

Unit 7: Rational Expressions & Equations

7.3 Adding & Subtracting Rational Expressions

Recall: When adding or subtracting fractions, we need common denominator

ex. $\frac{5}{9} + \frac{2 \cdot 3}{3 \cdot 3}$ $9 = 3 \cdot 3$ $3 = 3$ $\frac{5}{6} - \frac{2}{15}$ $6 = 2 \cdot 3$ $15 = 3 \cdot 5$ $LC = 3 \cdot 2 \cdot 5$

$$= \frac{5}{9} + \frac{6}{9} = \boxed{\frac{11}{9}}$$

$$= \frac{25}{30} - \frac{4}{30} = \frac{21}{30} = \boxed{\frac{7}{10}}$$

Adding & Subtracting Rational Expressions with Common Denominators:

ex. Find each sum or difference. State the non-permissible values.

a) $\frac{2a}{b} - \frac{a-1}{b}$

$$= \frac{2a - (a-1)}{b}$$

$$= \frac{2a - a + 1}{b} = \boxed{\frac{a+1}{b}}$$

$b \neq 0$

b) $\frac{2x}{x+4} + \frac{8}{x+4}$

$$= \frac{2x+8}{x+4}$$

$$= \frac{2(x+4)}{x+4}$$

$$= \boxed{2}$$

$x \neq -4$

c) $\frac{x^2}{x-2} + \frac{4x}{x-2} - \frac{8}{x-2}$

$$= \frac{x^2 + 4x - 8}{x-2}$$

$x \neq 2$

Adding & Subtracting Rational Expressions with Different Denominators:

ex. Rewrite each pair of rational expressions with common denominators:

a) $\frac{4}{5x}, \frac{y}{x^2}$ $LC = 5 \cdot x^2$

$$\frac{4x}{5x^2}, \frac{5y}{x^2}$$

$LC = 5 \cdot x \cdot x^2 = 5x^3$

b) $\frac{a}{b}, \frac{2a}{b+1}$ $LC = b(b+1)$

$$\frac{a(b+1)}{b(b+1)}, \frac{2a \cdot b}{b(b+1)}$$

$LC = (x-3)(x+3)$

c) $\frac{3x}{x^2-9}, \frac{x+2}{x+3}$

$$\frac{3x}{(x-3)(x+3)}, \frac{(x+2) \cdot (x-3)}{(x+3) \cdot (x-3)}$$

$LC = (x-3)(x+3)$

$$\frac{3x}{(x-3)(x+3)}, \frac{x^2 - x - 6}{(x-3)(x+3)}$$

ex. Simplify. State the non-permissible values.

a) $\left(\frac{2x}{xy}\right) + \frac{4}{x^2} - \frac{3}{1}$

$LC = x^2y$

$$= \frac{2}{y} + \frac{4}{x^2} - \frac{3}{1}$$

$$= \frac{2x^2}{x^2y} + \frac{4y}{x^2y} - \frac{3x^2y}{x^2y}$$

$$= \boxed{\frac{2x^2 + 4y - 3x^2y}{x^2y}}$$

$x \neq 0$
 $y \neq 0$

b) $\frac{x^2-20}{x^2-4} + \frac{x-2}{x+2} \cdot \frac{(x-1)}{(x-1)}$

$LC = (x-2)(x+2)$

$$= \frac{x^2-20}{(x-2)(x+2)} + \frac{x^2-4x+4}{(x-2)(x+2)}$$

$$= \frac{2x^2 - 4x - 16}{(x-2)(x+2)}$$

$$= \frac{2(x^2 - 2x - 8)}{(x-2)(x+2)}$$

$$= \frac{2(x-4)(x+2)}{(x-2)(x+2)} = \boxed{\frac{2(x-4)}{x-2}}$$

$x \neq \pm 2$

$$\begin{aligned}
 \text{c) } & \frac{x-1}{x^2-5x-6} - \frac{x-2}{x^2+4x+3} \\
 &= \frac{x-1}{(x+1)(x-6)} - \frac{x-2}{(x+1)(x+3)} \quad \text{LCD} = (x+1)(x-6)(x+3) \\
 &= \frac{(x-1)(x+3)}{(x+1)(x-6)(x+3)} - \frac{(x-2)(x-6)}{(x+1)(x+3)(x-6)} \\
 &= \frac{x^2+2x-3}{(x+1)(x-6)(x+3)} - \frac{(x^2-8x+12)}{(x+1)(x+3)(x-6)} \\
 &= \frac{x^2+2x-3}{(x+1)(x-6)(x+3)} - \frac{x^2+8x+12}{(x+1)(x-6)(x+3)} \\
 &= \frac{10x-15}{(x+1)(x-6)(x+3)} \\
 &= \frac{5(2x-3)}{(x+1)(x-6)(x+3)} \quad \boxed{x \neq -1, 6, -3}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & \frac{\frac{1}{1} + \frac{1}{x}}{\frac{x}{1} - \frac{1}{x}} = \frac{\frac{x}{x} + \frac{1}{x}}{\frac{x^2}{x} - \frac{1}{x}} \\
 &= \frac{\frac{x+1}{x}}{\frac{x^2-1}{x}} \\
 &= \frac{x+1}{x} \times \frac{x}{(x+1)(x-1)} \\
 &= \frac{1}{x-1} \quad \boxed{x \neq 0, \pm 1}
 \end{aligned}$$

Practice: pg. 336/ #3, 5abef, 6bcfd, 7ab, 8, 10ac, 12, 13, 15, 16, 18