

Unit 7: Rational Expressions & Equations

7.1 Rational Expressions

Rational Expression: An algebraic fraction with a numerator and a denominator that are polynomials

ex. $\frac{1}{x}$, $\frac{m}{m+1}$, $\frac{y^2-1}{y^2+2y+1}$, $\frac{x^2-4}{x^2-4}$

Non-Permissible Value:

- any value for a variable that makes an expression undefined
- for rationals, a value that results in a denominator of 0

ex. State the non-permissible values for each rational expression.

a) $\frac{5z}{x^2y}$

$x \neq 0, y \neq 0$

b) $\frac{3x+2}{5}$

no restrictions
OR
 $x \in \mathbb{R}$

c) $\frac{4x^2}{3(2x+1)}$

$3(2x+1) \neq 0$
 $x \neq -\frac{1}{2}$

d) $\frac{a-2}{a^2-7a+10}$

$a^2-7a+10 \neq 0$
 $(a-2)(a-5) \neq 0$
 $a \neq 2, 5$

e) $\frac{7z}{z^2+1}$

$z^2+1 \neq 0$
 $z^2+1 \geq 0 \checkmark$
no restrictions

Like fractions we can simplify rational expressions by cancelling common factors from all terms in the numerator and denominator. To do this we need to factor.

ex. Simplify. State any non-permissible values.

a) $\frac{9}{15}$

$\frac{3 \cdot 3}{3 \cdot 5} = \frac{3}{5}$

b) $\frac{a^3b}{a^4b^2}$

$\frac{a^3b}{a^4b^2} = \frac{a \cdot a \cdot a \cdot b}{a \cdot a \cdot a \cdot a \cdot b \cdot b} = \frac{1}{ab}$
OR
 $\frac{a^{3-4}b^{1-2}}{a^1b^{-1}} = \frac{1}{ab}$
 $a \neq 0, b \neq 0$

c) $\frac{12x^2y}{-24x^2z^3}$

$\frac{y}{-2z^3}$
 $x \neq 0, z \neq 0$

d) $\frac{4}{2x-3}$

$2x-3 \neq 0$
 $x \neq \frac{3}{2}$

e) $\frac{3x-6}{2x^2+x-10}$

$\frac{3(x-2)}{(x-2)(2x+5)}$

$\frac{3}{2x+5}$

$2x^2+x-10 \rightarrow \begin{matrix} 2x^2+x-10 & \ominus -10 \\ 2x^2-4x+5x-10 & \oplus 1 \end{matrix}$
 $\leftarrow 2x(x-2)+5(x-2)$

$x \neq 2, -\frac{5}{2}$

f) $\frac{1-x}{x^2-1}$

$\frac{-x+1}{(x-1)(x+1)}$

$\frac{-1}{(x-1)(x+1)}$

$\frac{-1}{x+1}$

$x \neq \pm 1$

original expression determines non-permissible

$$g) \frac{2x^2-3x-2}{x^2+5x-14} \rightarrow \frac{2x^2-4x+x-2}{2x(x-2)+1(x-2)}$$

$$\frac{(x-2)(2x+1)}{(x+7)(x-2)}$$

$$\boxed{\frac{2x+1}{x+7}}$$

$$\boxed{x \neq 2, -7}$$

$$h) \frac{2m-6}{9-m^2}$$

$$\frac{-2(\cancel{x-3})}{(\cancel{3-m})(3+m)}$$

$$\boxed{\frac{-2}{3+m}}$$

$$\boxed{m \neq \pm 3}$$

ex. Given the expression $\frac{16x^2-9y^2}{8x-6y}$

a) What expression represents the non-permissible values for x ?

$$8x-6y \neq 0$$

$$8x \neq 6y$$

$$x \neq \frac{6y}{8}$$

$$\boxed{x \neq \frac{3y}{4}}$$

b) Simplify.

$$\frac{(4x+3y)(\cancel{4x-3y})}{2(\cancel{4x-3y})}$$

$$\boxed{\frac{4x+3y}{2}}$$

c) Evaluate for $x = 2.6$ and $y = 1.2$

$$\frac{(4(2.6) + 3(1.2))}{2}$$

$$= \boxed{7}$$