PreCalculus 11 Date _____

Unit 8: Systems of Equations

8.2 Solving Systems of Equations Algebraically

Recall: Systems of Linear Equations – two algebraic solution methods

1) Substitution

$4x + 5y = 26$
$3x - y = -9$

$y = 3x + 9$

$4x + 5(3x + 9) = 26$
$4x + 15x + 45 = 26$
$19x = -19$
$x = -1$

$y = 3(-1) + 9$
$y = 6$

$(-1, 6)$

2) Elimination

$6a + 5b = 24$ $\times 2$ $\Rightarrow$ $12a + 10b = 48$
$4a + 3b = 12$ $\times 3$ $\Rightarrow$ $12a + 9b = 36$

$b = 12$

$4a + 3(12) = 12$
$4a = -24$
$a = -6$

$(-6, 12)$

Same methods can be applied to Linear-Quadratic or Quadratic-Quadratic Systems

ex. Solve:

$$5x - y = 10$$
$$x^2 + x - 2y = 0$$

Method 1 – Substitution

$5x - y = 10 \Rightarrow y = 5x - 10$

$x^2 + x - 2(5x - 10) = 0$
$x^2 + x - 10x + 20 = 0$
$x^2 - 9x + 20 = 0$
$(x - 4)(x - 5) = 0$

$x = 4$ OR $x = 5$

$y = 5(4) - 10$ $y = 5(5) - 10$
$= 10$ $= 15$

$(4, 10)$ OR $(5, 15)$

Method 2 – Elimination

$5x - y = 10 \Rightarrow$ $\times 2$ $10x - 2y = 20$
$x^2 + x - 2y = 0$

$-x^2 + 9x = 20$

$0 = x^2 - 9x + 20$

ex. Solve:

$$3x^2 - x - y - 2 = 0$$
$$6x^2 + 4x - y = 4$$

Method 1 – Substitution

$3x^2 - x - y - 2 = 0 \Rightarrow y = 3x^2 - x - 2$

$6x^2 + 4x - (3x^2 - x - 2) = 4$

$6x^2 + 4x - 3x^2 + x + 2 = 4$

$3x^2 + 5x - 2 = 0$

$3x^2 + 6x - x - 2 = 0$

$3x(x+2) - 1(x+2) = 0$

$(x+2)(3x-1) = 0$

$x = -2$ or $1/3$

$y = 3(-2)^2 - (-2) - 2$

$y = 12$

$(-2, 12)$

$y = 3(1/3)^2 - (1/3) - 2$

$= 1/3 - 1/3 - 2$

$= -2$

$(1/3, -2)$

Method 2 – Elimination

$-3x^2 + x + y + 2 = 0$

$6x^2 + 4x - y - 4 = 0$ +

$3x^2 + 5x - 2 = 0$

ex. Determine two integers such that the sum of the smaller number and twice the larger number is 46. Also, when the square of the smaller number is decreased by three times the larger, the result is 93.

Let x = smaller #

y = larger #

$x + 2y = 46 \Rightarrow 3x + 6y = 138$ (×3)

$x^2 - 3y = 93 \Rightarrow 2x^2 - 6y = 186$ (×2) +

$2x^2 + 3x = 324$

$2x^2 + 3x - 324 = 0$

$x = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(-324)}}{2(2)}$

$= \dfrac{-3 \pm \sqrt{2601}}{4}$

$= \dfrac{-3 \pm 51}{4}$

$x = 12$ or $-27/2$ not integer

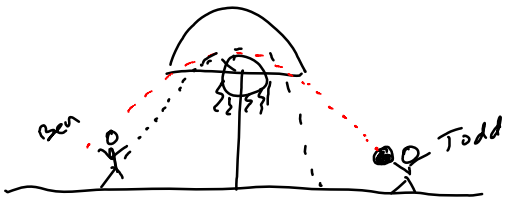
$12 + 2y = 46$

$2y = 34$

$y = 17$

The numbers are 12 & 17

ex. During a basketball game, Ben completes an impressive "alley-oop". Ben jumps up, catches the ball and tips it in the basket. The path of the ball thrown by his teammate can be modeled by the equation $d^2 - 2d + 3h = 9$ where *d* is the horizontal distance of the ball from the center of the hoop, in meters, and *h* is the height of the ball above the floor, in meters. The path of Ben's jump can be modeled by the equation $5d^2 - 10d + h = 0$, where *d* is his horizontal distance from the center of the hoop, in meters, and *h* is the height of his hands above the floor, in meters. Solve this system to the nearest hundredth and interpret the results.



$d^2 - 2d + 3h = 9 \xrightarrow{\times 5} 5d^2 - 10d + 15h = 45$

$5d^2 - 10d + h = 0 \rightarrow 5d^2 - 10d + h = 0$

$14h = 45$

$h = 45/14 \approx 3.21$

$d^2 - 2d + 3(45/14) = 9$

$14(d^2 - 2d + 135/14) = (9)14$

$14d^2 - 28d + 135 = 126$

$14d^2 - 28d + 9 = 0$

$d = \dfrac{-(-28) \pm \sqrt{(-28)^2 - 4(14)(9)}}{2(14)}$

$= \dfrac{28 \pm \sqrt{280}}{28}$

$d = 1.60$ or $d = 0.40$

$d = 1.60$
$h = 3.21$
ball travelling up
* too far from hoop

OR

$d = 0.40$
$h = 3.21$
Ben makes contact with the ball 0.40 m from hoop at a height of 3.21 m.

Practice: pg. 451/ #1, 3, 4, 8 – 10, 13, 19