $m^2 + \frac{7}{2}m + 5n = 7$, determine which solution is correct: $\left(\frac{1}{2}, 1\right)$ or $\left(\frac{1}{2}, -1\right)$.

10. Solve each system algebraically, giving exact answers. Explain why you chose the method you used.

    a) $p = 3k + 1$
    $p = 6k^2 + 10k - 4$

    b) $4x^2 + 3y = 1$
    $3x^2 + 2y = 4$

    c) $\frac{w^2}{2} + \frac{w}{4} - \frac{z}{2} = 3$
    $\frac{w^2}{3} - \frac{3w}{4} + \frac{z}{6} + \frac{1}{3} = 0$

    d) $2y - 1 = x^2 - x$
    $x^2 + 2x + y - 3 = 0$

11. The approximate height, h, in metres, travelled by golf balls hit with two different clubs over a horizontal distance of d metres is given by the following functions:

    seven-iron: $h(d) = -0.002d^2 + 0.3d$
    nine-iron: $h(d) = -0.004d^2 + 0.5d$

    a) At what distances is the ball at the same height when either of the clubs is used?

    b) What is this height?

12. Manitoba has many biopharmaceutical companies. Suppose scientists at one of these companies grow two different cell cultures in an identical nutrient-rich medium. The rate of increase, S, in square millimetres per hour, of the surface area of each culture after t hours is modelled by the following quadratic functions:

    First culture: $S(t) = -0.007t^2 + 0.05t$
    Second culture: $S(t) = -0.0085t^2 + 0.06t$

    a) What information would the scientists gain by solving the system of related equations?

    b) Solve the system algebraically.

    c) Interpret your solution.

Chapter 8 Practice Test

Multiple Choice

For #1 to #5, choose the best answer.

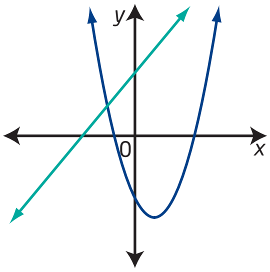
1. The graph for a system of equations is shown. In which quadrant(s) is there a solution to the system?

A I only

B II only

C I and II only

D II and III only



2. The system $y = \frac{1}{2}(x - 6)^2 + 2$ and $y = 2x + k$ has no solution. How many solutions does the system $y = -\frac{1}{2}(x - 6)^2 + 2$ and $y = 2x + k$ have?

A none

B one

C two

D infinitely many

3. Tables of values are shown for two different quadratic functions. What conclusion can you make about the related system of equations?

x	y
1	6
2	–3
3	–6
4	–3
5	6

x	y
1	–6
2	–3
3	–2
4	–3
5	–6

A It does not have a solution.

B It has at least two real solutions.

C It has an infinite number of solutions.

D It is quadratic-quadratic with a common vertex.

4. What is the solution to the following system of equations?

$y = (x + 2)^2 - 2$
$y = \frac{1}{2}(x + 2)^2$

A no solution

B $x = 2$

C $x = -4$ and $x = 2$

D $x = -4$ and $x = 0$

5. Connor used the substitution method to solve the system

$5m - 2n = 25$
$3m^2 - m + n = 10$

Below is Connor's solution for m. In which line did he make an error?

Connor's solution:
Solve the second equation for n:
$n = 10 - 3m^2 + m$ line 1
Substitute into the first equation:
$5m - 2(10 - 3m^2 + m) = 25$ line 2
$5m - 20 + 6m^2 - 2m = 25$
$6m^2 + 3m - 45 = 0$ line 3
$2m^2 + m - 15 = 0$
$(2m + 5)(m - 3) = 0$ line 4
$m = 2.5$ or $m = -3$

A line 1 **B** line 2

C line 3 **D** line 4

Short Answer

Where necessary, round your answers to the nearest hundredth.

6. A student determines that one solution to a system of quadratic-quadratic equations is (2, 1). What is the value of n if the equations are

$4x^2 - my = 10$
$mx^2 + ny = 20$

7. Solve algebraically.

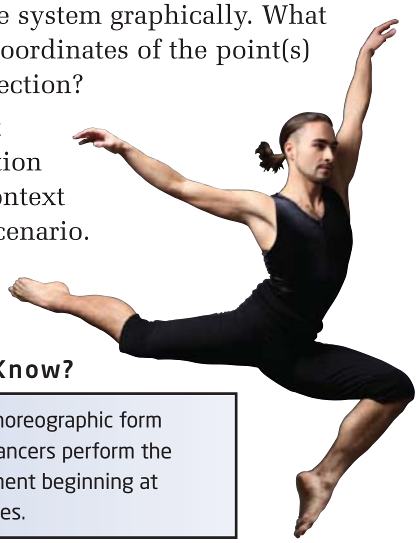
a) $5x^2 + 3y = -3 - x$
$2x^2 - x = -4 - 2y$

b) $y = 7x - 11$
$5x^2 - 3x - y = 6$

8. For a dance routine, the choreographer has arranged for two dancers to perform jeté jumps in canon. Sophie leaps first, and one count later Noah starts his jump. Sophie's jump can by modelled by the equation $h = -4.9t^2 + 5.1t$ and Noah's by the equation $h = -4.9(t - 0.5)^2 + 5.3(t - 0.5)$. In both equations, t is the time in seconds and h is the height in metres.

a) Solve the system graphically. What are the coordinates of the point(s) of intersection?

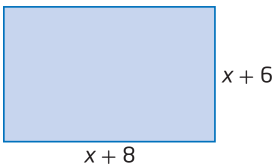
b) Interpret the solution in the context of this scenario.



Did You Know?

Canon is a choreographic form where the dancers perform the same movement beginning at different times.

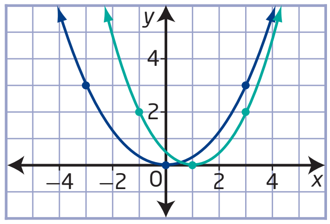
9. The perimeter of the rectangle is represented by $8y$ metres and the area is represented by $(6y + 3)$ square metres.



a) Write two equations in terms of x and y: one for the perimeter and one for the area of the rectangle.

b) Determine the perimeter and the area.

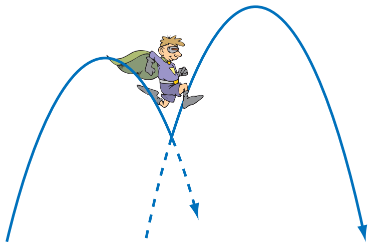
10. a) Determine a system of quadratic equations for the functions shown.



b) Solve the system algebraically.

Extended Response

11. Computer animators design game characters to have many different abilities. The double-jump mechanic allows the character to do a second jump while in mid-air and change its first trajectory to a new one.

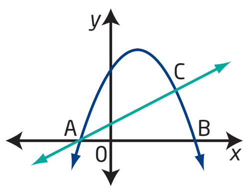


During a double jump, the first part of the jump is modelled by the equation $h = -12.8d^2 + 6.4d$, and the second part is modelled by the equation $h = -\frac{248}{15}(d - 0.7)^2 + 2$. In both equations, d is the horizontal distance and h is the height, in centimetres.

a) Solve the system of quadratic-quadratic equations by graphing.

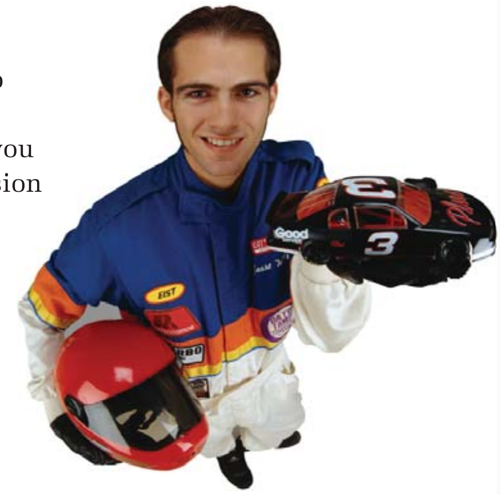
b) Interpret your solution.

12. The parabola $y = -x^2 + 4x + 26.5$ intersects the x-axis at points A and B. The line $y = 1.5x + 5.25$ intersects the parabola at points A and C. Determine the approximate area of $\triangle ABC$.



Unit 4 Project

Nanotechnology



This part of your project will require you to be creative and to use your math skills. Combining your knowledge of parabolas and quadratic systems with the nanotechnology information you have gathered in this chapter, you will design a futuristic version of a every-day object. The object should have some linear and parabolic design lines.

Chapter 8 Task

Choose an object that you feel could be improved using nanotechnology. Look at the information presented in this chapter's Project Corners to give you ideas.

- Explain how the object you have chosen will be enhanced by using nanotechnology.
- Create a new design for your chosen object. Your design must include intersections of parabolic and linear design curves.
- Your design will inevitably go through a few changes as you develop it. Keep a well-documented record of the evolution of your design.
- Select a part of your design that involves an intersection of parabolas or an intersection of parabolas and lines. Determine model equations for each function involved in this part of your design.
- Using these equations, determine any points of intersection.
- What is the relevance of the points of intersection to the design of the object? How is it helpful to have model equations and to know the coordinates of the points of intersection?

Check Your Understanding

Practise

1. Which of the ordered pairs are solutions to the given inequality?
   a) $y < x + 3$,
   $\{(7, 10), (-7, 10), (6, 7), (12, 9)\}$
   b) $-x + y \leq -5$,
   $\{(2, 3), (-6, -12), (4, -1), (8, -2)\}$
   c) $3x - 2y > 12$,
   $\{(6, 3), (12, -4), (-6, -3), (5, 1)\}$
   d) $2x + y \geq 6$,
   $\{(0, 0), (3, 1), (-4, -2), (6, -4)\}$

2. Which of the ordered pairs are *not* solutions to the given inequality?
   a) $y > -x + 1$,
   $\{(1, 0), (-2, 1), (4, 7), (10, 8)\}$
   b) $x + y \geq 6$,
   $\{(2, 4), (-5, 8), (4, 1), (8, 2)\}$
   c) $4x - 3y < 10$,
   $\{(1, 3), (5, 1), (-2, -3), (5, 6)\}$
   d) $5x + 2y \leq 9$,
   $\{(0, 0), (3, -1), (-4, 2), (1, -2)\}$

3. Consider each inequality.
   - Express y in terms of x, if necessary. Identify the slope and the y-intercept.
   - Indicate whether the boundary should be a solid line or a dashed line.

   a) $y \leq x + 3$
   b) $y > 3x + 5$
   c) $4x + y > 7$
   d) $2x - y \leq 10$
   e) $4x + 5y \geq 20$
   f) $x - 2y < 10$

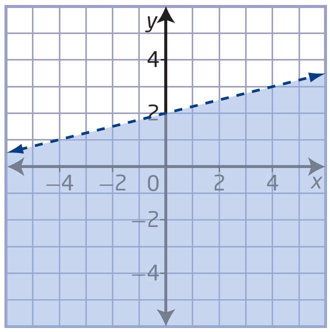
4. Graph each inequality without using technology.
   a) $y \leq -2x + 5$
   b) $3y - x > 8$
   c) $4x + 2y - 12 \geq 0$
   d) $4x - 10y < 40$
   e) $x \geq y - 6$

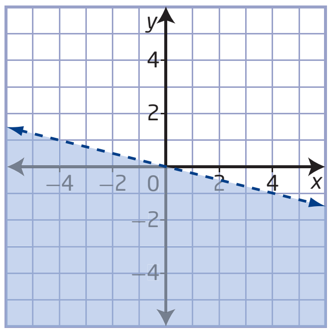
5. Graph each inequality using technology.
   a) $6x - 5y \leq 18$
   b) $x + 4y < 30$
   c) $-5x + 12y - 28 > 0$
   d) $x \leq 6y + 11$
   e) $3.6x - 5.3y + 30 \geq 4$

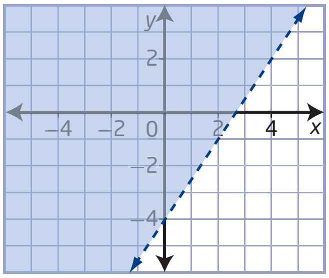
6. Determine the solution to $-5y \leq x$.

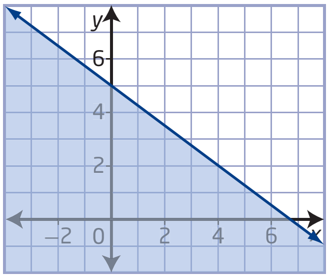
7. Use graphing technology to determine the solution to $7x - 2y > 0$.

8. Graph each inequality. Explain your choice of graphing methods.
   a) $6x + 3y \geq 21$
   b) $10x < 2.5y$
   c) $2.5x < 10y$
   d) $4.89x + 12.79y \leq 145$
   e) $0.8x - 0.4y > 0$

9. Determine the inequality that corresponds to each graph.
   a)


   b)


c) 

d) 

Apply

10. Express the solution to $x + 0y > 0$ graphically and in words.

11. Amaruq has a part-time job that pays her \$12/h. She also sews baby moccasins and sells them for a profit of \$12 each. Amaruq wants to earn at least \$250/week.

a) Write an inequality that represents the number of hours that Amaruq can work and the number of baby moccasins she can sell to earn at least \$250. Include any restrictions on the variables.

b) Graph the inequality.

c) List three different ordered pairs in the solution.

d) Give at least one reason that Amaruq would want to earn income from her part-time job as well as her sewing business, instead of focusing on one method only.

12. The Alberta Foundation for the Arts provides grants to support artists. The Aboriginal Arts Project Grant is one of its programs. Suppose that Camille has received a grant and is to spend at most \$3000 of the grant on marketing and training combined. It costs \$30/h to work with an elder in a mentorship program and \$50/h for marketing assistance.

a) Write an inequality to represent the number of hours working with an elder and receiving marketing assistance that Camille can afford. Include any restrictions on the variables.

b) Graph the inequality.

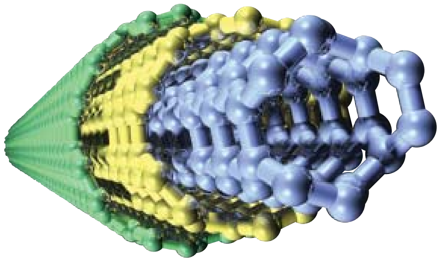
Mother Eagle by Jason Carter, artist chosen to represent Alberta at the Vancouver 2010 Olympics. Jason is a member of the Little Red River Cree Nation.

Web Link

To learn more about the Alberta Foundation for the Arts, go to www.mhrprecalc11.ca and follow the links.

13. Mariya has purchased a new smart phone and is trying to decide on a service plan. Without a plan, each minute of use costs \$0.30 and each megabyte of data costs \$0.05. A plan that allows unlimited talk and data costs \$100/month. Under which circumstances is the plan a better choice for Mariya?

14. Suppose a designer is modifying the tabletop from Example 4. The designer wants to replace the aluminum used in the table with a nanomaterial made from nanotubes. The budget for the project remains \$50, the cost of glass is still \$60/m², and the nanomaterial costs \$45/kg. Determine all possible combinations of material available to the designer.



Multi-walled carbon nanotube

15. Speed skaters spend many hours training on and off the ice to improve their strength and conditioning. Suppose a team has a monthly training budget of \$7000. Ice rental costs \$125/h, and gym rental for strength training costs \$55/h. Determine the solution region, or all possible combinations of training time that the team can afford.

Olympic gold medalist Christine Nesbitt

Did You Know?

Canadian long-track and short-track speed skaters won 10 medals at the 2010 Olympic Winter Games in Vancouver, part of an Olympic record for the most gold medals won by a country in the history of the Winter Games.

Extend

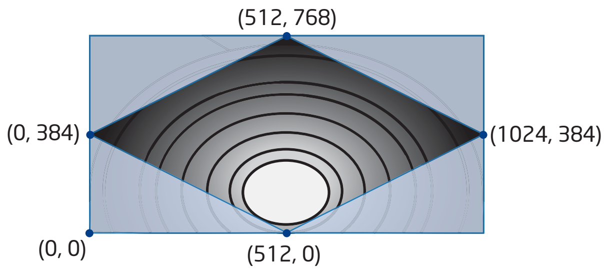
16. Drawing a straight line is not the only way to divide a plane into two regions.

a) Determine one other relation that when graphed divides the Cartesian plane into two regions.

b) For your graph, write inequalities that describe each region of the Cartesian plane in terms of your relation. Justify your answer.

c) Does your relation satisfy the definition of a solution region? Explain.

17. Masha is a video game designer. She treats the computer screen like a grid. Each pixel on the screen is represented by a coordinate pair, with the pixel in the bottom left corner of the screen as (0, 0). For one scene in a game she is working on, she needs to have a background like the one shown.



The shaded region on the screen is made up of four inequalities. What are the four inequalities?

18. MINI LAB Work in small groups.

In April 2008, Manitoba Hydro agreed to provide Wisconsin Public Service with up to 500 MW (megawatts) of hydroelectric power over 15 years, starting in 2018. Hydroelectric projects generate the majority of power in Manitoba; however, wind power is a method of electricity generation that may become more common. Suppose that hydroelectric power costs \$60/MWh (megawatt hour) to produce, wind power costs \$90/MWh, and the total budget for all power generation is \$35 000/h.

Step 1 Write the inequality that represents the cost of power generation. Let x represent the number of megawatt hours of hydroelectric power produced. Let y represent the number of megawatt hours of wind power produced.

Step 2 Graph and solve the inequality for the cost of power generation given the restrictions imposed by the hydroelectric agreement. Determine the coordinates of the vertices of the solution region. Interpret the intercepts in the context of this situation.

Step 3 Suppose that Manitoba Hydro can sell the hydroelectric power for \$95/MWh and the wind power for \$105/MWh. The equation $R = 95x + 105y$ gives the revenue, R, in dollars, from the sale of power. Use a spreadsheet to find the revenue for a number of different points in the solution region. Is it possible to find the revenue for all possible combinations of power generation? Can you guarantee that the point giving the maximum possible revenue is shown on your spreadsheet?

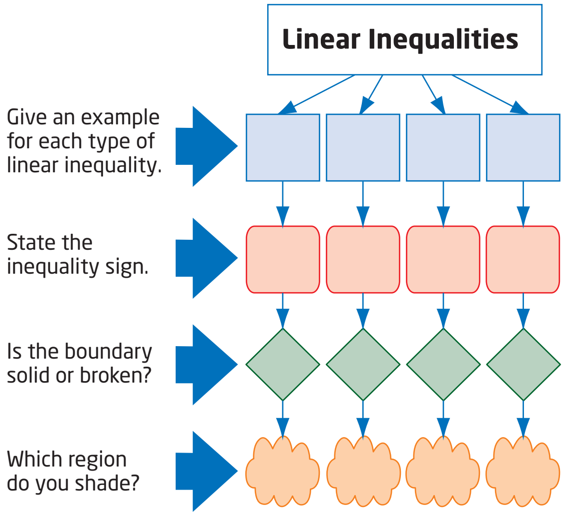
	A	B	C
1	Hydroelectric (MWH)	Wind (MWH)	Revenue (\$)
2	0	0	0
3	50	25	7375
4	100	50	14750
5	150	75	22125
6	200	100	29500
7			
8			

Step 4 It can be shown that the maximum revenue is always obtained from one of the vertices of the solution region. What combination of wind and hydroelectric power leads to the highest revenue?

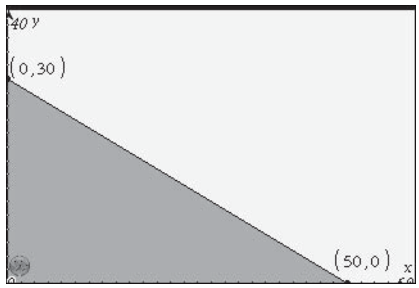
Step 5 With your group, discuss reasons that a combination other than the one that produces the maximum revenue might be chosen.

Create Connections

19. Copy and complete the following mind map.



20. The graph shows the solution to a linear inequality.



a) Write a scenario that has this region as its solution. Justify your answer.

b) Exchange your scenario with a partner. Verify that the given solution fits each scenario.

21. The inequality $2x - 3y + 24 > 0$, the positive y-axis, and the negative x-axis define a region in quadrant II.

a) Determine the area of this region.

b) How does the area of this region depend on the y-intercept of the boundary of the inequality $2x - 3y + 24 > 0$?

c) How does the area of this region depend on the slope of the boundary of the inequality $2x - 3y + 24 > 0$?

d) How would your answers to parts b) and c) change for regions with the same shape located in the other quadrants?

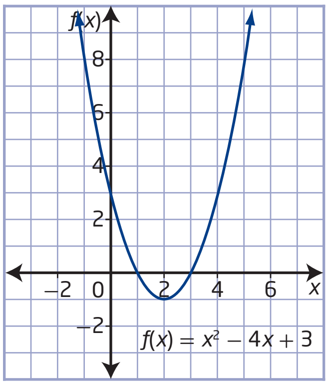
Key Ideas

- The solution to a quadratic inequality in one variable is a set of values.
- To solve a quadratic inequality, you can use one of the following strategies:
  - Graph the corresponding function, and identify the values of x for which the function lies on, above, or below the x-axis, depending on the inequality symbol.
  - Determine the roots of the related equation, and then use a number line and test points to determine the intervals that satisfy the inequality.
  - Determine when each of the factors of the quadratic expression is positive, zero, or negative, and then use the results to determine the sign of the product.
  - Consider all cases for the required product of the factors of the quadratic expression to find any x-values that satisfy both factor conditions in each case.
- For inequalities with the symbol $\geq$ or $\leq$, include the x-intercepts in the solution set.

Check Your Understanding

Practise

1. Consider the graph of the quadratic function $f(x) = x^2 - 4x + 3$.



What is the solution to

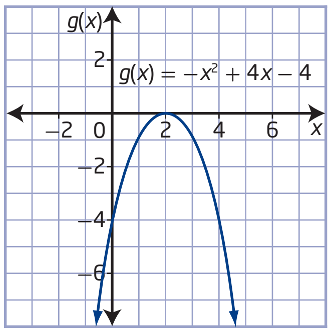
a) $x^2 - 4x + 3 \leq 0$?

b) $x^2 - 4x + 3 \geq 0$?

c) $x^2 - 4x + 3 > 0$?

d) $x^2 - 4x + 3 < 0$?

2. Consider the graph of the quadratic function $g(x) = -x^2 + 4x - 4$.



What is the solution to

a) $-x^2 + 4x - 4 \leq 0$?

b) $-x^2 + 4x - 4 \geq 0$?

c) $-x^2 + 4x - 4 > 0$?

d) $-x^2 + 4x - 4 < 0$?

3. Is the value of x a solution to the given inequality?

a) $x = 4$ for $x^2 - 3x - 10 > 0$

b) $x = 1$ for $x^2 + 3x - 4 \geq 0$

c) $x = -2$ for $x^2 + 4x + 3 < 0$

d) $x = -3$ for $-x^2 - 5x - 4 \leq 0$

4. Use roots and test points to determine the solution to each inequality.

a) $x(x + 6) \geq 40$

b) $-x^2 - 14x - 24 < 0$

c) $6x^2 > 11x + 35$

d) $8x + 5 \leq -2x^2$

5. Use sign analysis to determine the solution to each inequality.

a) $x^2 + 3x \leq 18$

b) $x^2 + 3 \geq -4x$

c) $4x^2 - 27x + 18 < 0$

d) $-6x \geq x^2 - 16$

6. Use case analysis to determine the solution to each inequality.

a) $x^2 - 2x - 15 < 0$

b) $x^2 + 13x > -12$

c) $-x^2 + 2x + 5 \leq 0$

d) $2x^2 \geq 8 - 15x$

7. Use graphing to determine the solution to each inequality.

a) $x^2 + 14x + 48 \leq 0$

b) $x^2 \geq 3x + 28$

c) $-7x^2 + x - 6 \geq 0$

d) $4x(x - 1) > 63$

8. Solve each of the following inequalities. Explain your strategy and why you chose it.

a) $x^2 - 10x + 16 < 0$

b) $12x^2 - 11x - 15 \geq 0$

c) $x^2 - 2x - 12 \leq 0$

d) $x^2 - 6x + 9 > 0$

9. Solve each inequality.

a) $x^2 - 3x + 6 \leq 10x$

b) $2x^2 + 12x - 11 > x^2 + 2x + 13$

c) $x^2 - 5x < 3x^2 - 18x + 20$

d) $-3(x^2 + 4) \leq 3x^2 - 5x - 68$

Apply

10. Each year, Dauphin, Manitoba, hosts the largest ice-fishing contest in Manitoba. Before going on any ice, it is important to know that the ice is thick enough to support the intended load. The solution to the inequality $9h^2 \geq 750$ gives the thickness, h, in centimetres, of ice that will support a vehicle of mass 750 kg.

a) Solve the inequality to determine the minimum thickness of ice that will safely support the vehicle.

b) Write a new inequality, in the form $9h^2 \geq$ mass, that you can use to find the ice thickness that will support a mass of 1500 kg.

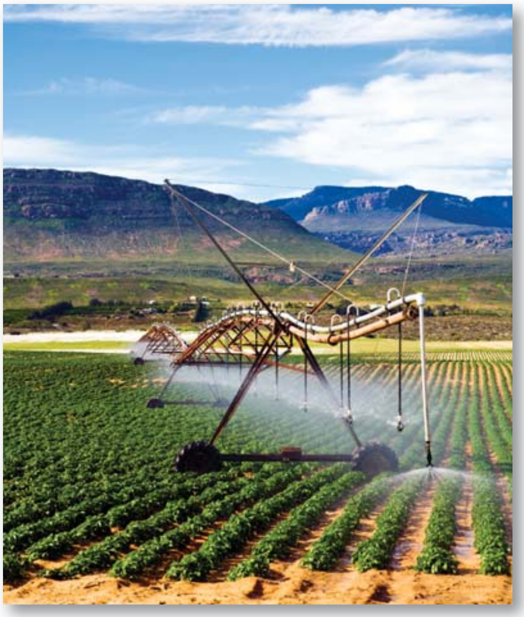
c) Solve the inequality you wrote in part b).

d) Why is the thickness of ice required to support 1500 kg not twice the thickness needed to support 750 kg? Explain.

Did You Know?

Conservation efforts at Dauphin Lake, including habitat enhancement, stocking, and education, have resulted in sustainable fish stocks and better fishing for anglers.

11. Many farmers in Southern Alberta irrigate their crops. A centre-pivot irrigation system spreads water in a circular pattern over a crop.



a) Suppose that Murray has acquired rights to irrigate up to 63 ha (hectares) of his land. Write an inequality to model the maximum circular area, in square metres, that he can irrigate.

b) What are the possible radii of circles that Murray can irrigate? Express your answer as an exact value.

c) Express your answer in part b) to the nearest hundredth of a metre.

Did You Know?

The hectare is a unit of area defined as 10 000 m^2. It is primarily used as a measurement of land area.

12. Suppose that an engineer determines that she can use the formula $-t^2 + 14 \leq P$ to estimate when the price of carbon fibre will be P dollars per kilogram or less in t years from the present.

a) When will carbon fibre be available at \$10/kg or less?

b) Explain why some of the values of t that satisfy the inequality do not solve the problem.

c) Write and solve a similar inequality to determine when carbon fibre prices will drop below \$5/kg.

Did You Know?

Carbon fibre is prized for its high strength-to-mass ratio. Prices for carbon fibre were very high when the technology was new, but dropped as manufacturing methods improved.

13. One leg of a right triangle is 2 cm longer than the other leg. How long should the shorter leg be to ensure that the area of the triangle is greater than or equal to 4 cm^2?

Extend

14. Use your knowledge of the graphs of quadratic functions and the discriminant to investigate the solutions to the quadratic inequality $ax^2 + bx + c \geq 0$.

a) Describe all cases where all real numbers satisfy the inequality.

b) Describe all cases where exactly one real number satisfies the inequality.

c) Describe all cases where infinitely many real numbers satisfy the inequality and infinitely many real numbers do not satisfy the inequality.

15. For each of the following, give an inequality that has the given solution.

a) $-2 \leq x \leq 7$

b) $x < 1$ or $x > 10$

c) $\frac{5}{3} \leq x \leq 6$

d) $x < -\frac{3}{4}$ or $x > -\frac{1}{5}$

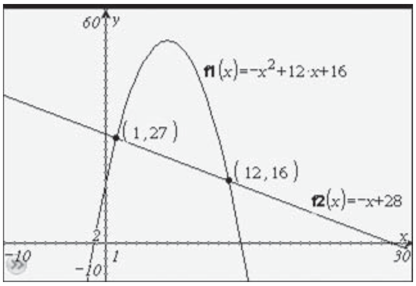
e) $x \leq -3 - \sqrt{7}$ or $x \geq -3 + \sqrt{7}$

f) $x \in R$

g) no solution

16. Solve $|x^2 - 4| \geq 2$.

17. The graph shows the solution to the inequality $-x^2 + 12x + 16 \geq -x + 28$.



a) Why is $1 \leq x \leq 12$ the solution to the inequality?

b) Rearrange the inequality so that it has the form $q(x) \geq 0$ for a quadratic $q(x)$.

c) Solve the inequality you determined in part b).

d) How are the solutions to parts a) and c) related? Explain.

Create Connections

18. In Example 3, the first step in the solution was to rearrange the inequality $2x^2 - 7x > 12$ into $2x^2 - 7x - 12 > 0$. Which solution methods require this first step and which do not? Show the work that supports your conclusions.

19. Compare and contrast the methods of graphing, roots and test points, sign analysis, and case analysis. Explain which of the methods you prefer to use and why.

20. Devan needs to solve $x^2 + 5x + 4 \leq -2$. His solutions are shown.

Devan's solution:

Begin by rewriting the inequality: $x^2 + 5x + 6 \geq 0$

Factor the left side: $(x + 2)(x + 3) \geq 0$. Then, consider two cases:

Case 1:

$(x + 2) \leq 0$ and $(x + 3) \leq 0$

Then, $x \leq -2$ and $x \leq -3$, so the solution is $x \leq -3$.

Case 2:

$(x + 2) \geq 0$ and $(x + 3) \geq 0$

Then, $x \geq -2$ and $x \geq -3$, so the solution is $x \geq -2$.

From the two cases, the solution to the inequality is $x \leq -3$ or $x \geq -2$.

a) Decide whether his solution is correct. Justify your answer.

b) Use a different method to confirm the correct answer to the inequality.

Project Corner — Financial Considerations

- Currently, the methods of nanotechnology in several fields are very expensive. However, as is often the case, it is expected that as technology improves, the costs will decrease. Nanotechnology seems to have the potential to decrease costs in the future. It also promises greater flexibility and greater precision in the manufacturing of goods.
- What changes in manufacturing might help lower the cost of nanotechnology?

Your Turn

Sports climbers use a rope that is longer and supports less mass than manila rope. The rope can safely support a mass, M, in pounds, modelled by the inequality $M \leq 1240(d - 2)^2$, where d is the diameter of the rope, in inches. Graph the inequality to examine how the mass that the rope supports is related to the diameter of the rope.

Key Ideas

- A quadratic inequality in two variables represents a region of the Cartesian plane containing the set of points that are solutions to the inequality.
- The graph of the related quadratic function is the boundary that divides the plane into two regions.
  - When the inequality symbol is $\leq$ or $\geq$, include the points on the boundary in the solution region and draw the boundary as a solid line.
  - When the inequality symbol is $<$ or $>$, do not include the points on the boundary in the solution region and draw the boundary as a dashed line.
- Use a test point to determine the region that contains the solutions to the inequality.

Check Your Understanding

Practise

1. Which of the ordered pairs are solutions to the inequality?

a) $y < x^2 + 3$,
$\{(2, 6), (4, 20), (-1, 3), (-3, 12)\}$

b) $y \leq -x^2 + 3x - 4$,
$\{(2, -2), (4, -1), (0, -6), (-2, -15)\}$

c) $y > 2x^2 + 3x + 6$,
$\{(-3, 5), (0, -6), (2, 10), (5, 40)\}$

d) $y \geq -\frac{1}{2}x^2 - x + 5$,
$\{(-4, 2), (-1, 5), (1, 3.5), (3, 2.5)\}$

2. Which of the ordered pairs are *not* solutions to the inequality?

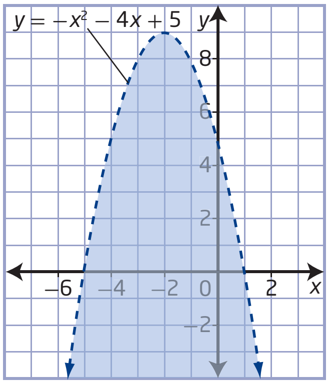
a) $y \geq 2(x - 1)^2 + 1$,
$\{(0, 1), (1, 0), (3, 6), (-2, 15)\}$

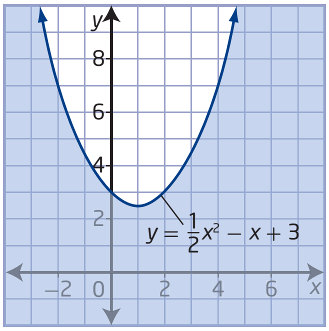
b) $y > -(x + 2)^2 - 3$,
$\{(-3, 1), (-2, -3), (0, -8), (1, 2)\}$

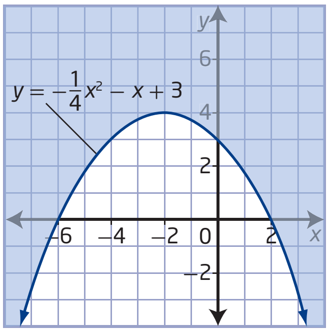
c) $y \leq \frac{1}{2}(x - 4)^2 + 5$,
$\{(0, 4), (3, 1), (4, 5), (2, 9)\}$

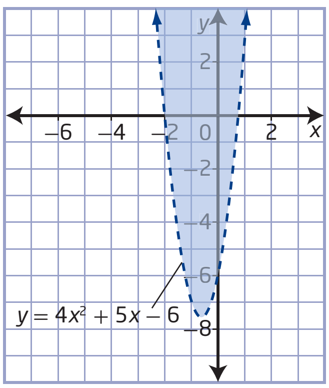
d) $y < -\frac{2}{3}(x + 3)^2 - 2$,
$\{(-2, 2), (-1, -5), (-3, -2), (0, -10)\}$

3. Write an inequality to describe each graph, given the function defining the boundary parabola.

a) 

b) 

c) 

d) 

4. Graph each quadratic inequality using transformations to sketch the boundary parabola.

a) $y \geq 2(x + 3)^2 + 4$

b) $y > -\frac{1}{2}(x - 4)^2 - 1$

c) $y < 3(x + 1)^2 + 5$

d) $y \leq \frac{1}{4}(x - 7)^2 - 2$

5. Graph each quadratic inequality using points and symmetry to sketch the boundary parabola.

a) $y < -2(x - 1)^2 - 5$

b) $y > (x + 6)^2 + 1$

c) $y \geq \frac{2}{3}(x - 8)^2$

d) $y \leq \frac{1}{2}(x + 7)^2 - 4$

6. Graph each quadratic inequality.

a) $y \leq x^2 + x - 6$

b) $y > x^2 - 5x + 4$

c) $y \geq x^2 - 6x - 16$

d) $y < x^2 + 8x + 16$

7. Graph each inequality using graphing technology.

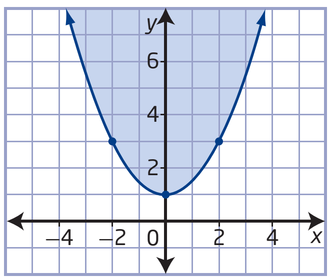
a) $y < 3x^2 + 13x + 10$

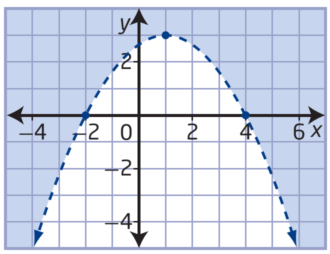
b) $y \geq -x^2 + 4x + 7$

c) $y \leq x^2 + 6$

d) $y > -2x^2 + 5x - 8$

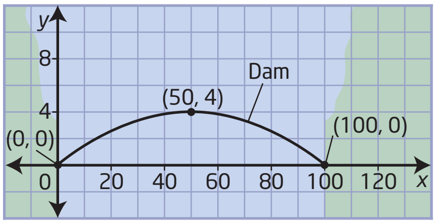
8. Write an inequality to describe each graph.

a) 

b) 

Apply

9. When a dam is built across a river, it is often constructed in the shape of a parabola. A parabola is used so that the force that the river exerts on the dam helps hold the dam together. Suppose a dam is to be built as shown in the diagram.



a) What is the quadratic function that models the parabolic arch of the dam?

b) Write the inequality that approximates the region below the parabolic arch of the dam.

Did You Know?

The Mica Dam, which spans the Columbia River near Revelstoke, British Columbia, is a parabolic dam that provides hydroelectric power to Canada and parts of the United States.

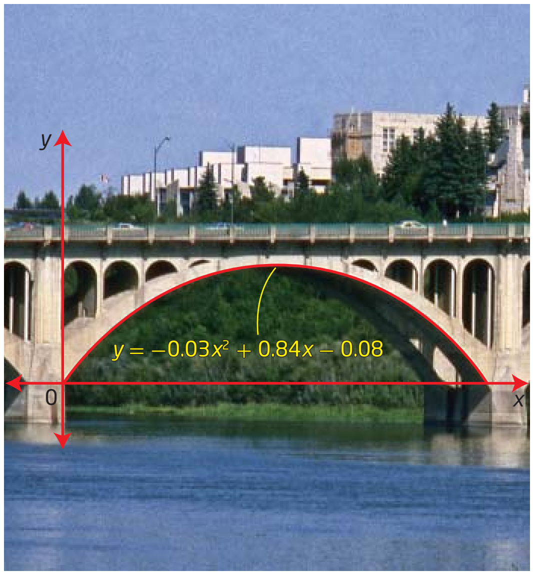
10. In order to get the longest possible jump, ski jumpers need to have as much lift area, L, in square metres, as possible while in the air. One of the many variables that influences the amount of lift area is the hip angle, a, in degrees, of the skier. The relationship between the two is given by $L \geq -0.000\,125a^2 + 0.040a - 2.442$.

a) Graph the quadratic inequality.

b) What is the range of hip angles that will generate lift area of at least $0.50\ \text{m}^2$?

Canadian ski jumper Stefan Read

11. The University Bridge in Saskatoon is supported by several parabolic arches. The diagram shows how a Cartesian plane can be applied to one arch of the bridge. The function $y = -0.03x^2 + 0.84x - 0.08$ approximates the curve of the arch, where x represents the horizontal distance from the bottom left edge and y represents the height above where the arch meets the vertical pier, both in metres.



a) Write the inequality that approximates the possible water levels below the parabolic arch of the bridge.

b) Suppose that the normal water level of the river is at most 0.2 m high, relative to the base of the arch. Write and solve an inequality to represent the normal river level below the arch.

c) What is the width of the river under the arch in the situation described in part b)?

12. In order to conduct microgravity research, the Canadian Space Agency uses a Falcon 20 jet that flies a parabolic path. As the jet nears the vertex of the parabola, the passengers in the jet experience nearly zero gravity that lasts for a short period of time. The function $h = -2.944t^2 + 191.360t + 6950.400$ models the flight of a jet on a parabolic path for time, t, in seconds, since weightlessness has been achieved and height, h, in metres.

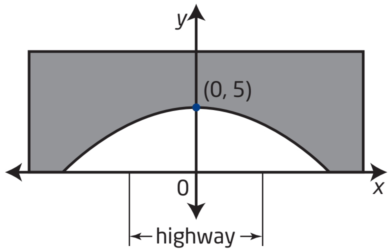
Canadian Space Agency astronauts David Saint-Jacques and Jeremy Hansen experience microgravity during a parabolic flight as part of basic training.

a) The passengers begin to experience weightlessness when the jet climbs above 9600 m. Write an inequality to represent this information.

b) Determine the time period for which the jet is above 9600 m.

c) For how long does the microgravity exist on the flight?

13. A highway goes under a bridge formed by a parabolic arch, as shown. The highest point of the arch is 5 m high. The road is 10 m wide, and the minimum height of the bridge over the road is 4 m.



a) Determine the quadratic function that models the parabolic arch of the bridge.

b) What is the inequality that represents the space under the bridge in quadrants I and II?

Extend

14. Tavia has been adding advertisements to her Web site. Initially her revenue increased with each additional ad she included on her site. However, as she kept increasing the number of ads, her revenue began to drop. She kept track of her data as shown.

Number of Ads	0	10	15
Revenue ($)	0	100	75

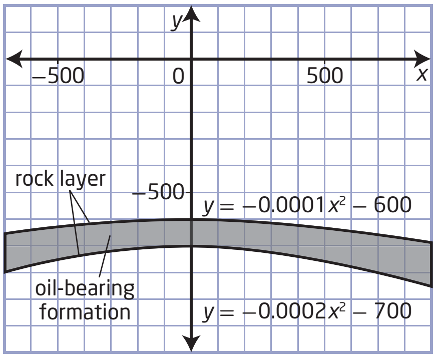
a) Determine the quadratic inequality that models Tavia's revenue.

b) How many ads can Tavia include on her Web site to earn revenue of at least $50?

Did You Know?

The law of diminishing returns is a principle in economics. The law states the surprising result that when you continually increase the quantity of one input, you will eventually see a decrease in the output.

15. Oil is often recovered from a formation bounded by layers of rock that form a parabolic shape. Suppose a geologist has discovered such an oil-bearing formation. The quadratic functions that model the rock layers are $y = -0.0001x^2 - 600$ and $y = -0.0002x^2 - 700$, where x represents the horizontal distance from the centre of the formation and y represents the depth below ground level, both in metres. Write the inequality that describes the oil-bearing formation.



Create Connections

16. To raise money, the student council sells candy-grams each year. From past experience, they expect to sell 400 candy-grams at a price of \$4 each. They have also learned from experience that each \$0.50 increase in the price causes a drop in sales of 20 candy-grams.

a) Write an equality that models this situation. Define your variables.

b) Suppose the student council needs revenue of at least \$1800. Solve an inequality to find all the possible prices that will achieve the fundraising goal.

c) Show how your solution would change if the student council needed to raise \$1600 or more.

17. An environmentalist has been studying the methane produced by an inactive landfill. To approximate the methane produced, p, as a percent of peak output compared to time, t, in years, after the year 2000, he uses the inequality $p \le 0.24t^2 - 8.1t + 74$.

a) For what time period is methane production below 10% of the peak production?

b) Graph the inequality used by the environmentalist. Explain why only a portion of the graph is a reasonable model for the methane output of the landfill. Which part of the graph would the environmentalist use?

c) Modify your answer to part a) to reflect your answer in part b).

d) Explain how the environmentalist can use the concept of domain to make modelling the situation with the quadratic inequality more reasonable.

18. Look back at your work in Unit 2, where you learned about quadratic functions. Working with a partner, identify the concepts and skills you learned in that unit that have helped you to understand the concepts in this unit. Decide which concept from Unit 2 was most important to your understanding in Unit 4. Find another team that chose a different concept as the most important. Set up a debate, with each team defending its choice of most important concept.

Chapter 9 Review

9.1 Linear Inequalities in Two Variables, pages 464–475

1. Graph each inequality without using technology.

a) $y \leq 3x - 5$

b) $y > -\frac{3}{4}x + 2$

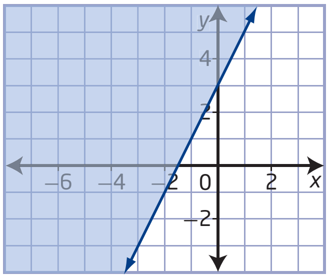
c) $3x - y \geq 6$

d) $4x + 2y \leq 8$

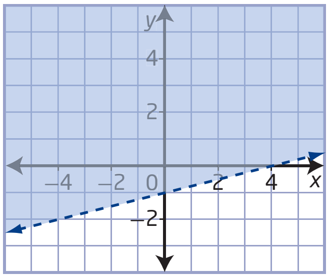
e) $10x - 4y + 3 < 11$

2. Determine the inequality that corresponds to each of the following graphs.

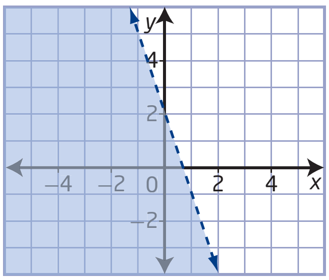
a)



b)



c)



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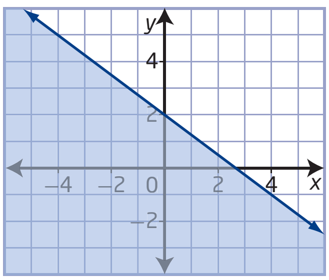


3. Graph each inequality using technology.

a) $4x + 5y > 22$

b) $10x - 4y + 52 \geq 0$

c) $-3.2x + 1.1y < 8$

d) $12.4x + 4.4y > 16.5$

e) $\frac{3}{4}x \leq 9y$

4. Janelle has a budget of \$120 for entertainment each month. She usually spends the money on a combination of movies and meals. Movie admission, with popcorn, is \$15, while a meal costs \$10.

a) Write an inequality to represent the number of movies and meals that Janelle can afford with her entertainment budget.

b) Graph the solution.

c) Interpret your solution. Explain how the solution to the inequality relates to Janelle's situation.

5. Jodi is paid by commission as a salesperson. She earns 5% commission for each laptop computer she sells and 8% commission for each DVD player she sells. Suppose that the average price of a laptop is \$600 and the average price of a DVD player is \$200.

a) What is the average amount Jodi earns for selling each item?

b) Jodi wants to earn a minimum commission this month of \$1000. Write an inequality to represent this situation.

c) Graph the inequality. Interpret your results in the context of Jodi's earnings.

9.2 Quadratic Inequalities in One Variable, pages 476–487

6. Choose a strategy to solve each inequality. Explain your strategy and why you chose it.

a) $x^2 - 2x - 63 > 0$

b) $2x^2 - 7x - 30 \geq 0$

c) $x^2 + 8x - 48 < 0$

d) $x^2 - 6x + 4 \geq 0$

7. Solve each inequality.

a) $x(6x + 5) \leq 4$

b) $4x^2 < 10x - 1$

c) $x^2 \leq 4(x + 8)$

d) $5x^2 \geq 4 - 12x$

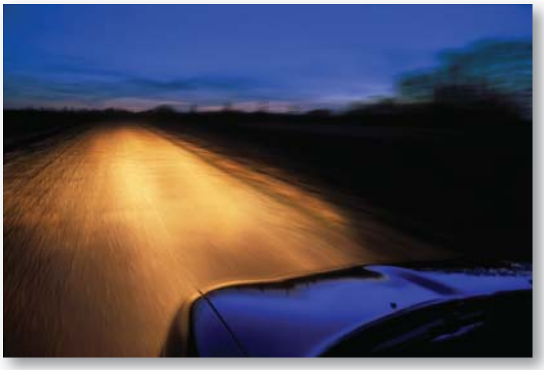
8. A decorative fountain shoots water in a parabolic path over a pathway. To determine the location of the pathway, the designer must solve the inequality $-\frac{3}{4}x^2 + 3x \leq 2$, where x is the horizontal distance from the water source, in metres.

a) Solve the inequality.

b) Interpret the solution to the inequality for the fountain designer.

9. A rectangular storage shed is to be built so that its length is twice its width. If the maximum area of the floor of the shed is 18 m^2, what are the possible dimensions of the shed?

10. David has learned that the light from the headlights reaches about 100 m ahead of the car he is driving. If v represents David's speed, in kilometres per hour, then the inequality $0.007v^2 + 0.22v \leq 100$ gives the speeds at which David can stop his vehicle in 100 m or less.



a) What is the maximum speed at which David can travel and safely stop his vehicle in the 100-m distance?

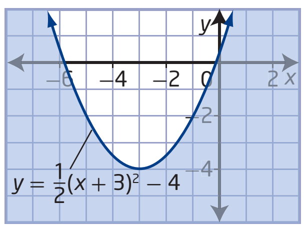
b) Modify the inequality so that it gives the speeds at which a vehicle can stop in 50 m or less.

c) Solve the inequality you wrote in part b). Explain why your answer is not half the value of your answer for part a).

9.3 Quadratic Inequalities in Two Variables, pages 488–500

11. Write an inequality to describe each graph, given the function defining the boundary parabola.

a)



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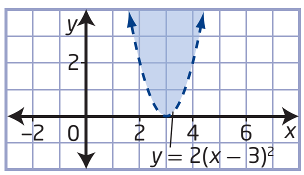


12. Graph each quadratic inequality.

a) $y < x^2 + 2x - 15$

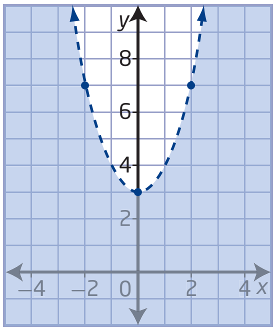
b) $y \geq -x^2 + 4$

c) $y > 6x^2 + x - 12$

d) $y \leq (x - 1)^2 - 6$

13. Write an inequality to describe each graph.

a)



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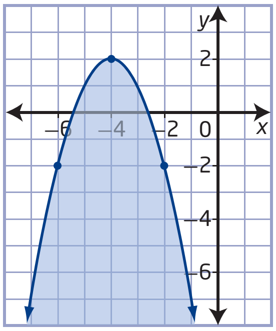


14. You can model the maximum Saskatchewan wheat production for the years 1975 to 1995 with the function $y = 0.003t^2 - 0.052t + 1.986$, where t is the time, in years, after 1975 and y is the yield, in tonnes per hectare.

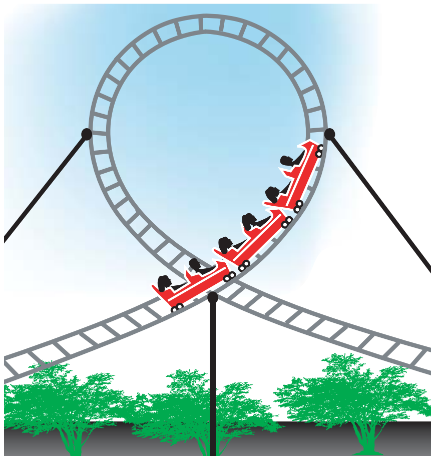
a) Write and graph an inequality to model the potential wheat production during this period.

b) Write and solve an inequality to represent the years in which production is at most 2 t/ha.

Did You Know?

Saskatchewan has 44% of Canada's total cultivated farmland. Over 10% of the world's total exported wheat comes from this province.

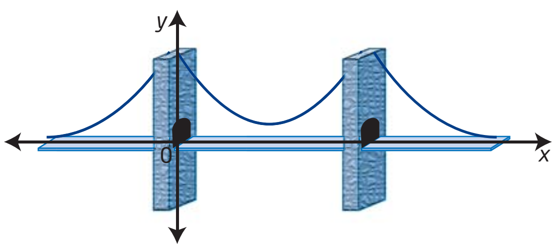
15. An engineer is designing a roller coaster for an amusement park. The speed at which the roller coaster can safely complete a vertical loop is approximated by $v^2 \geq 10r$, where v is the speed, in metres per second, of the roller coaster and r is the radius, in metres, of the loop.



a) Graph the inequality to examine how the radius of the loop is related to the speed of the roller coaster.

b) A vertical loop of the roller coaster has a radius of 16 m. What are the possible safe speeds for this vertical loop?

16. The function $y = \frac{1}{20}x^2 - 4x + 90$ models the cable that supports a suspension bridge, where x is the horizontal distance, in metres, from the base of the first support and y is the height, in metres, of the cable above the bridge deck.



a) Write an inequality to determine the points for which the height of the cable is at least 20 m.

b) Solve the inequality. What does the solution represent?

Chapter 9 Practice Test

Multiple Choice

For #1 to #5, choose the best answer.

1. An inequality that is equivalent to $3x - 6y < 12$ is

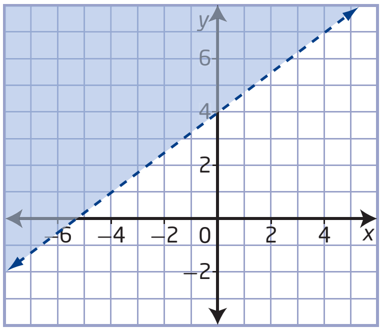
A $y < \frac{1}{2}x - 2$

B $y > \frac{1}{2}x - 2$

C $y < 2x - 2$

D $y > 2x - 2$

2. What linear inequality does the graph show?



A $y > \frac{3}{4}x + 4$

B $y \geq \frac{3}{4}x + 4$

C $y < \frac{4}{3}x + 4$

D $y \leq \frac{4}{3}x + 4$

3. What is the solution set for the quadratic inequality $6x^2 - 7x - 20 < 0$?

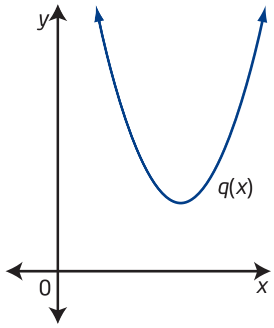
A $\left\{x \mid x \leq -\frac{4}{3} \text{ or } x \geq \frac{5}{2}, x \in \mathrm{R}\right\}$

B $\left\{x \mid -\frac{4}{3} \leq x \leq \frac{5}{2}, x \in \mathrm{R}\right\}$

C $\left\{x \mid -\frac{4}{3} < x < \frac{5}{2}, x \in \mathrm{R}\right\}$

D $\left\{x \mid x < -\frac{4}{3} \text{ or } x > \frac{5}{2}, x \in \mathrm{R}\right\}$

4. For the quadratic function $q(x)$ shown in the graph, which of the following is true?



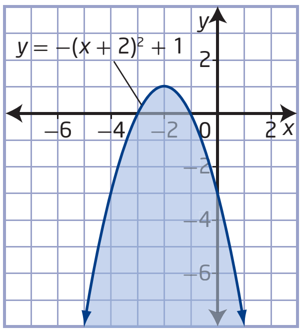
A There are no solutions to $q(x) > 0$.

B All real numbers are solutions to $q(x) \geq 0$.

C All real numbers are solutions to $q(x) \leq 0$.

D All positive real numbers are solutions to $q(x) < 0$.

5. What quadratic inequality does the graph show?



A $y < -(x + 2)^2 + 1$

B $y \geq -(x + 2)^2 + 1$

C $y \leq -(x + 2)^2 + 1$

D $y > -(x + 2)^2 + 1$

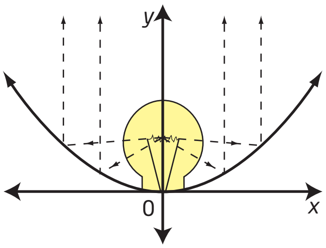
Short Answer

6. Graph $8x \geq 2(y - 5)$.

7. Solve $12x^2 < 7x + 10$.

8. Graph $y > (x - 5)^2 + 4$.

9. Stage lights often have parabolic reflectors to make it possible to focus the beam of light, as indicated by the diagram.



Suppose the reflector in a stage light is represented by the function $y = 0.02x^2$. What inequality can you use to model the region illuminated by the light?

10. While on vacation, Ben has \$300 to spend on recreation. Scuba diving costs \$25/h and sea kayaking costs \$20/h. What are all the possible ways that Ben can budget his recreation money?

Extended Response

11. Malik sells his artwork for different prices depending on the type of work. Pen and ink sketches sell for \$50, and watercolours sell for \$80.

a) Malik needs an income of at least \$1200 per month. Write an inequality to model this situation.

b) Graph the inequality. List three different ordered pairs in the solution.

c) Suppose Malik now needs at least \$2400 per month. Write an inequality to represent this new situation. Predict how the answer to this inequality will be related to your answer in part b).

d) Solve the new inequality from part c) to check your prediction.

12. Let $f(x)$ represent a quadratic function.

a) State a quadratic function for which the solution set to $f(x) \leq 0$ is $\{x \mid -3 \leq x \leq 5, x \in \mathrm{R}\}$. Justify your answer.

b) Describe all quadratics for which solutions to $f(x) \leq 0$ are of the form $m \leq x \leq n$ for some real numbers m and n.

c) For your answer in part b), explain whether it is more convenient to express quadratic functions in the form $f(x) = ax^2 + bx + c$ or $f(x) = a(x - p)^2 + q$, and why.

13. The normal systolic blood pressure, p, in millimetres of mercury (mmHg), for a woman a years old is given by $p = 0.01a^2 + 0.05a + 107$.

a) Write an inequality that expresses the ages for which you expect systolic blood pressure to be less than 120 mmHg.

b) Solve the inequality you wrote in part a).

c) Are all of the solutions to your inequality realistic answers for this problem? Explain why or why not.

Cumulative Review, Chapters 8–9

Chapter 8 Systems of Equations

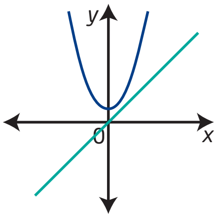
1. Examine each system of equations and match it with a possible sketch of the system. You do not need to solve the systems to match them.

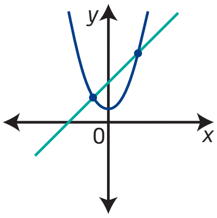
A $y = x^2 + 1$
$y = -x^2 + 1$

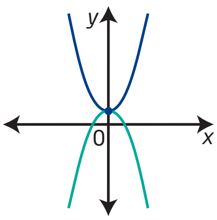
B $y = x^2 + 1$
$y = x$

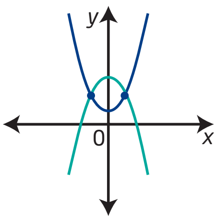
C $y = x^2 + 1$
$y = -x^2 + 4$

D $y = x^2 + 1$
$y = x + 4$

a)


b)


c)


d)


2. Solve the system of linear-quadratic equations graphically. Express your answers to the nearest tenth.

$3x + y = 4$
$y = x^2 - 3x - 1$

3. Consider the system of linear-quadratic equations

$y = -x^2 + 4x + 1$
$3x - y - 1 = 0$

a) Solve the system algebraically.

b) Explain, in graphical terms, what the ordered pairs from part a) represent.

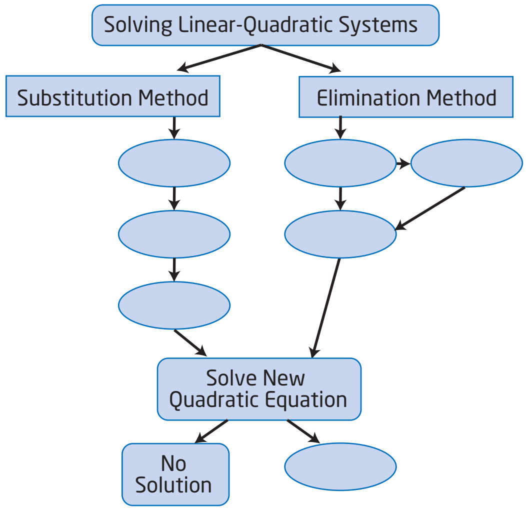
4. Given the quadratic function $y = x^2 + 4$ and the linear function $y = x + b$, determine all the possible values of b that would result in a system of equations with

a) two solutions

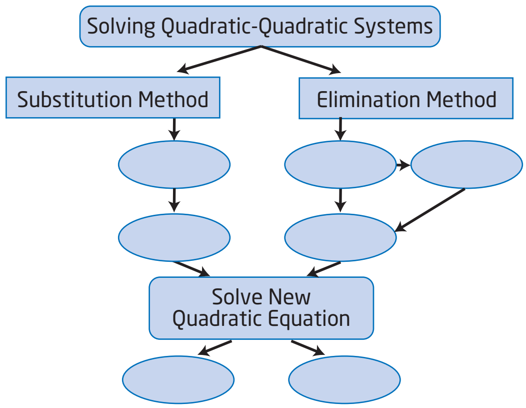
b) exactly one solution

c) no solution

5. Copy and complete the flowchart for solving systems of linear-quadratic equations.



6. Copy and complete the flowchart for solving systems of quadratic-quadratic equations.



7. The price, P, in dollars, per share, of a high-tech stock has fluctuated over a 10-year period according to the equation $P = 14 + 12t - t^2$, where t is time, in years. The price of a second high-tech stock has shown a steady increase during the same time period according to the relationship $P = 2t + 30$. Algebraically determine for what values the two stock prices will be the same.

8. Explain how you could determine if the given system of quadratic-quadratic equations has zero, one, two, or an infinite number of solutions without solving or using technology.

$y = (x - 4)^2 + 2$
$y = -(x + 3)^2 - 1$

9. Solve the system of quadratic-quadratic equations graphically. Express your answers to the nearest tenth.

$y = -2x^2 + 6x - 1$
$y = -4x^2 + 4x + 2$

10. Algebraically determine the solution(s) to each system of quadratic-quadratic equations.

a) $y = 2x^2 + 9x - 5$
$y = 2x^2 - 4x + 8$

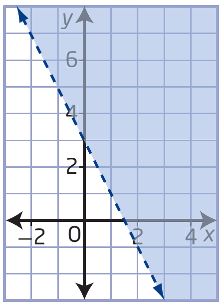
b) $y = 12x^2 + 17x - 5$
$y = -x^2 + 30x - 5$

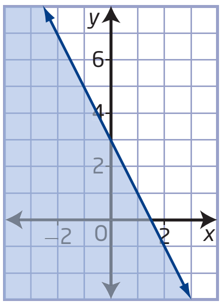
Chapter 9 Linear and Quadratic Inequalities

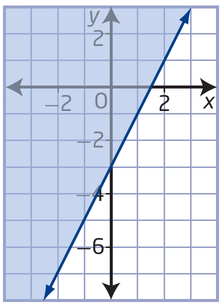
11. Match each inequality with its graph.

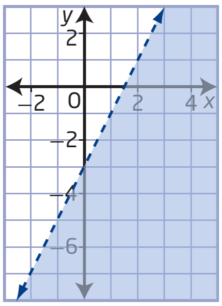
A $2x + y < 3$ **B** $2x - y \leq 3$

C $2x - y \geq 3$ **D** $2x + y > 3$

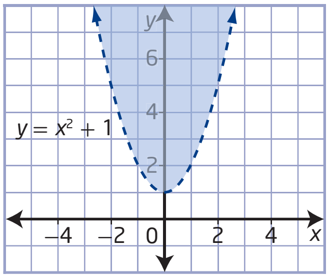
a)


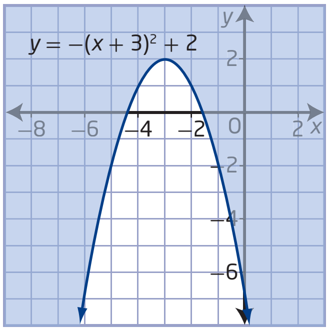
b)


c)


d)


12. Write an inequality to describe each graph, given the function defining the boundary parabola.

a)


b)


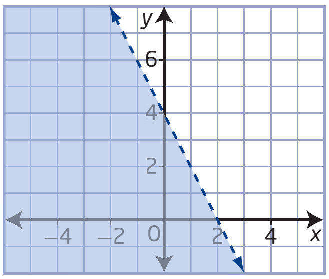
13. Explain how each test point can be used to determine the solution region that satisfies the inequality $y > x - 2$.

a) $(0, 0)$

b) $(2, -5)$

c) $(-1, 1)$

14. What linear inequality is shown in the graph?



15. Sketch the graph of $y \geq x^2 - 3x - 4$. Use a test point to verify the solution region.

16. Use sign analysis to determine the solution of the quadratic inequality $2x^2 + 9x - 33 \geq 2$.

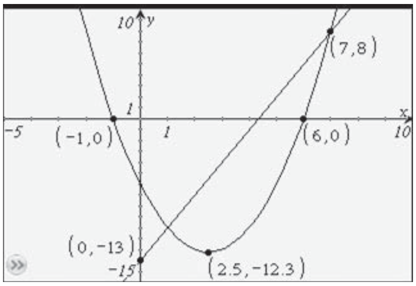
17. Suppose a rectangular area of land is to be enclosed by 1000 m of fence. If the area is to be greater than 60 000 m², what is the range of possible widths of the rectangle?

Unit 4 Test

Multiple Choice

For #1 to #9, choose the best answer.

1. Which of the following ordered pairs is a solution to the system of linear-quadratic equations?



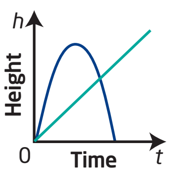
A (2.5, −12.3) **B** (6, 0)

C (7, 8) **D** (0, −13)

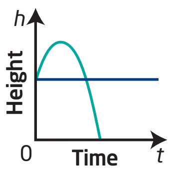
2. Kelowna, British Columbia, is one of the many places in western Canada with bicycle motocross (BMX) race tracks for teens.

Which graph models the height versus time of two the racers travelling over one of the jumps?

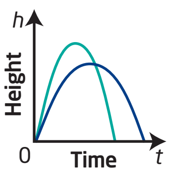
A



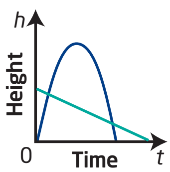
B



C



D



3. The ordered pairs (1, 3) and (−3, −5) are the solutions to which system of linear-quadratic equations?

A $y = 3x + 5$
$y = x^2 - 2x - 1$

B $y = 2x + 1$
$y = x^2 + 4x - 2$

C $y = x + 2$
$y = x^2 + 2$

D $y = 4x - 1$
$y = x^2 - 3x + 5$

4. How many solutions are possible for the following system of quadratic-quadratic equations?

$y - 5 = 2(x + 1)^2$
$y - 5 = -2(x + 1)^2$

A zero

B one

C two

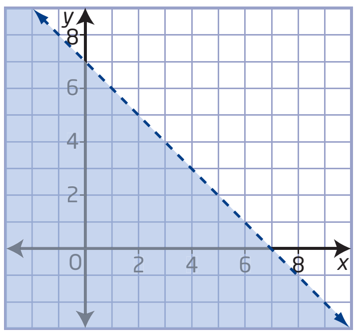
D an infinite number

5. Which point cannot be used as a test point to determine the solution region for $4x - y \leq 5$?

A (−1, 1) **B** (2, 5)

C (3, 1) **D** (2, 3)

6. Which linear inequality does the graph show?



A $y \leq -x + 7$

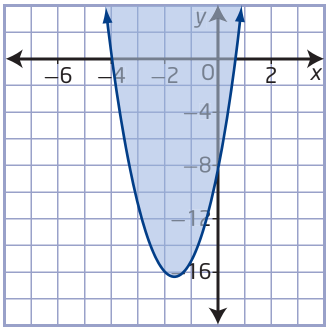
B $y \geq -x + 7$

C $y > -x + 7$

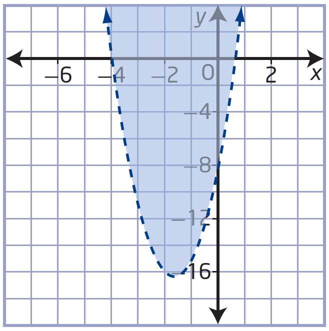
D $y < -x + 7$

7. Which graph represents the quadratic inequality $y \geq 3x^2 + 10x - 8$?

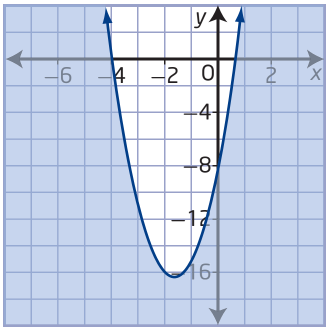
A



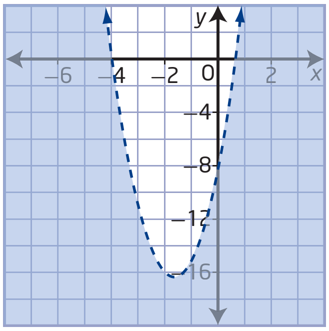
B



C



D



8. Determine the solution(s), to the nearest tenth, for the system of quadratic-quadratic equations.

$y = -\frac{2}{3}x^2 + 2x + 3$
$y = x^2 - 4x + 5$

A (3.2, 2.5)

B (3.2, 2.5) and (0.4, 3.7)

C (0.4, 2.5) and (2.5, 3.7)

D (0.4, 3.2)

9. What is the solution set for the quadratic inequality $-3x^2 + x + 11 < 1$?

A $\left\{x \mid x < -\frac{5}{3} \text{ or } x > 2, x \in \mathrm{R}\right\}$

B $\left\{x \mid x < -\frac{5}{3} \text{ or } x \geq 2, x \in \mathrm{R}\right\}$

C $\left\{x \mid -\frac{5}{3} < x < 2, x \in \mathrm{R}\right\}$

D $\left\{x \mid -\frac{5}{3} \leq x \leq 2, x \in \mathrm{R}\right\}$

Numerical Response

Copy and complete the statements in #10 to #12.

10. One of the solutions for the system of linear-quadratic equations $y = x^2 - 4x - 2$ and $y = x - 2$ is represented by the ordered pair $(a, 3)$, where the value of a is ■.

11. The solution of the system of quadratic-quadratic equations represented by $y = x^2 - 4x + 6$ and $y = -x^2 + 6x - 6$ with the greater coordinates is of the form (a, a), where the value of a is ■.

12. On a forward somersault dive, Laurie's height, h, in metres, above the water t seconds after she leaves the diving board is approximately modelled by $h(t) = -5t^2 + 5t + 4$. The length of time that Laurie is above 4 m is ■.

Written Response

13. Professional golfers, such as Canadian Mike Weir, make putting look easy to spectators. New technology used on a television sports channel analyses the greens conditions and predicts the path of the golf ball that the golfer should putt to put the ball in the hole. Suppose the straight line from the ball to the hole is represented by the equation $y = 2x$ and the predicted path of the ball is modelled by the equation $y = \frac{1}{4}x^2 + \frac{3}{2}x$.

a) Algebraically determine the solution to the system of linear-quadratic equations.

b) Interpret the points of intersection in this context.

14. Two quadratic functions, $f(x) = x^2 - 6x + 5$ and $g(x)$, intersect at the points $(2, -3)$ and $(7, 12)$. The graph of $g(x)$ is congruent to the graph of $f(x)$ but opens downward. Determine the equation of $g(x)$ in the form $g(x) = a(x - p)^2 + q$.

15. Algebraically determine the solutions to the system of quadratic-quadratic equations. Verify your solutions.

$4x^2 + 8x + 9 - y = 5$
$3x^2 - x + 1 = y + x + 6$

16. Dolores solved the inequality $3x^2 - 5x - 10 > 2$ using roots and test points. Her solution is shown.

$$\begin{aligned} 3x^2 - 5x - 10 &> 2 \\ 3x^2 - 5x - 8 &> 0 \\ 3x^2 - 5x - 8 &= 0 \\ (3x - 8)(x + 1) &= 0 \\ 3x - 8 = 0 \quad &\text{or} \quad x + 1 = 0 \\ 3x = 8 \quad & \quad\quad x = -1 \\ x = \frac{8}{3} & \end{aligned}$$

Choose test points -2, 0, and 3 from the intervals $x < -1$, $-1 < x < \frac{8}{3}$, and $x > \frac{8}{3}$, respectively.

The values of x less than -1 satisfy the inequality $3x^2 - 5x - 10 > 2$.

a) Upon verification, Dolores realized she made an error. Explain the error and provide a correct solution.

b) Use a different strategy to determine the solution to $3x^2 - 5x - 10 > 2$.

17. A scoop in field hockey occurs when a player lifts the ball off the ground with a shovel-like movement of the stick, which is placed slightly under the ball. Suppose a player passes the ball with a scoop modelled by the function $h(t) = -4.9t^2 + 10.4t$, where h is the height of the ball, in metres, and t represents time, in seconds. For what length of time, to the nearest hundredth of a second, is the ball above 3 m?