

Unit 8: Systems of Equations; Linear & Quadratic Inequalities

8.4 Linear Inequalities in Two Variables

Recall – Linear Inequalities in One Variable

Solve $2x + 5 > 3x - 8$

$$\begin{array}{r} 2x + 5 > 3x - 8 \\ -3x \quad -3x \quad -15 \\ \hline -x > -13 \end{array}$$

$$-x > -13$$

$$\boxed{x < 13}$$

inequality statement

$$\{x \mid x < 13, x \in \mathbb{R}\} \quad \text{set notation}$$

$$(-\infty, 13)$$

interval notation

$$x < 13$$

$$(-\infty, 13]$$

Linear Inequalities in 2 Variables:

$$Ax + By < C$$

$$Ax + By \leq C$$

$$Ax + By > C$$

$$Ax + By \geq C$$

Describes a region in the Cartesian (coordinate) plane

The ordered pair (x, y) is a solution of the linear inequality if $x + y$ satisfies the inequality statement

Solution Region:

- All the points in the Cartesian plane that satisfy an inequality
- the solution set

Boundary:

- line or curve that separates the plane into 2 regions (solution region & non-solution region)

$\geq$ or $\leq$ solid boundary - it is part of the solution

$>$ or $<$ dashed boundary - it is not part of the solution

ex. Graph $2x + 3y \geq 6$

① determine & graph the boundary

$$3y \geq -2x + 6$$

$$y \geq -\frac{2}{3}x + 2$$

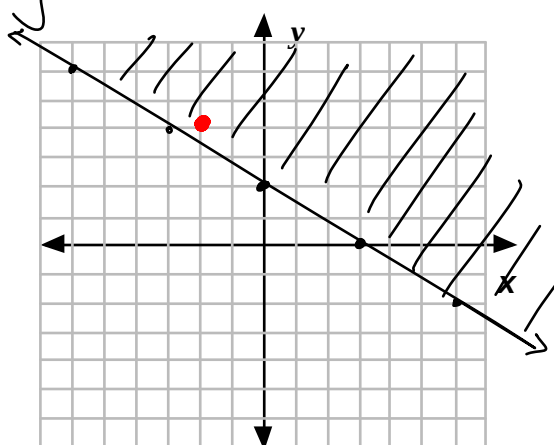
② use test point to determine the solution region

$$(0, 0) \quad 2(0) + 3(0) \geq 6$$

$$0 \not\geq 6 \quad \therefore (0, 0) \text{ is not part of the solution}$$

Is $(-2, 4)$ part of the solution?

Yes, it is in the shaded solution region



check

$$2(-2) + 3(4) \geq 6$$

$$-4 + 12 \geq 6$$

$$8 \geq 6 \quad \checkmark$$

ex. Graph $10x - 5y > 0$

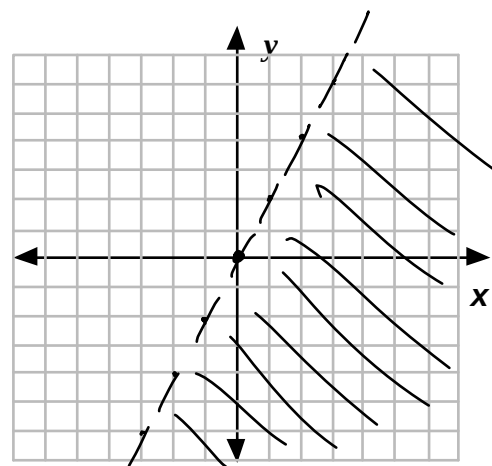
$$-5y > -10x$$

$$y < 2x$$

Test $(-1, 1)$

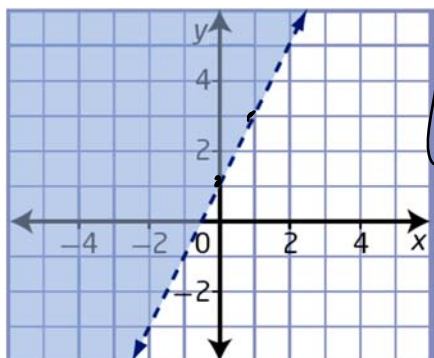
$$10(-1) - 5(1) > 0$$

$$-15 > 0 \quad \times$$

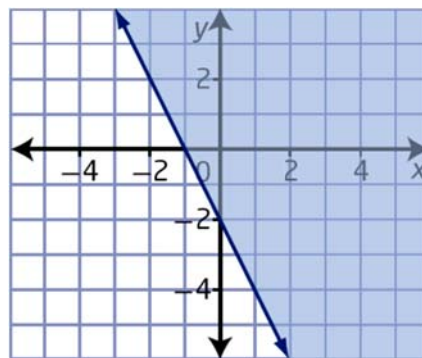


Shortcut? once isolated y
 $>$ or $>$, region above boundary
 $<$ or $\leq$, region below boundary

ex. Write an inequality for each graph below:



$$y > 2x + 1$$



$$y \geq -2x - 2$$

ex. Suppose you are constructing a tabletop using an aluminum frame and glass top. The most that you can spend on materials is \$50. Laminated safety glass costs \$60/m², and aluminum costs \$1.75/ft. Find all possible combinations of materials sufficient to make the tabletop.

Let $x = \text{area of glass}$

Let $y = \text{length of aluminum}$

$$60x + 1.75y \leq 50$$

$$1.75y \leq -60x + 50$$

$$y \leq \frac{-60x + 50}{1.75}$$

