

Unit 7: Rational Expressions & Equations

7.4 Rational Equations

Recall:

Solve $\frac{2}{3}x - 6 = \frac{3}{8}$

$$4x - 6 = 15$$

$$4x = 21$$

$$x = \frac{21}{4}$$

ex. Solve (determine the roots/zeros).

a) $\frac{2}{x^2-4} + \frac{10}{6x+12} = \frac{1}{x-2}$

$$\frac{2}{(x-2)(x+2)} + \frac{5}{3(x+2)} = \frac{1}{x-2}$$

$$\left(\frac{2}{(x-2)(x+2)}\right) \cdot \frac{3(x-2)(x+2)}{3(x-2)(x+2)} + \left(\frac{5}{3(x+2)}\right) \cdot \frac{3(x-2)(x+2)}{3(x-2)(x+2)} = \left(\frac{1}{x-2}\right) \cdot \frac{3(x-2)(x+2)}{3(x-2)(x+2)}$$

$$6 + 5x - 10 = 3x + 6$$

$$2x = 10$$

$$x = 5$$

Use same technique for rational equations.

- factor denominators
- identify non-permissible values
- multiply through by the LCD & eliminate den.
- solve & check against the restrictions

b) $\frac{4k-1}{k+2} - \frac{k+1}{k-2} = \frac{k^2-4k+24}{(k-2)(k+2)}$

LCD = $3(x-2)(x+2)$

$x \neq \pm 2$

$$\frac{(4k-1)(k-2)}{(k+2)(k-2)} - \frac{(k+1)(k+2)}{(k-2)(k+2)} = \frac{(k^2-4k+24)}{(k-2)(k+2)}$$

$$(4k-1)(k-2) - (k+1)(k+2) = k^2 - 4k + 24$$

$$4k^2 - 9k + 2 - (k^2 + 3k + 2) = k^2 - 4k + 24$$

$$4k^2 - 9k + 2 - k^2 - 3k - 2 = k^2 - 4k + 24$$

$$3k^2 - 12k = k^2 - 4k + 24$$

$$2k^2 - 8k - 24 = 0$$

$$k^2 - 4k - 12 = 0$$

$$(k-6)(k+2) = 0$$

$$k = 6 \text{ or } -2$$

extraneous root

ex. Two friends share a paper route. Sheena can deliver the papers in 40 minutes. Jeff can cover the same route in 50 minutes. How long (nearest minute) does the route take if they work together?

	Time to deliver papers (min)	Fraction of work done in 1 min	Fraction of work done in t minutes
Sheena	40	$\frac{1}{40}$	$\frac{1}{40} \cdot t = \frac{t}{40}$
Jeff	50	$\frac{1}{50}$	$\frac{1}{50} \cdot t = \frac{t}{50}$
Together	t	$\frac{1}{t}$	$\frac{1}{t} \cdot t = 1$

$$\frac{t}{40} + \frac{t}{50} = 1$$

$$5t + 4t = 200$$

$$9t = 200$$

$$t = 200/9 = 22.2$$

22 minutes

ex. A sled dog race runs non-stop from The Pas to Flin Flon and back, a distance of 70 km each way. Bad weather caused the winner's average speed to decrease by 6 km/h on the return trip. If the total time for the trip was 8.5 hours. What was the winning team's average speed on the way to Flin Flon?

	Distance (km)	Rate (km/h)	Time (h) $t = \frac{d}{s}$
The Pas to Flin Flon	70	x	$\frac{70}{x}$
Flin Flon to The Pas	70	$x - 6$	$\frac{70}{x - 6}$

$$\frac{70}{x} + \frac{70}{x - 6} = 8.5$$

$$70x - 420 + 70x = 8.5x^2 - 51x$$

$$0 = 8.5x^2 - 121x + 420$$

$$x = \frac{-(-121) \pm \sqrt{(-121)^2 - 4(8.5)(420)}}{2(8.5)}$$

$$= \frac{121 \pm \sqrt{22201}}{17}$$

$$x = 20$$

~~$x = 24.7$~~

20 km/h